International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 2 No. 2 Feb. 2013, pp. 131-137

http://www.issr-journals.org/ijias/

A Fuzzy Production-Distribution Inventory Model with Shortage

***G. M. Arun Prasath*[1] *and C. V. Seshaiah*[2]**

[1]Department of Mathematics,
CMS College of Engineering,
Namakkal, Tamilnadu, India

[2]Department of Mathematics,
Sri Ramakrishna Engineering College,
Coimbatore, Tamilnadu, India

ABSTRACT: A Production-Distribution inventory model with shortages and unit cost dependent demand has been formulated along with possible constraints. In most of the real world situations, the cost parameters are imprecise in nature. Hence, the unit cost is imposed here in fuzzy environment. Due to complexity, the proposed model has been solved by LINGO software. The model is also solved for without shortages as the special case. The model is illustrated with a numerical example.

KEYWORDS: fuzzy inventory model, warehouse location problem, triangular fuzzy number, membership function, secondary warehouse, LINGO software.

1 INTRODUCTION

Silver [6] designed the classical inventory problems by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature. But in practical situation, unit price and demand rate of an item may be related to each other. When the demand of an item is high, an item is produced in large numbers and fixed costs of production are spread over a large number of items. Hence the unit price of an item inversely relates to the demand of that item.

Nirmal Kumar Mandal et al [4] formulate the multi objective fuzzy inventory model with three constraints and solved the model by using geometric programming method. Debdulal Panda et al [2] formulate the multi-item stochastic and fuzzy-stochastic inventory models under imprecise goal and chance constraints with warehouse space as area and demand is dependent on unit cost.

Manas Kumar Maiti et al [3] showed that the formulation of fuzzy inventory model with two warehouses one in the heart of the market place and other slightly away from the market place. The warehouse placed slightly away from the market place is named as secondary warehouse. If the organization maintains two warehouses for one selling store then the maintenance cost of the warehouses will be more. But if the organization separately assigns one secondary warehouse to one or more than one selling stores then the total maintenance cost of the secondary warehouses will be reduced and consequently, the total expenditure of the organization will be decreased. To execute this idea the warehouse location problem has been taken to find the optimum number of warehouses and total cost of the secondary warehouses.

Panneerselvam [5] showed that the warehouse location problem for the organization can assign the warehouse for the nearby selling stores to find that the optimal distance. Consider the selling stores of an organization situated in different places of markets and the secondary warehouse wherefrom the required materials are to be shipped to the selling stores. Warehouse space available in the selling store can be considered in terms of area and/or volume, but most of the researchers consider only the area of the warehouse space.

Corresponding Author: G. M. Arun Prasath (arunprasathgm@gmail.com)

Arun Prasath G.M. et al [1] showed that to minimize the total expenditure of the organization by reducing the number of warehouses allocating to the selling stores with warehouse space in area and the materials transferred from warehouse to selling store in known quantity, If the warehouse space is taken in terms of area, then the maximum area of the warehouse space will be covered by the goods arranged in the ground level. But, if the warehouse space is taken in terms of volume, then the minimum volume will be covered by the same amount of goods arranged in rows and columns. Because of this reason, in this paper, the warehouse space is considered in terms of volume. Here, a combined model of fuzzy inventory model and warehouse location problem is formulated with possible constraints. The unit cost and lot size are decision variables.

2 MODEL FORMULATION

2.1 MODEL AND ASSUMPTIONS

2.1.1 ASSUMPTIONS AND NOTATIONS

The basic assumptions about the model are:

(i) Production rate is instantaneous
(ii) Shortage is allowed
(iii) Lead time is zero
(iv) The warehouse space is taken in terms of volume
(v) Lot size is considered as a required material for the selling store from the secondary warehouse
(vi) The unit cost is taken in fuzzy environment
(vii) Demand to be dependent on unit cost and it is related to the unit price

As: $D_i = \dfrac{C_i}{p_i^{e_i}}$

Where C_i (>0) and $e_i\left(0 < e_i < 1\right)$ are constants and real numbers selected to provide the best fit of the estimated price function. While $C_i > 0$ is an obvious condition since both D_i and p_i must be non-negative.

