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New Weighted Gruss Type Inequalities Via (α, β) Fractional q-Integral Inequalities

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ABSTRACT: In this paper, we use the fractional q-Integrals on a specific time scales to generate some new inequalities of Gruss type. For this paper, some classical results can be deduced as some special case.

KEYWORDS: Fractional q-calculus, Integral inequalities, Gruss inequality.

1 INTRODUCTION

In 1935, G. Gruss [1] proved the following classical integral inequality:

$$\frac{1}{b-a}\int_a^b f(x)g(x)dx-\left(\frac{1}{b-a}\int_a^b f(x)dx\right)\left(\frac{1}{b-a}\int_a^b g(x)dx\right)\le\frac{(\Phi-\phi)(\Psi-\psi)}{4} \qquad (1)$$

provided that f and g are two integrable functions on $[a,b]$ and satisfying the conditions

$$\phi\le f(x)\le\Phi,\ \psi\le g(x)\le\Psi;\quad \Phi,\phi,\Psi,\psi\in\mathbb{R}, x\in[a,b] \quad (2)$$

In [2], Dragomir proved that:

$$|T(f,g,p)|\le\frac{(\Phi-\phi)(\Psi-\psi)}{4}\left(\int_a^b p(x)\right)^2 \qquad (3)$$

where:

$$T(f,g,p):=\int_a^b p(x)dx\int_a^b p(x)f(x)g(x)dx-\left(\int_a^b p(x)f(x)dx\right)\left(\int_a^b p(x)g(x)dx\right) \quad (4)$$

and p is a positive function on $[a,b]$, and f and g are two integrable functions on $[a,b]$ satisfying (2).

In the case of fractional integrals [3], G. Anastassiou established another fractional integral inequality of Gruss type. Other papers dealing with various generalizations related to the Riemann-Liouville fractional integrals and to the q-fractional integrals have appeared in the literature. For more details, we refer the reader to ([4], [5], [6], [7]).

In this paper, we use the fractional q-integrals on time scales to establish new inequalities related to (1) and (3). Our results have some relationships with those obtained in ([5], [6]) and mentioned above. For these results, Theorem 3.1 of [4] can be deduced as a particular case.

2 Notations and Preliminaries

We give a summary of the mathematical notations and definitions used in this paper. For more details, one can consult [8]. Let $t_0 \in R.$ We define:

$$T_{t_0} := \{t \, : \, t = t_0 q^n, n \in N\} \cup \{0\}, 0 < q < 1 \qquad (5)$$

For a function $f \, : \, T_{t_0} \to R,$ the ∇ q-derivative of f is:

$$\nabla_q f(t) = \frac{f(qt) - f(t)}{(q-1)t} \qquad (6)$$

For all $t \in T/\{0\}$ and its ∇q -integral is defined by:

$$\int_0^t f(\tau) \nabla \tau = (1-q)t \sum_{i=0}^{\infty} q^i f(tq^i) \qquad (7)$$

The fundamental theorem of calculus applies to the q-derivative and q-integral. In particular, we have:

$$\nabla_q \int_0^t f(\tau) \nabla \tau = f(t) \qquad (8)$$

If f is continuous at 0, then:

$$\int_0^t \nabla_q f(\tau) \nabla \tau = f(t) - f(0) \qquad (9)$$

Let T_{t_1}, T_{t_2} denote two time scales. Let $f \, : \, T_{t_1} \to R$ be continuous let $g \, : \, T_{t_1} \to T_{t_2}$ be q-differentiable, strictly increasing, and $g(0) = 0$. Then for $b \in T_{t_1},$ we have:

$$\int_0^b f(t) \nabla_q g(t) \nabla t = \int_0^{g(b)} (f \circ g^{-1})(s) \nabla s \qquad (10)$$

The q-factorial function is defined as follows:

If n is a positive integer, then:

$$(t-s)^{\underline{(n)}} = (t-s)(t-qs)(t-q^2 s)...(t-q^{n-1}s) \qquad (11)$$

If n is not a positive integer, then:

$$(t-s)^{\underline{(n)}} = t^n \prod_{k=0}^{\infty} \frac{1-(\frac{s}{t})q^k}{1-(\frac{s}{t})q^{n+k}} \qquad (12)$$

