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Estimation of Errors: Mathematical Expressions of Temperature, Substrate Concentration and Enzyme Concentration based Formulas for obtaining intermediate values of the Rate of Enzymatic Reaction

Nizam Uddin

M. B. Khalsa College,
Indore, Madhya Pradesh, India

ABSTRACT: This research paper is based on the estimation of errors in the formulas which are used to obtaining intermediate values of the rate of enzymatic reaction. The rate of enzymatic reaction is affected by concentration of substrate, Temperature, concentration of enzyme and other factors. The rise in Temperature accelerates an Enzyme reaction. At certain Temperature known as the optimum Temperature the activity is maximum. The concentration of substrate is the limiting factor, as the substrate concentration increases, the Enzyme reaction rate increases. Assuming a sufficient concentration of substrate is available, increasing Enzyme concentration will increase the enzymatic reaction rate. These formulas are derived from temperature, substrate concentration and enzyme concentration based mathematical functions. These formulas are used to obtaining intermediate values of the rate of enzymatic reaction. Formulas which are derived using Newton's method for interpolation are worked in conditions which are depending on the point lies. If the point lies in the upper half then used Newton's forward interpolation formula. If the point lies in the lower half then we used Newton's backward interpolation formula. And when the interval is not equally spaced then used Newton's divide difference interpolation formula. When the tabulated values of the function are not equidistant then used Lagrangian polynomial. Mathematical expressions are derived for estimation of errors using intermediate values and formulas. All expressions are worked in n limit which is the optimum limit.

KEYWORDS: Estimation of Errors, Intermediate values, Mathematical Expressions, Rate of enzymatic reaction.

1 INTRODUCTION

Temperature, concentration of substrate, concentration of enzyme and other factors are affected the rate of enzymatic reaction [1]. The rise in Temperature accelerates an Enzyme reaction but at the same time causes inactivation of the protein. At certain Temperature known as the optimum Temperature the activity is maximum. The concentration of substrate is the limiting factor, as the substrate concentration increases, the Enzyme reaction rate increases. Assuming a sufficient concentration of substrate is available, increasing Enzyme concentration will increase the enzymatic reaction rate. Temperature, concentration of substrate and concentration of enzyme are increased the rate of enzymatic reaction at a limit which is called optimum limit [2]. On the basis of this concept, there are three mathematical functions [1] [2]:

$$V = f(T)$$
$$V = f(S)$$
$$V = f(E)$$

Where V is the rate of enzymatic reaction, T is the temperature, S is the concentration of substrate and E is the concentration of enzyme. These functions are be in n intervals and n is the optimum limit [1]. Some formulas are derived on the basis of these functions using Newton's method for interpolation and Lagrangian polynomial. These formulas are used to obtaining intermediate values of the rate of enzymatic reaction. Formulas which are derived using Newton's method for interpolation are worked in conditions which are depending on the point lies. If the point lies in the upper half then used Newton's forward interpolation formula. If the point lies in the lower half then we used Newton's backward interpolation formula. And when the interval is not equally spaced then used Newton's divide difference interpolation formula. When the tabulated values of the function are not equidistant then used Lagrangian polynomial [3]-[14].

2 FORMULAS FOR OBTAINING INTERMEDIATE VALUES OF *V* WITH TEMPERATURE

Let $V = f(T)$ be a function defined by n points $(T_0,V_0),(T_1,V_1)..........(T_n,V_n)$. Where V is the rate of reaction and T is the Temperature of reaction. And other factors are to be constant [1][2]. When $T_1, T_2, T_3 .... T_n$ are equally spaced with interval h.

2.1 IF THE POINT LIES IN THE UPPER HALF

In this condition we used following formula [1]:

$$V(T) = V_0 + \Delta V_0(u) + \frac{\Delta^2 V_0}{2!}(u)[(u-1)].....+\frac{\Delta^n V_0}{n!}[(u)\{(u-1)\}.......\{(u-n\}]$$

[Where Δ is forward difference operator and $\frac{T-T_0}{h} = u$]

2.1.1 ESTIMATION OF ERROR

Let $V = f(T)$ be a function defined by $(n+1)$ points $(T_0,V_0),(T_1,V_1).........(T_n,V_n)$. When $T_1, T_2, T_3 .... T_n$ are equally spaced with interval h and this function is continuous and differentiable $(n+1)$ times.