We use the following notations in proposed model:

(i) n = number of items

(ii) I = Total investment cost for replenishment

(iii) L - Inside length of the warehouse

(iv) B - Inside breadth of the warehouse

(v) M - Height of the shelf

(vi) V – Volume of the warehouse space

(vii) Z – Total Expenditure

Parameters for the i^{th} (i = 1, 2, ….,n) item are

(viii) $D_i = D_i(p_i)$ demand rate (function of cost price)

(ix) Q_i = lot size (a decision variable)

(x) S_i = set up cost per cycle

(xi) H_i = inventory holding cost per unit item

(xii) p_i = price per unit item (a fuzzy decision variable)

(xiii) M_i - Shortage level

(xiv) δ_i – Shortage cost per unit item *i*

(xv) l_i - Length of the unit item *i*

(xvi) b_i - Breadth of the unit item *i*

(xvii) h_i - Height of the unit item *i*

(xviii) v_i -Volume of the unit item *i*

(xix) V_W - Percentage of utilization of volume of the warehouse

(xx) *t*- Number of orders

(xxi) α - Total number of stores

(xxii) β - Total number of proposed warehouses

(xxiii) γ - Maximum number of warehouses to be located for serving α stores $(\gamma \le \beta)$.

(xxiv) S_{c_k} - Set up Cost of the warehouse

(xxv) A_{jk} - Distance between the store *j* and warehouse *k*

(xxvi) d – Maximum allowable distance between the warehouse *k* and selling store *j*.

(xxvii) t_c - Transportation cost of unit item from the warehouse to the selling store. The transportation cost is fixed when the distance between warehouse and selling store is less than *d*.

Volume of the unit item is defined by $v_i = l_i \times b_i \times h_i$

To calculate the volume of the warehouse space inside the selling store, multiply the lengths of the dimensions of the inside of the warehouse, that is, multiply the inside length, inside breadth and maximum shelf height.

i.e., Volume of the warehouse space inside the selling store is defined by $V = L \times B \times M$

2.1.2 Mathematical Model

The amount of stock for the i^{th} item *(i = 1,2,….,n)* be R_i at time *t = 0*. In the interval $(0, T_i (= t_{1i} + t_{2i})$, the inventory level gradually decreases to meet demands. By this process the inventory level reaches zero level at time t_{1i} and the shortages are allowed to occur in the interval (t_{1i}, T_i). The cycle then repeats itself (Fig. 1).

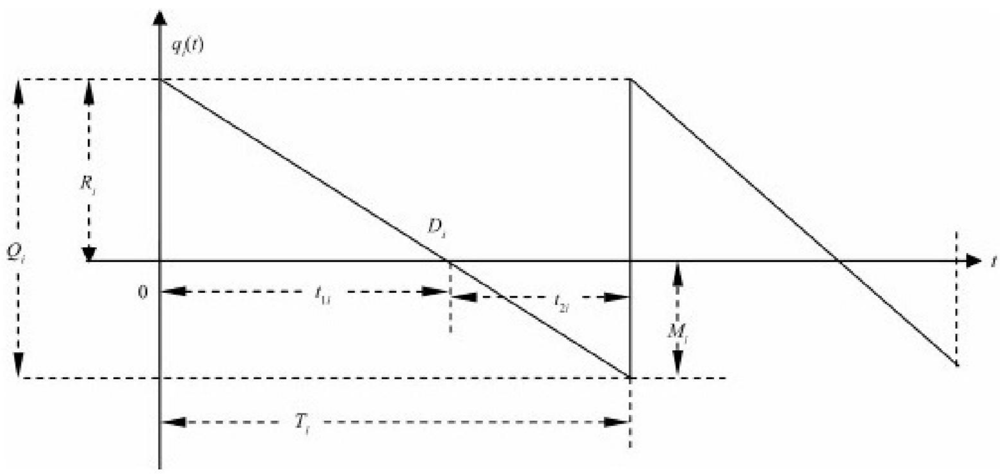

Fig. 1.

The differential equation for the instantaneous inventory $q_i(t)$ at time t in $(0, T_i)$ is given by

$$\frac{dq_i(t)}{dt} = \begin{cases} -D_i & \text{for} \quad 0 \le t \le t_{1i} \\ -D_i & \text{for} \quad t_{1i} \le t \le T_i \end{cases} \qquad \text{........(1)}$$

With the initial conditions $q_i(0) = R_i (= Q_i - M_i), q_i(T_i) = -M_i,\ q_i(t_{1i}) = 0$

For each period a fixed amount of shortage is allowed and there is a penalty cost c_{2i} per items of unsatisfied demand per unit time.

From (1),

$$q_i(t) = \begin{cases} R_i - D_i t & \text{for } 0 \le t \le t_{1i} \\ D_i(t_{1i} - t) & \text{for } t_{1i} \le t \le T_i \end{cases}$$

So, $D_i t_{1i} = R_i,\ M_i = D_i t_{2i},\ Q_i = D_i T_i$.