The q-derivative of the q-factorial function with respect to t is:

$$\nabla_q (t-s)^{\underline{(n)}} = \frac{1-q^n}{1-q} (t-s)^{\underline{(n-1)}} \qquad (13)$$

And the q-derivative of the q-factorial function with respect to s is:

$$\nabla_q (t-s)^{\underline{(n)}} = -\frac{1-q^n}{1-q}(t-qs)^{\underline{(n-1)}} \qquad (14)$$

The q-exponential function is defined as:

$$e_q(t) = \prod_{k=0}^{\infty}(1-q^k t), e_q(0) = 1 \qquad (15)$$

The fractional q-integral operator of order $\alpha \geq 0$, for a function f is defined as:

$$\nabla_q^{-\alpha} f(t) = \tfrac{1}{\Gamma_q(\alpha)}\int_0^t (t-q\tau)^{\underline{\alpha-1}} f(\tau)\nabla\tau; \quad \alpha > 0, t > 0 \qquad (16)$$

Where: $\Gamma_q(\alpha) := \frac{1}{1-q}\int_0^1 (\frac{u}{1-q})^{\alpha-1} e_q(qu)\nabla u$

3 Main Results

Our first result is the following theorem. This result can be found in [4]. Here, we propose another method to prove it.

Theorem 3.1: *Let f and g be two integrable functions on $[0,\infty[$ satisfying the condition (2) on $[0,\infty[$ and let p be a positive function on $[0,\infty[$. Then for all $t > 0, \alpha > 0,$ we have:*

$$\left|\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha} pfg(t) - \nabla_q^{-\alpha} pf(t)\nabla_q^{-\alpha} pg(t)\right| \leq \left(\frac{\nabla_q^{-\alpha} p(t)}{2}\right)^2 (\Phi-\phi)(\Psi-\psi) \quad (17)$$

Proof: Let us consider the quantity:

$$H(\tau,\rho) := (f(\tau)-f(\rho))(g(\tau)-g(\rho)); \; \tau,\rho \in (0,t) \qquad (18)$$

It is easy to see that:

$$\int_0^t\int_0^t \tfrac{(t-q\tau)^{\underline{\alpha-1}}(t-q\rho)^{\underline{\alpha-1}}}{\Gamma_q^2(\alpha)} p(\tau)p(\rho)H(\tau,\rho)\nabla\tau\nabla\rho = 2\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha} pfg(t) - 2\nabla_q^{-\alpha} pf(t)\nabla_q^{-\alpha} pg(t) \qquad (19)$$

Thanks to the weighted Cauchy Schwartz integral inequality, we can write:

$$\left(\int_0^t\int_0^t \tfrac{(t-q\tau)^{\underline{\alpha-1}}(t-q\rho)^{\underline{\alpha-1}}}{\Gamma_q^2(\alpha)} p(\tau)p(\rho)H(\tau,\rho)\nabla\tau\nabla\rho\right)^2$$
$$\leq \int_0^t\int_0^t \tfrac{(t-q\tau)^{\underline{\alpha-1}}(t-q\rho)^{\underline{\alpha-1}}}{\Gamma_q^2(\alpha)} p(\tau)p(\rho)(f(\tau)-f(\rho))^2\nabla\tau\nabla\rho \quad (20)$$
$$\times\int_0^t\int_0^t \tfrac{(t-q\tau)^{\underline{\alpha-1}}(t-q\rho)^{\underline{\alpha-1}}}{\Gamma_q^2(\alpha)} p(\tau)p(\rho)(g(\tau)-g(\rho))^2\nabla\tau\nabla\rho$$

Using (16) we can develop the right hand side of (20) as follows:

$$\int_0^t\int_0^t \frac{(t-q\tau)^{\alpha-1}(t-q\rho)^{\alpha-1}}{\Gamma_q^2(\alpha)} p(\tau)p(\rho)(f(\tau)-f(\rho))^2\nabla\tau\nabla\rho = 2\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pf^2(t)-2(\nabla_q^{-\alpha}pf(t))^2 \qquad (21)$$

And:

$$\int_0^t\int_0^t \frac{(t-q\tau)^{\alpha-1}(t-q\rho)^{\alpha-1}}{\Gamma_q^2(\alpha)} p(\tau)p(\rho)(g(\tau)-g(\rho))^2\nabla\tau\nabla\rho = 2\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pg^2(t)-2(\nabla_q^{-\alpha}pg(t))^2 \qquad (22)$$