Let $V = f(T)$ be approximated by a polynomial $P_n(T)$ of degree not exceeding a such that

$$P_n(T_i) = V_i \qquad \text{[Where } i = 1,2,3..........n \text{]} \tag{1}$$

Since the expression $f(T) - P_n(T)$ vanishes for $T_1, T_2, T_3 .... T_n$,

We put $f(T) - P_n(T) = K\phi(T)$ (2)

Where $\phi(T) = (T - T_0)(T - T_1)...............(T - T_n)$ (3)

And K is to be determined in such a way that equation (2) holds for any intermediate values of T, say $T - T'$ [where $T_0 \le T' \le T_n$].

Therefore from (2)

$$K = \frac{f(T') - P(T')}{\varphi(T')} \tag{4}$$

Now we construct a function $f(T)$ such that

$$f(T_0) = f(T_1) - P_n(T) - K\varphi(T)$$

Where K is given by equation (4).

It is clear that

$$f(T_0) = f(T_1) = f(T_2) = f(T_3) = ...f(T_n) = f(T') = 0 \tag{5}$$

Let $f(T)$ vanishes $(n+2)$ times in the interval $T_0 \le T \le T_n$; consequently, by the repeated application of Rolle's Theorem [15] [16], $f'(T)$ must vanish $(n+1)$ times, $f''(T)$ must vanish n times etc in the interval $T_0 \le T \le T_n$.

Particularly, $f^{(n+1)}(T)$ must vanish once in the interval $T_0 \le T \le T_n$. Let this point be $T = Z$, $T_0 < Z < T_n$.

Now differentiating equation (5) $(n+1)$ times with respect to T and putting $T = Z$, we got:

$$f^{(n+1)}(Z) - K(n+1)! = 0$$

Or $$K = \frac{f^{(n+1)}(Z)}{(n+1)!} \tag{6}$$

Putting this value of K in equation (4), we got:

$$\frac{f^{(n+1)}(Z)}{(n+1)!} = \frac{f(T') - P_n(T')}{\varphi(T')}$$

Or $$f(T') - P_n(T') = \frac{f^{(n+1)}(Z)}{(n+1)!}\varphi(T'), \qquad T_0 < Z < T_n$$

Since T' is arbitrary therefore on dropping the prime on T' we got:

$$f(T) - P_n(T) = \frac{f^{(n+1)}(Z)}{(n+1)!}\varphi(T), \quad T_0 < Z < T_n \tag{7}$$

Now we use Taylor's theorem [17] [18]:

$$f(T+h) = f(Z) + hf'(Z) + \frac{h^2}{2!}f''(Z) + ... + \frac{h^n}{n!}f^n(Z) + ... \tag{8}$$

Neglecting the terms containing second and higher powers of h in equation (8), we got:

$$f(Z+h) = f(Z) + hf'(Z)$$

Or $$f'(Z) = \frac{f(Z+h) - f(Z)}{h} \tag{9}$$

Or $$f'(Z) = \frac{1}{h}\Delta f(Z) \qquad [\therefore \Delta f(T+h) f(T)]$$

$$Df(Z) = \frac{1}{h}\Delta f(Z) \qquad [\therefore D = \frac{d}{dZ}]$$

$$D = \frac{1}{h}\Delta \qquad \text{[Because } f(Z) \text{ is arbitrary]}$$

$$\therefore D^{n+1} = \frac{1}{h^{n+1}}\Delta^{n+1}$$

From equation (9), we got:

$$f^{(n+1)}(Z) = \frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Z)$$

Putting the values of $f^{(n+1)}(Z)$ in equation (7), we got

$$f(T) - P_n(T) = \left[\frac{\varphi(T)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Z)\right]$$

$$f(T) - P_n(T) = \left[\frac{(T-T_0)(T-T_1)(T-T_2)................(T-T_0)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Z)\right] \tag{10}$$

If $\frac{T-T_0}{h}=u$

Then

$$T-T_0=hu$$
$$T-T_1=T-(T_0+h)=(T-T_0)-h=(hu-h)=h(u-1)$$

Similarly $T-T_2=h(u-2)$

:

:

:

Similarly $T-T_n=h(u-n)$

Putting these values in equation (10), we got:

$$f(T)-P_n(T)=\left[\frac{(hu)\{h(u-1)\}\{h(u-2)\}\{h(u-3)\}.............\{(u-n)\}}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Z)\right]$$

$$f(T)-P_n(T)=\left[\frac{u(u-1)(u-2)(u-3).........(u-n)}{(n+1)!}\right]\left[\Delta^{(n+1)}f(Z)\right]$$

This is mathematical expression for estimation of error, if the point lies in the upper half.