Holding cost = $H_i \int_0^{t_{1i}} q_i(t)dt$ = $\dfrac{H_i (Q_i - M_i)^2}{2Q_i} T_i$

Shortage cost of the store = $\delta_i \int_{t_{1i}}^{T_i} (-q_i(t))\ dt$ = $\dfrac{\delta_i M_i^{\ 2}}{2Q_i} T_i$

Production cost = $p_i Q_i$

2.1.3 Objective Function of the Mathematical Model in Fuzzy Environment

Total Expenditure (T.E.) = [(Number of selling stores $\times$ Total Annual cost of the store)+ Total Annual cost of the secondary warehouses]

T.E.= {[Number of selling stores $\times$(Production cost + Set up cost of the selling store + Holding cost+ Shortage cost)]+(Transportation cost + Set up cost of the secondary warehouse) }

$$Min\ \ Z = \left\{ \alpha \sum_{i=1}^{n} \left[C_i \tilde{p}_i^{\,1-e_i} + \frac{C_i}{\tilde{p}_i^{\,e_i}} \frac{S_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{\delta_i M_i^{\ 2}}{2Q_i} \right] \right.$$

$$\left. + \sum_{i=1}^{n} \sum_{j=1}^{\alpha} \sum_{k=1}^{\beta} (A_{jk} \times Q_i \times t_c \times x_{jk}) + \sum_{k=1}^{\beta} (S_{c_k} \times y_k) \right\}$$

2.1.4 Constraints of the Model

(i) The limitation on the available warehouse space in the store, $\sum_{i=1}^{n} v_i Q_i \le V$

(ii) The upper limit of the total amount investment, $\sum_{i=1}^{n} \tilde{p}_i Q_i \le I$

(iii) The upper limit on the number of orders can be made in a time cycle on the system, $\sum_{i=1}^{n} \dfrac{C_i}{\tilde{p}_i^{\,e_i} Q_i} \le t$

(iv) Percentage of utilization of volume of the warehouse, $\dfrac{V \times V_W}{\left(\sum_{i=1}^{n} v_i Q_i \right) \times 100} = 1$

(v) Restricts the total number of sites that are assigned warehouses to a maximum of γ , $\sum_{k=1}^{\beta} y_k \leq \gamma$

(vi) Restricts that each store is served by only one warehouse, $\sum_{k=1}^{\beta} x_{jk} = 1,\ j = 1,2,3,\ldots,\alpha$

(vii) Makes sure that each site which is assigned with a warehouse serves at least one store,

$$\sum_{j=1}^{\alpha} x_{jk} - y_k \geq 0,\ k = 1,2,3,\ldots\beta$$

(viii) Makes sure that each site which is not assigned with a warehouse does not serve any of the stores,

$$\sum_{j=1}^{\alpha} x_{jk} - \alpha y_k \leq 0,\ k = 1,2,3,\ldots\beta$$

(ix) Restricts the maximum distance between the warehouse and selling store, $\sum_{j=1}^{\alpha} A_{jk} x_{jk} \leq d,\ k = 1,2,3,\ldots\beta$

In the above constraints, constraint (vii) will be inactive when (viii) is active, similarly, constraint (viii) will be inactive when (vii) is active.

Where $x_{jk} = \begin{cases} 1, & \text{if a warehouse at site k} \\ 0, & \text{otherwise} \end{cases}$ and

$y_k = \begin{cases} 1, & \text{if a warehouse is open at site k} \\ 0, & \text{otherwise} \end{cases}$ and $\tilde{p}_i, Q_i > 0\ (i=1,2,\ldots\ldots,n), 0 \leq V_W \leq 100$

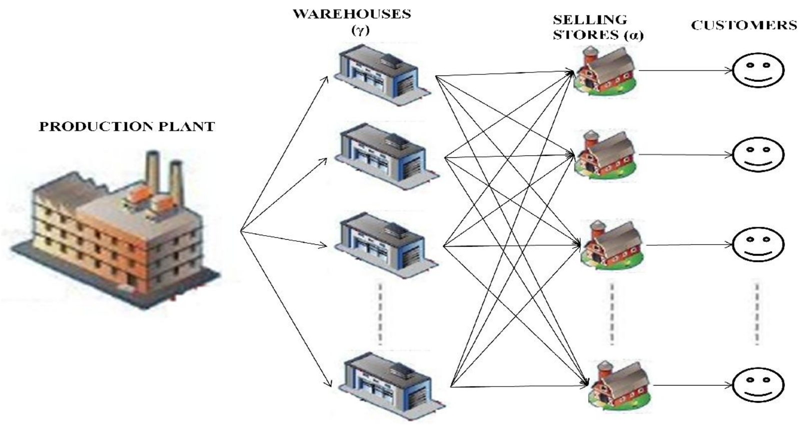

Fig. 1. The proposed configuration of supply network

2.1.5 Membership Function

The membership function for the triangular fuzzy number $\tilde{p}_i = (k_{u_i}, k_{m_i}, k_{o_i})$, $i = 1,2,\ldots n$ is

$$\mu_{p_i}(x) = \begin{cases} \dfrac{p_i - k_{u_i}}{k_{m_i} - k_{u_i}}, & k_{u_i} \leq p_i \leq k_{m_i} \\ \dfrac{k_{o_i} - p_i}{k_{o_i} - k_{m_i}}, & k_{m_i} \leq p_i \leq k_{o_i} \\ 0, & \text{otherwise} \end{cases}$$