Thanks to (19) (21) and (22) we can write (20) as follows:

$$\begin{aligned}&\left(\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pfg(t)-\nabla_q^{-\alpha}pf(t)\nabla_q^{-\alpha}pg(t)\right)^2\\ &\le\left(\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pf^2(t)-(\nabla_q^{-\alpha}pf(t))^2\right)\left(\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pg^2(t)-(\nabla_q^{-\alpha}pg(t))^2\right)\end{aligned} \qquad (23)$$

On the other hand, we have:

$$\begin{aligned}&\left(\Phi p(\rho)-f(\rho)p(\rho)\right)\left(p(\tau)f(\tau)-\phi p(\tau)\right)+\left(\Phi p(\tau)-f(\tau)p(\tau)\right)\left(p(\rho)f(\rho)-\phi p(\rho)\right)\\ &-p(\tau)\left(\Phi-f(\tau)\right)\left(f(\tau)-\phi\right)p(\rho)-p(\rho)\left(\Phi-f(\rho)\right)\left(f(\rho)-\phi\right)p(\tau)\\ &=p(\rho)f^2(\tau)p(\tau)+p(\tau)f^2(\rho)p(\rho)-2p(\tau)f(\tau)f(\rho)p(\rho)\end{aligned} \qquad (24)$$

Which implies that:

$$\begin{aligned}&\left(\Phi p(\rho)-f(\rho)p(\rho)\right)\left(\nabla_q^{-\alpha}p(t)f(t)-\phi\nabla_q^{-\alpha}p(t)\right)+\left(\Phi\nabla_q^{-\alpha}p(t)-\nabla_q^{-\alpha}f(t)p(t)\right)\left(p(\rho)f(\rho)-\phi p(\rho)\right)\\ &-p(\rho)\nabla_q^{-\alpha}\left(p(t)(\Phi-f(t))(f(t)-\phi)\right)-p(\rho)\left(\Phi-f(\rho)\right)\left(f(\rho)-\phi\right)\nabla_q^{-\alpha}p(t)\\ &=p(\rho)\nabla_q^{-\alpha}f^2(t)p(t)+p(\rho)f^2(\rho)\nabla_q^{-\alpha}p(t)-2p(\rho)f(\rho)\nabla_q^{-\alpha}\left(f(t)p(t)\right)\end{aligned} \qquad (25)$$

Now, multiplying both sides of (25) by $\frac{(t-q\rho)^{\alpha-1}}{\Gamma_q(\alpha)}$; $\rho\in(0,t)$ and integrating the resulting identity with respect to ρ over $(0,t),$ we have:

$$\begin{aligned}&\left(\nabla_q^{-\alpha}p(t)f(t)-\phi\nabla_q^{-\alpha}p(t)\right)\int_0^t\frac{(t-q\rho)^{\alpha-1}}{\Gamma_q(\alpha)}\left(\Phi p(\rho)-f(\rho)p(\rho)\right)\nabla\rho\\ &+\left(\Phi\nabla_q^{-\alpha}p(t)-\nabla_q^{-\alpha}f(t)p(t)\right)\int_0^t\frac{(t-q\rho)^{\alpha-1}}{\Gamma_q(\alpha)}\left(p(\rho)f(\rho)-\phi p(\rho)\right)\nabla\rho\\ &-\nabla_q^{-\alpha}\left(p(t)(\Phi-f(t))(f(t)-\phi)\right)\int_0^t\frac{(t-q\rho)^{\alpha-1}}{\Gamma_q(\alpha)}p(\rho)\nabla\rho\\ &-\nabla_q^{-\alpha}p(t)\int_0^t\frac{(t-q\rho)^{\alpha-1}}{\Gamma_q(\alpha)}p(\rho)(\Phi-f(\rho))(f(\rho)-\phi)\nabla\rho\\ &=\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pf^2(t)+\nabla_q^{-\alpha}pf^2(t)\nabla_q^{-\alpha}p(t)-2\nabla_q^{-\alpha}pf(t)\nabla_q^{-\alpha}pf(t)\end{aligned} \qquad (26)$$