2.2 IF THE POINT LIES IN THE LOWER HALF

In this condition we used following formula [1]:

$$V(T)=V_n+\nabla V_n(z)+\frac{\nabla^2 V_n}{2!}[\{z(z+1)\}]+.....+\frac{\nabla^n V_n}{n!}[(z)\{(z+1)\}..\{z+(n\text{-}1)\}]$$

[Where ∇ is backward difference operator and $\frac{T-T_n}{h}=z$]

2.2.1 ESTIMATION OF ERROR

Let $V=f(T)$ be a function defined by $(n+1)$ points $(T_0,V_0),(T_1,V_1).........(T_n,V_n)$. When $T_1,T_2,T_3.............T_n$ are equally spaced with interval h and this function is continuous and differentiable $(n+1)$ times.

Let $V=f(T)$ be approximated by a polynomial $P_n(T)$ of degree not exceeding a such that

$P_n(T_i)=V_i$ [Where $i=1,2,3..........n$] (11)

Since the expression $f(T)-P_n(T)$ vanishes for $T_1,T_2,T_3.............T_n$,

We put $f(T)-P_n(T)=K\phi(T)$ (12)

Where $\phi(T)=(T-T_n)(T-T_{n-1}).............(T-T_0)$ (13)

And K is to be determined in such a way that equation (12) holds for any intermediate values of T, say $T-T'$ [where $T_0\le T'\le T_n$].

Therefore from equation (12),

$$K = \frac{f(T') - P(T')}{\varphi_1(T')} \tag{14}$$

Now we construct a function $f(T)$ such that

$$f(T_0) = f(T_1) - P_n(T) - K\varphi_1(T)$$

Where K is given by equation (14).

It is clear that

$$f(T_0) = f(T_1) = f(T_2) = f(T_3) = ...f(T_n) = f(T') = 0 \tag{15}$$

Let $f(T)$ vanishes (n+2) times in the interval $T_0 \le T \le T_n$; consequently, by the repeated application of Rolle's Theorem [15] [16], $f'(T)$ must vanish $(n+1)$ times, $f''(T)$ must vanish n times etc in the interval $T_0 \le T \le T_n$.

Particularly, $f^{(n+1)}(T)$ must vanish once in the interval $T_0 \le T \le T_n$. Let this point be $T = Y,\ T_0 < Z < T_n$.

Now differentiating equation (15) $(n+1)$ times with respect to T and putting $T = Y,$, we got:

$$f^{(n+1)}(Y) - K(n+1)! = 0$$

Or
$$K = \frac{f^{(n+1)}(Y)}{(n+1)!} \tag{16}$$

Putting this value of K in equation (14), we got:

$$\frac{f^{(n+1)}(Y)}{(n+1)!} = \frac{f(T') - P_n(T')}{\varphi_1(T')}$$

Or
$$f(T') - P_n(T') = \frac{f^{(n+1)}(Y)}{(n+1)!}\varphi_1(T'), \qquad T_0 < Y < T_n$$

Since T' is arbitrary therefore on dropping the prime on T' we got:

$$f(T) - P_n(T) = \frac{f^{(n+1)}(Y)}{(n+1)!}\varphi_1(T), \quad T_0 < Y < T_n \tag{17}$$

Now we use Taylor's theorem [17] [18]:

$$f(T+h) = f(Y) + hf'(Y) + \frac{h^2}{2!}f''(Y) + ... + \frac{h^n}{n!}f^n(Y) + ... \tag{18}$$

Neglecting the terms containing second and higher powers of h in equation (18), we got:

$$f(Y+h) = f(Y) + hf'(Y)$$

Or
$$f'(Z) = \frac{f(Y+h) - f(Y)}{h} \tag{19}$$

Or
$$f'(Z) = \frac{1}{h}\Delta f(Y) \qquad [\therefore \Delta f(T+h)f(T)]$$

$$Df(Y) = \frac{1}{h}\Delta f(Y) \qquad [\therefore D = \frac{d}{dY}]$$

$$D = \frac{1}{h}\Delta \qquad \text{[Because } f(Y) \text{ is arbitrary]}$$

$$\therefore D^{n+1} = \frac{1}{h^{n+1}}\Delta^{n+1}$$

From equation (19), we got:

$$f^{(n+1)}(Y) = \frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(Y)$$

Putting the values of $f^{(n+1)}(Y)$ in equation (17), we got:

$$f(T) - P_n(T) = \left[\frac{\varphi_1(T)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(Y)\right]$$

$$f(T) - P_n(T) = \left[\frac{(T-T_0)(T-T_1)(T-T_2)................(T-T_0)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(Y)\right] \tag{20}$$

If $\dfrac{T-T_n}{h} = z$

Then

$$T - T_n = hz$$

$$T - T_{n-1} = T - (T_n - h) = (T - T_n) + h = (hz + h) = h(z+1)$$

Similarly $T - T_{n-2} = h(z+2)$

:

:

:

Similarly $T - T_0 = h(z+n)$

Putting these values in equation (20), we got:

$$f(T) - P_n(T) = \left[\frac{(hz)\{h(z-1)\}\{h(z-2)\}\{h(z-3)\}.............\{(z-n)\}}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(Y)\right]$$

This is mathematical expression for estimation of error, if the point lies in the lower half.