3 NUMERICAL EXAMPLE

Assume that *n=1*, C_1*=113*, $S_1 = \$100$, $t = 4$, $H_1 = \$1$, $l_1 = 2m$, $b_1 = 1m$, $h_1 = 4m$, $L = 10m$, $B = 12m$, $M = 30m$, $\tilde{p}_1 = \$(10,15,20)$ and $I = \$1400$, $\alpha = 5$, $\beta = 5$ $d = 7$, t_c = \$2 *(fixed)*, $d = 7kms$, $\gamma = 5$. The distance between the stores and warehouses and the volume of materials required for the stores are given in table1.

Table 1. Distance Matrix [A_{jk}] in Kms

	Proposed warehouse (*k*)					
	-	1	2	3	4	5
Store (*j*)	1	5	3	7	6	8
	2	3	8	10	9	6
	3	10	9	6	5	9
	4	1	7	4	8	10
	5	9	8	10	9	4
Set up cost (S_{c_k})		100	120	110	140	150

The example is solved by using LINGO 8.0 software, and the result of the example is given in table 2.

Table 2. Result of the example

e_1	p_1	μ_{p_1}	Q_1	V_W	M_1	Z	x_{jk}	y_k
0.660	10.69	0.138	18.16	4.03	4.54	3077.28	$x_{12} = 1$, $x_{21} = 1$	y_1=1
0.670	11.64	0.328	17.44	3.88	4.36	3031.31	$x_{34} = 1$, $x_{41} = 1$	y_2=1
0.680	12.71	0.542	16.72	3.72	4.18	2984.22	$x_{55} = 1$	y_3 = 0
0.690	13.91	0.782	16.00	3.56	4.00	2936.01	& all remaining	y_4=1
0.695	14.56	0.912	15.65	3.48	3.91	2911.50	$x_{jk} = 0$	y_5=1
0.700	15.26	0.948	15.29	3.40	3.82	2886.72		
0.710	16.79	0.642	14.58	3.24	3.64	2836.37		
0.715	17.64	0.472	14.22	3.16	3.56	2810.80		
0.720	18.54	0.292	13.87	3.08	3.47	2784.98		
0.725	19.50	0.100	13.52	3.00	3.38	2758.91		

4 SPECIAL CASE (THE PROPOSED MODEL WITHOUT SHORTAGES)

Here we derive the result for the above said inventory model (2) for without shortage (i.e., $M_i = 0$). The result is presented in table 3.

Table 3. Result of the proposed model without shortages

e_1	p_1	μ_{p_1}	Q_1	V_W	Z	x_{jk}	y_k
0.660	10.27	0.054	18.90	4.20	3042.55		
0.670	11.18	0.236	18.16	4.04	2997.95	$x_{12} = 1$, $x_{21} = 1$	y_1=1
0.680	12.20	0.440	17.41	3.87	2952.23	$x_{34} = 1$, $x_{41} = 1$	y_2=1
0.690	13.35	0.670	16.67	3.70	2905.39	$x_{55} = 1$	y_3 = 0
0.695	13.98	0.796	16.30	3.62	2881.56	& all remaining	y_4=1
0.700	14.64	0.928	15.93	3.54	2857.46	$x_{jk} = 0$	y_5=1
0.710	16.11	0.778	15.20	3.38	2808.46		
0.715	16.92	0.616	14.83	3.30	2783.57		
0.720	17.78	0.444	14.47	3.21	2758.42		
0.725	18.69	0.262	14.10	3.13	2733.02		

5 CONCLUSION

In this paper we have proposed a concept of the optimal solution of the production-distribution inventory model with shortage. Here, a combined model is developed to optimize the total expenditure of the organization. The unit cost is taken in fuzzy environment. The constraint goals are restricted up to certain values and the warehouse space is considered in terms of volume. The proposed model is illustrated with a numerical example. The management can achieve to his target varying the level of unit cost (p_i) from 10 and 20. Due to complexity, the model is solved by LINGO 8.0 optimization software. We have also solved the model for without shortages as a special case of the original problem. In real life inventory control system, the cost parameters such as holding cost, ordering cost, production cost etc., are imprecise in nature. Similarly, in practical situations, resources like warehouse space, number of orders etc. are also imprecise in nature. Hence, in future, any of the above cost parameters or resources can be taken in fuzzy environment. The proposed model is more general and can be applied to the real inventory problems faced by the practitioners in the industry and other areas.

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