Which gives:

$$\begin{aligned}&\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}pf^2(t)-\left(\nabla_q^{-\alpha}pf(t)\right)^2\\ &=\left(\Phi\nabla_q^{-\alpha}p(t)-\nabla_q^{-\alpha}pf(t)\right)\left(\nabla_q^{-\alpha}pf(t)-\phi\nabla_q^{-\alpha}p(t)\right)\\ &-\nabla_q^{-\alpha}p(t)\nabla_q^{-\alpha}\left((\Phi-f(t))(f(t)-\phi)p(t)\right)\end{aligned} \qquad (27)$$

Applying (27) with $g = f$ we obtain:

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha} pg^2(t) - \left(\nabla_q^{-\alpha} pg(t)\right)^2 \\ &= \left(\Psi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pg(t)\right)\left(\nabla_q^{-\alpha} pg(t) - \psi\nabla_q^{-\alpha} p(t)\right) - \nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha}\left((\Psi - g(t))(g(t)-\psi)p(t)\right)\end{aligned} \quad (28)$$

Since:

$$-\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha}\left((\Phi - f(t))(f(t)-\varphi)p(t)\right) \leq 0$$

And:

$$-\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha}\left((\Psi - g(t))(g(t)-\psi)p(t)\right) \leq 0$$

Then we have respectively:

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha} pf^2(t) - \left(\nabla_q^{-\alpha} pf(t)\right)^2 \\ &\leq \left(\Phi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pf(t)\right)\left(\nabla_q^{-\alpha} pf(t) - \phi\nabla_q^{-\alpha} p(t)\right)\end{aligned} \quad (29)$$

And:

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha} pg^2(t) - \left(\nabla_q^{-\alpha} pg(t)\right)^2 \\ &\leq \left(\Psi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pg(t)\right)\left(\nabla_q^{-\alpha} pg(t) - \psi\nabla_q^{-\alpha} p(t)\right)\end{aligned} \quad (30)$$

Now using (29) and (30) we can estimate the inequality (23) as follows:

$$\begin{aligned}&\left(\nabla_q^{-\alpha} p(t)\nabla_q^{-\alpha} pfg(t) - \nabla_q^{-\alpha} pf(t)\nabla_q^{-\alpha} pg(t)\right)^2 \\ &\leq \left(\Phi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pf(t)\right)\left(\nabla_q^{-\alpha} pf(t) - \phi\nabla_q^{-\alpha} p(t)\right) \\ &\times\left(\Psi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pg(t)\right)\left(\nabla_q^{-\alpha} pg(t) - \psi\nabla_q^{-\alpha} p(t)\right)\end{aligned} \quad (31)$$

By the inequality $4rs \leq (r+s)^2, r, s \in \mathbb{R},$ we obtain:

$$4\left(\Phi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pf(t)\right)\left(\nabla_q^{-\alpha} pf(t) - \phi\nabla_q^{-\alpha} p(t)\right) \leq \left((\Phi - \phi)\nabla_q^{-\alpha} p(t)\right)^2 \quad (32)$$

And:

$$4\left(\Psi\nabla_q^{-\alpha} p(t) - \nabla_q^{-\alpha} pg(t)\right)\left(\nabla_q^{-\alpha} pg(t) - \psi\nabla_q^{-\alpha} p(t)\right) \leq \left((\Psi - \psi)\nabla_q^{-\alpha} p(t)\right)^2 \quad (33)$$

Thanks to (31) (32) and (33) we obtain (17).

Our second result is the following theorem in which we generalize Theorem 3.1 of [4].

THEOREM 3.2: *Let* f *and* g *be two integrable functions on* $[0,\infty[$ *satisfying the condition (2) on* $[0,\infty[$ *and let* p *be a positive function on* $[0,\infty[$. *Then for all* $t>0,\alpha>0,\beta>0,$ *we have:*