2.3 If Intervals are not be equally spaced

In this condition we used following formula [1]:

$$\mathrm{V(T)} = \mathrm{V_1} + \Delta_{\mathrm{d}}\mathrm{V_1}(\mathrm{T} - T_1) + \Delta_{\mathrm{d}}^2\mathrm{V_1}(\mathrm{T} - T_1)(\mathrm{T} - T_2) + ...+ \Delta_{\mathrm{d}}^{\mathrm{n}}\mathrm{V_1}[(\mathrm{T} - \mathrm{T_1})(\mathrm{T} - T_2)...(\mathrm{T} - \mathrm{T_n})]$$

[Where Δ_d is divide difference operator]

2.3.1 Estimation of error

Let $f(T)$ be a real-valued function define n interval and $(n+1)$ times differentiable on (a,b). If $P_n(T)$ is the polynomial. Which interpolates $f(T)$ at the $(n+1)$ distinct points $T_0, T_1 \ldots\ldots T_n \in (a,b)$, then for all $\bar{T} \in [a,b]$,there exists $\xi = \xi(\bar{T}) \in (a,b)$

$$e_n(\bar{T}) = f(\bar{T}) - P_n(\bar{T})$$
$$= \frac{f^{(n+1)}(\xi)}{(n+1)} \prod_{j=0}^{n} (\bar{T} - T_j) \tag{21}$$

This is mathematical expression for estimation of error, if intervals are not be equally spaced.

2.4 When the tabulated values of the function are not equidistant

In this condition we used following formula [2]:

$$\mathrm{V(T)} = \sum_{i=1}^{n} \mathrm{V}_i \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(\mathrm{T}-\mathrm{T}_j)}{(\mathrm{T}_i-\mathrm{T}_j)}$$

2.4.1 Estimation of error

Since the approximating polynomial $f(T)$ given by Lagrangian formula has the same values $f(T_0)$ $f(T_1)$ $f(T_2)$ $f(T_3)$ $f(T_4)$ .... $f(T_n)$ as does $V = f(T)$ for the arguments T_0, T_1, T_2, T_3, T_4 ...., T_n the error term must have zeros at these $(n+1)$ points.

Therefore $(T - T_0)$ $(T - T_1)$ $(T - T_2)$ $(T - T_3)$ .... $(T - T_n)$ must be factors of the error and we can write:

$$F(T) = f(T) + \frac{(T-T_0)(T-T_1)(T-T_2)(T-T_3)\ldots\ldots\ldots\ldots\ldots\ldots(T-T_n)}{(n+1)!} K(T) \tag{22}$$

Let T to be fixed in value and consider the function

$$W(x) = F(x) - f(x) \frac{(x-T_0)(x-T_1)(x-T_2)(x-T_3)\ldots\ldots\ldots\ldots\ldots(x-T_n)}{(n+1)!} K(T) \tag{23}$$

Then $W(x)$ has zero $x = T_0, T_1, T_2, T_3 \ldots\ldots\ldots\ldots T_n$ and T .

Since the $(n+1)^{th}$ derivative of the n^{th} degree polynomial $f(T)$ is zero.

$$W^{(n+1)}(x) = F^{(n+1)}(x) - K(T) \tag{24}$$

As a consequence of Rolle's Theorem [15] [16], the $(n+1)^{th}$ derivative of $W(x)$ has at least one real zero $x = \xi$ in the range $T_0 < \xi < T_n$

Therefore substituting $x = \xi$ in equation (24)

$$W^{(n+1)}(\xi) = F^{(n+1)}(\xi) - K(T)$$

Or
$$K(T) = F^{(n+1)}(\xi) - W^{(n+1)}(\xi)$$
$$= F^{(n+1)}(\xi)$$

Using this expression for $K(T)$ and writing out $f(T)$

$$f(T)= \frac{(T-T_1)(T-T_2)............(T-T_n)}{(T_0-T_1)(T_0-T_2)............(T_0-T_n)}f(T_0)+\frac{(T-T_0)(T-T_2)............(T-T_n)}{(T_1-T_0)(T_1-T_2)............(T_1-T_n)}f(T_1)+.......$$
$$.......+\frac{(T-T_0)(T-T_1)............(T-T_{n-1})}{(T_n-T_0)(T_n-T_1)............(T_n-T_{n-1})}f(T_n)+\frac{(T-T_0)(T-T_1)............(T-T_n)}{(n+1)!}f^{(n+1)}(\xi)$$

Where $T_0 < \xi < T_n$

This is mathematical expression for estimation of error, if the tabulated values of the function are not equidistant.