$$\begin{aligned}&\Big(\nabla_q^{-\alpha}p(t)\nabla_q^{-\beta}pfg(t)+\nabla_q^{-\beta}p(t)\nabla_q^{-\alpha}pfg(t)-\nabla_q^{-\alpha}pf(t)\nabla_q^{-\beta}pg(t)-\nabla_q^{-\beta}pf(t)\nabla_q^{-\alpha}pg(t)\Big)^2\\&\leq\Big[\Big(\Phi\nabla_q^{-\alpha}p(t)-\nabla_q^{-\alpha}pf(t)\Big)\Big(\nabla_q^{-\beta}pf(t)-\varphi\nabla_q^{-\beta}p(t)\Big)\\&+\Big(\nabla_q^{-\alpha}pf(t)-\varphi\nabla_q^{-\alpha}p(t)\Big)\Big(\Phi\nabla_q^{-\beta}p(t)-\nabla_q^{-\beta}pf(t)\Big)\Big]\\&\times\Big[\Big(\Psi\nabla_q^{-\alpha}p(t)-\nabla_q^{-\alpha}pf(t)\Big)\Big(\nabla_q^{-\beta}pf(t)-\psi\nabla_q^{-\beta}p(t)\Big)\\&+\Big(\nabla_q^{-\alpha}pf(t)-\psi\nabla_q^{-\alpha}p(t)\Big)\Big(\Psi\nabla_q^{-\beta}p(t)-\nabla_q^{-\beta}pf(t)\Big)\Big]\end{aligned}\qquad(34)$$

Proof: Multiplying (18) by $\frac{(t-q\tau)^{\alpha-1}(t-q\rho)^{\beta-1}}{\Gamma_q(\alpha)\Gamma_q(\beta)}\ p(\tau)p(\rho)\ ;\ \tau,\rho\in(0,t),$ integrating the resulting identity with respect to τ and ρ over $(0,t)^2$, then applying the Cauchy-Schwarz inequality for double integrals, we obtain:

$$\begin{aligned}&\Big(\nabla_q^{-\alpha}p(t)\nabla_q^{-\beta}pfg(t)+\nabla_q^{-\beta}p(t)\nabla_q^{-\alpha}pfg(t)-\nabla_q^{-\alpha}pf(t)\nabla_q^{-\beta}pg(t)-\nabla_q^{-\beta}pf(t)\nabla_q^{-\alpha}pg(t)\Big)^2\\&\leq\Big(\nabla_q^{-\alpha}p(t)\nabla_q^{-\beta}pf^2(t)+\nabla_q^{-\beta}p(t)\nabla_q^{-\alpha}pf^2(t)-2\nabla_q^{-\alpha}pf(t)\nabla_q^{-\beta}pf(t)\Big)\\&\times\Big(\nabla_q^{-\alpha}p(t)\nabla_q^{-\beta}pg^2(t)+\nabla_q^{-\beta}p(t)\nabla_q^{-\alpha}pg^2(t)-2\nabla_q^{-\alpha}pg(t)\nabla_q^{-\beta}pg(t)\Big)\end{aligned}\qquad(35)$$

Multiplying both sides of (25) by $\frac{(t-q\rho)^{\beta-1}}{\Gamma_q(\beta)}\ ;\rho\in(0,t)$ and integrating the resulting identity with respect to ρ from 0 to $t,$ we have:

$$\begin{aligned}&\Big(\nabla_q^{-\alpha}pf(t)-\phi\nabla_q^{-\alpha}p(t)\Big)\int_0^t\tfrac{(t-q\rho)^{\beta-1}}{\Gamma_q(\beta)}p(\rho)\Big(\Phi-f(\rho)\Big)\nabla\rho\\&+\Big(\Phi\nabla_q^{-\alpha}p(t)-\nabla_q^{-\alpha}pf(t)\Big)\int_0^t\tfrac{(t-q\rho)^{\beta-1}}{\Gamma_q(\beta)}p(\rho)\Big(f(\rho)-\phi\Big)\nabla\rho\\&-\nabla_q^{-\alpha}\Big((\Phi-f(t))(f(t)-\phi)p(t)\Big)\int_0^t\tfrac{(t-q\rho)^{\beta-1}}{\Gamma_q(\beta)}p(\rho)\nabla\rho\\&-\nabla_q^{-\alpha}p(t)\int_0^t\tfrac{(t-q\rho)^{\beta-1}}{\Gamma_q(\beta)}p(\rho)\Big(\Phi-f(\rho)\Big)\Big(f(\rho)-\phi\Big)\nabla\rho\\&=\nabla_q^{-\beta}p(t)\nabla_q^{-\alpha}pf^2(t)+\nabla_q^{-\alpha}pf^2(t)\nabla_q^{-\beta}p(t)-2\nabla_q^{-\alpha}pf(t)\nabla_q^{-\beta}pf(t)\end{aligned}\qquad(36)$$