3 FORMULAS FOR OBTAINING INTERMEDIATE VALUES OF *V* WITH CONCENTRATION OF SUBSTRATE

Let $V = f(S)$ be a function defined by n points $(S_0,V_0),(S_1,V_1).........(S_n,V_n)$. When $S_1,S_2,S_3..............S_n$ are equally spaced with interval h. Where V is the rate of enzymatic reaction and S is the concentration of substrate. And other factors are be constant [1] [2].

3.1 IF THE POINT LIES IN THE UPPER HALF

In this condition we used following formula [1]:

$$\mathrm{V(S)=V_0+\Delta V_0(w)+\frac{\Delta^2V_0}{2!}(w)[(w-1)].....+\frac{\Delta^nV_0}{n!}[(w)\{(w-1)\}.......\{(w-n\}]}$$

[Where Δ is forward difference operator and $\frac{S-S_0}{h}=w$]

3.1.1 ESTIMATION OF ERROR

Let $V = f(S)$ be a function defined by $(n+1)$ points $(S_0,V_0),(S_1,V_1).........(S_n,V_n)$. When $S_1,S_2,S_3..............S_n$ are equally spaced with interval h and this function is continuous and differentiable $(n+1)$ times.

Let $V = f(S)$ be approximated by a polynomial $P_n(S)$ of degree not exceeding a such that

$P_n(S_i)=V_i$ [Where $i = 1,2,3..........n$] (25)

Since the expression $f(S)-P_n(S)$ vanishes for $S_1,S_2,S_3..............S_n$,

We put $f(S)-P_n(S)=K\varphi(S)$ (26)

Where $\varphi(S)=(S-S_0)(S-S_1).............(S-S_n)$ (27)

And K is to be determined in such a way that equation (26) holds for any intermediate values of S, say $S-S'$ [where $S_0 \le S' \le S_n$].

Therefore from equation (26)

$$K=\frac{f(S')-P(S')}{\varphi(S')} \quad (28)$$

Now we construct a function $f(S)$ such that

$$f(S_0)=f(S_1)-P_n(S)-K\varphi(S)$$

Where K is given by equation (27).

It is clear that:

$$f(S_0)=f(S_1)=f(S_2)=f(S_3)=...........f(S_n)=f(S')=0 \quad (29)$$

Let $f(S)$ vanishes $(n+2)$ times in the interval $S_0 \leq S \leq S_n$; consequently, by the repeated application of Rolle's Theorem [15] [16], $f'(S)$ must vanish $(n+1)$ times, $f''(S)$ must vanish n times etc in the interval $S_0 \leq S \leq S_n$.

Particularly, $f^{(n+1)}(S)$ must vanish once in the interval $S_0 \leq S \leq S_n$. Let this point be $S = Q,\ S_0 \leq S \leq S_n$.

Now differentiating equation (29) $(n+1)$ times with respect to S and putting $S = Q$, we got:

$$f^{(n+1)}(Q) - K(n+1)! = 0$$

Or
$$K = \frac{f^{(n+1)}(Q)}{(n+1)!} \tag{30}$$

Putting this value of K in equation (28), we got:

$$\frac{f^{(n+1)}(Q)}{(n+1)!} = \frac{f(S') - P_n(S')}{\varphi(S')}$$

Or
$$f(S') - P_n(S') = \frac{f^{(n+1)}(Q)}{(n+1)!}\varphi(S')\,, \qquad S_0 < S < S_n$$

Since S' is arbitrary therefore on dropping the prime on S' we got:

$$f(S) - P_n(S) = \frac{f^{(n+1)}(Q)}{(n+1)!}\varphi(S)\,, \quad S_0 < S < S_n \tag{31}$$

Now we use Taylor's theorem [17] [18]:

$$f(S+h) = f(Q) + hf'(Q) + \frac{h^2}{2!}f''(Q) + \ldots\ldots\ldots + \frac{h^n}{n!}f^n(Q) + \ldots.. \tag{32}$$

Neglecting the terms containing second and higher powers of h in equation (32), we got:

$$f(Q+h) = f(Q) + hf'(Q)$$

Or
$$f'(Q) = \frac{f(Q+h) - f(Q)}{h} \tag{33}$$

Or
$$f'(Q) = \frac{1}{h}\Delta f(Q) \qquad [\therefore \Delta f(S+h) f(S)]$$

$$Df(Q) = \frac{1}{h}\Delta f(Q) \qquad [\therefore D = \frac{d}{dQ}]$$

$$D = \frac{1}{h}\Delta \qquad \text{[Because } f(Q) \text{ is arbitrary]}$$

$$\therefore D^{n+1} = \frac{1}{h^{n+1}}\Delta^{n+1}$$

From equation (33), we got:

$$f^{(n+1)}(Q) = \frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(Q)$$

Putting the values of $f^{(n+1)}(Q)$ in equation (31), we got

$$f(S)-P_n(S)=\left[\frac{\varphi(S)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Q)\right]$$

$$f(S)-P_n(S)=\left[\frac{(S-S_0)(S-S_1)(S-S_2)................(S-S_0)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Q)\right] \quad (34)$$

If $\frac{S-S_0}{h}=r$

Then

$$S-S_0=hr$$
$$S-S_1=S-(S_0+h)=(S-S_0)-h=(hr-h)=h(r-1)$$

Similarly $S-S_2=h(r-2)$

:

:

:

Similarly $S-S_n=h(r-n)$

Putting these values in equation (34), we got:

$$f(S)-P_n(S)=\left[\frac{(hr)\{h(r-1)\}\{h(r-2)\}\{h(r-3)\}.............\{(r-n)\}}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(Q)\right]$$

$$f(S)-P_n(S)=\left[\frac{r(r-1)(r-2)(r-3).........(r-n)}{(n+1)!}\right]\left[\Delta^{(n+1)}f(Q)\right]$$

This is mathematical expression for estimation of error, if the point lies in the upper half.

3.2 IF THE POINT LIES IN THE LOWER HALF

In this condition we used following formula [1]:

$$\mathrm{V(S)}=\mathrm{V_n}+\nabla\mathrm{V_n}\,(\mathrm{J})+\frac{\nabla^2\mathrm{V_n}}{2!}[\{\mathrm{J(J+1)}\}]+.....+\frac{\nabla^n\mathrm{V_n}}{\mathrm{n!}}[(\mathrm{J})\{(\mathrm{J}+1)\}..\{\mathrm{J}+(\mathrm{n}-1)\}]$$

[Where ∇ is backward difference operator and $\frac{S-S_n}{h}=J$]

3.2.1 ESTIMATION OF ERROR

Let $V=f(S)$ be a function defined by $(n+1)$ points $(S_0,V_0),(S_1,V_1)..........(S_n,V_n)$. When $S_1,S_2,S_3..............S_n$ are equally spaced with interval h and this function is continuous and differentiable $(n+1)$ times.

Let $V=f(S)$ be approximated by a polynomial $P_n(S)$ of degree not exceeding a such that

$$P_n(S_i)=V_i \qquad \text{[Where } i=1,2,3.........n\text{]} \quad (35)$$

Since the expression $f(S)-P_n(S)$ vanishes for $S_1,S_2,S_3..............S_n$,

We put $f(S)-P_n(S)=K\varphi(S)$ (36)

Where $\varphi(S) = (S - S_0)(S - S_1)............(S - S_n)$ (37)

And K is to be determined in such a way that equation (36) holds for any intermediate values of S, say $S - S'$ [where $S_0 \le S' \le S_n$].

Therefore from (2)

$$K = \frac{f(S') - P(S')}{\varphi_1(S')} \tag{38}$$

Now we construct a function $f(S)$ such that

$f(S_0) = f(S_1) - P_n(S) - K\varphi_1(S)$

Where K is given by equation (37).

It is clear that

$$f(S_0) = f(S_1) = f(S_2) = f(S_3) = ...f(S_n) = f(S') = 0 \tag{39}$$

Let $f(S)$ vanishes $(n+2)$ times in the interval $S_0 \le S \le S_n$; consequently, by the repeated application of Rolle's Theorem [15][16], $f'(S)$ must vanish $(n+1)$ times, $f''(S)$ must vanish n times etc in the interval $S_0 \le S \le S_n$.

Particularly, $f^{(n+1)}(S)$ must vanish once in the interval $S_0 \le S \le S_n$. Let this point be $S = R,\ S_0 \le S \le S_n$.