Therefore,

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta} pf^2(t)+\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha} pf^2(t)-2\nabla_q^{-\alpha} pf(t)\nabla_q^{-\beta} pf(t)\\&=\left(\Phi\nabla_q^{-\beta} p(t)-\nabla_q^{-\beta} pf(t)\right)\left(\nabla_q^{-\alpha} pf(t)-\phi\nabla_q^{-\alpha} p(t)\right)\\&+\left(\Phi\nabla_q^{-\alpha} p(t)-\nabla_q^{-\alpha} pf(t)\right)\left(\nabla_q^{-\beta} pf(t)-\phi\nabla_q^{-\beta} p(t)\right)\\&-\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha}\left((\Phi-f(t))(f(t)-\phi)p(t)\right)-\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta}\left((\Phi-f(t))(f(t)-\phi)p(t)\right)\end{aligned} \qquad (37)$$

Applying (37) with $g=f$ we obtain:

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta} pg^2(t)+\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha} pg^2(t)-2\nabla_q^{-\alpha} pg(t)\nabla_q^{-\beta} pg(t)\\&=\left(\Psi\nabla_q^{-\beta} p(t)-\nabla_q^{-\beta} pg(t)\right)\left(\nabla_q^{-\alpha} pg(t)-\psi\nabla_q^{-\alpha} p(t)\right)\\&+\left(\Psi\nabla_q^{-\alpha} p(t)-\nabla_q^{-\alpha} pg(t)\right)\left(\nabla_q^{-\beta} pg(t)-\psi\nabla_q^{-\beta} p(t)\right)\\&-\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha}\left((\Psi-g(t))(g(t)-\psi)p(t)\right)-\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta}\left((\Psi-g(t))(g(t)-\psi)p(t)\right)\end{aligned} \qquad (38)$$

Since $(\Phi-f(x))(f(x)-\phi)\geq 0$ and $(\Psi-g(x))(g(x)-\psi)\geq 0,$ then can write:

$$-\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta}\left((\Phi-f(t))(f(t)-\varphi)p(t)\right)-\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha}\left((\Phi-f(t))(f(t)-\varphi)p(t)\right)\leq 0 \qquad (39)$$

And:

$$-\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta}\left((\Psi-g(t))(g(t)-\psi)p(t)\right)-\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha}\left((\Psi-g(t))(g(t)-\psi)p(t)\right)\leq 0 \qquad (40)$$

Consequently,

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta} pf^2(t)+\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha} pf^2(t)-2\nabla_q^{-\alpha} pf(t)\nabla_q^{-\beta} pf(t)\\&\leq\left(\Phi\nabla_q^{-\beta} p(t)-\nabla_q^{-\beta} pf(t)\right)\left(\nabla_q^{-\alpha} pf(t)-\phi\nabla_q^{-\alpha} p(t)\right)\\&+\left(\Phi\nabla_q^{-\alpha} p(t)-\nabla_q^{-\alpha} pf(t)\right)\left(\nabla_q^{-\beta} pf(t)-\phi\nabla_q^{-\beta} p(t)\right)\end{aligned} \qquad (41)$$

And:

$$\begin{aligned}&\nabla_q^{-\alpha} p(t)\nabla_q^{-\beta} pg^2(t)+\nabla_q^{-\beta} p(t)\nabla_q^{-\alpha} pg^2(t)-2\nabla_q^{-\alpha} pg(t)\nabla_q^{-\beta} pg(t)\\&\leq\left(\Psi\nabla_q^{-\beta} p(t)-\nabla_q^{-\beta} pg(t)\right)\left(\nabla_q^{-\alpha} pg(t)-\psi\nabla_q^{-\alpha} p(t)\right)\\&+\left(\Psi\nabla_q^{-\alpha} p(t)-\nabla_q^{-\alpha} pg(t)\right)\left(\nabla_q^{-\beta} pg(t)-\psi\nabla_q^{-\beta} p(t)\right)\end{aligned} \qquad (42)$$

Thanks to (41) (42) and (35), we obtain (34).

REMARK: *1. Applying Theorem 3.2 for $\alpha=\beta$ we obtain Theorem 3.1*

2. Applying Theorem 3.2 for $\alpha=\beta$ we obtain Theorem 3.1 of [4] on $[0,t], t>0.$

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