Now differentiating equation (39) $(n+1)$ times with respect to S and putting $S = R$, we got:

$$f^{(n+1)}(R) - K(n+1)! = 0$$

Or

$$K = \frac{f^{(n+1)}(R)}{(n+1)!} \tag{40}$$

Putting this value of K in equation (38), we got:

$$\frac{f^{(n+1)}(R)}{(n+1)!} = \frac{f(S') - P_n(S')}{\varphi_1(S')}$$

Or

$$f(S') - P_n(S') = \frac{f^{(n+1)}(R)}{(n+1)!}\varphi_1(S'), \qquad S_0 < S < S_n$$

Since S' is arbitrary therefore on dropping the prime on S' we got:

$$f(S) - P_n(S) = \frac{f^{(n+1)}(R)}{(n+1)!}\varphi_1(S), \quad S_0 < S < S_n \tag{41}$$

Now we use Taylor's theorem [17][18]:

$$f(S+h) = f(R) + hf'(R) + \frac{h^2}{2!}f''(R) + ... + \frac{h^n}{n!}f^n(R) + ... \tag{42}$$

Neglecting the terms containing second and higher powers of h in equation (42), we got:

$$f(R+h) = f(R) + hf'(R)$$

Or

$$f'(R) = \frac{f(R+h) - f(R)}{h} \tag{43}$$

Or $$f'(R) = \frac{1}{h}\Delta f(R) \qquad [\therefore \Delta f(S+h) f(S)]$$

$$Df(R) = \frac{1}{h}\Delta f(R) \qquad [\therefore D = \frac{d}{dR}]$$

$$D = \frac{1}{h}\Delta \qquad \text{[Because } f(R) \text{ is arbitrary]}$$

$$\therefore D^{n+1} = \frac{1}{h^{n+1}}\Delta^{n+1}$$

From equation (43), we got:

$$f^{(n+1)}(R) = \frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(R)$$

Putting the values of $f^{(n+1)}(R)$ in equation (41), we got

$$f(S) - P_n(S) = \left[\frac{\varphi_1(S)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(R)\right]$$

$$f(S) - P_n(S) = \left[\frac{(S-S_0)(S-S_1)(S-S_2)................(S-S_0)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(R)\right] \qquad (44)$$

If $\frac{S-S_n}{h} = J$

Then

$$S - S_n = hJ$$
$$S - S_{n-1} = S - (S_n - h) = (S - S_n) + h = (hJ + h) = h(J+1)$$

Similarly $S - S_{n-2} = h(J+2)$

:

:

:

Similarly $S - S_0 = h(J+n)$

Putting these values in equation (44), we got:

$$f(S) - P_n(S) = \left[\frac{(hJ)\{h(J-1)\}\{h(J-2)\}\{h(J-3)\}.............\{(J-n)\}}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)} f(R)\right]$$

$$f(S) - P_n(S) = \left[\frac{J(J-1)(J-2)(J-3).........(J-n)}{(n+1)!}\right]\left[\Delta^{(n+1)} f(R)\right]$$

This is mathematical expression for estimation of error, if the point lies in the lower half.

3.3 IF INTERVALS ARE NOT BE EQUALLY SPACED

In this condition we used following formula [1]:

$$V(S)=V_1+\Delta_d V_1(S-S_1)+\Delta_d^2 V_1(S-S_1)(S-S_2)+.....+\Delta_d^n V_1[(S-S_1)(S-S_2)...(S-S_n)]$$

[Where Δd is divide difference operator]

3.3.1 Estimation of error

Let $f(S)$ be a real-valued function define n interval and $(n+1)$ times differentiable on (a,b). If $P_n(S)$ is the polynomial. Which interpolates $f(S)$ at the $(n+1)$ distinct points $S_0, S_1 ..... S_n \in (a,b)$, then for all $\bar{T} \in [a,b]$,there exists $\xi = \xi(\bar{S}) \in (a,b)$

$$e_n(\bar{S}) = f(\bar{S}) - P_n(\bar{S})$$

$$= \frac{f^{(n+1)}(\xi)}{(n+1)} \prod_{j=0}^{n} (\bar{S} - S_j) \tag{45}$$

This is mathematical expression for estimation of error, if intervals are not be equally spaced.

3.4 When the tabulated values of the function are not equidistant

In this condition we used following formula [2]:

$$V(S) = \sum_{i=1}^{n} V_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{(S - S_j)}{(S_i - S_j)}$$

3.4.1 Estimation of error

Since the approximating polynomial $f(S)$ given by Lagrangian formula has the same values $f(S_0)$ $f(S_1)$ $f(S_2)$ $f(S_3)$ $f(S_4)$ ………… $f(S_n)$ as does $V = f(S)$ for the arguments S_0, S_1, S_2, S_3, S_4 ………………….., S_n the error term must have zeros at these $(n+1)$ points.

Therefore $(S-S_0)$ $(S-S_1)$ $(S-S_2)$ $(S-S_3)$ ………………………… $(S-S_n)$ must be factors of the error and we can write:

$$F(S) = f(S) + \frac{(S-S_0)(S-S_1)(S-S_2)(S-S_3)..................(S-S_n)}{(n+1)!} K(S) \tag{46}$$

Let S to be fixed in value and consider the function

$$W(x) = F(x) - f(x) \frac{(x-S_0)(x-S_1)(x-S_2)(x-S_3)...............(x-S_n)}{(n+1)!} K(S) \tag{47}$$

Then $W(x)$ has zero $x = S_0, S_1, S_2, S_3 .... S_n$ and S .

Since the $(n+1)^{th}$ derivative of the n^{th} degree polynomial $f(S)$ is zero.

$$W^{(n+1)}(x) = F^{(n+1)}(x) - K(S) \tag{48}$$

As a consequence of Rolle's Theorem [15] [16], the $(n+1)^{th}$ derivative of $W(x)$ has at least one real zero $x = \xi$ in the range $S_0 < \xi < S_n$

Therefore substituting $x = \xi$ in equation (48)

$$W^{(n+1)}(\xi) = F^{(n+1)}(\xi) - K(S)$$

Or

$$K(S) = F^{(n+1)}(\xi) - W^{(n+1)}(\xi)$$

$$= F^{(n+1)}(\xi)$$

Using this expression for $K(S)$ and writing out $f(S)$

$$f(S)=\frac{(S-S_1)(S-S_2)............(S-S_n)}{(S_0-S_1)(S_0-S_2)............(S_0-S_n)}f(S_0)+\frac{(S-S_0)(S-S_2)............(S-S_n)}{(S_1-S_0)(S_1-S_2)............(S_1-S_n)}f(S_1)+.......$$
$$.......+\frac{(S-S_0)(S-S_1)............(S-S_{n-1})}{(S_n-S_0)(S_n-S_1)............(S_n-S_{n-1})}f(S_n)+\frac{(S-S_0)(S-S_1)............(S-S_n)}{(n+1)!}f^{(n+1)}(\xi)$$

Where $S_0<\xi<S_n$

This is mathematical expression for estimation of error, if the tabulated values of the function are not equidistant.

4 FORMULAS FOR OBTAINING INTERMEDIATE VALUES OF *V* WITH CONCENTRATION OF ENZYME

Let $V=f(E)$ be a function defined by n points $(E_0,V_0),(E_1,V_1)..........(E_n,V_n)$. When $E_1,E_2,E_3..............E_n$ are equally spaced with interval h. Where ***V*** is the rate of enzymatic reaction and E is the concentration of enzyme. And other factors are be constant [1] [2].

4.1 IF THE POINT LIES IN THE UPPER HALF

In this condition we used following formula [1]:

$$V(E)=V_0+\Delta V_0(X)+\frac{\Delta^2 V_0}{2!}(X)[(X-1)].....+\frac{\Delta^n V_0}{n!}[(X)\{(X-1)\}.......\{(X-n\}]$$

[Where Δ is forward difference operator and $\frac{E-E_0}{h}=X$]

4.1.1 ESTIMATION OF ERROR

Let $V=f(E)$ be a function defined by $(n+1)$ points $(E_0,V_0),(E_1,V_1)..........(E_n,V_n)$. When $E_1,E_2,E_3..............E_n$ are equally spaced with interval h and this function is continuous and differentiable $(n+1)$ times.

Let $V=f(E)$ be approximated by a polynomial $P_n(E)$ of degree not exceeding a such that

$$P_n(E_i)=V_i \qquad [\text{Where } i=1,2,3..........n] \tag{49}$$

Since the expression $f(E)-P_n(E)$ vanishes for $E_1,E_2,E_3..............E_n$,

We put $f(E)-P_n(E)=K\varphi(E)$ (50)

Where $\varphi(E)=(E-E_0)(E-E_1).............(E-E_n)$ (51)

And K is to be determined in such a way that equation (50) holds for any intermediate values of E, say $E-E'$ [where $E_0\le E'\le E_n$].

Therefore from (50)

$$K=\frac{f(E')-P(E')}{\varphi(E')} \tag{52}$$

Now we construct a function $f(E)$ such that

$$f(E_0)=f(E_1)-P_n(E)-K\varphi(E)$$

Where K is given by equation (52).

It is clear that : $f(E_0)=f(E_1)=f(E_2)=f(E_3)=...........f(E_n)=f(E')=0$ (53)

Let $f(E)$ vanishes $(n+2)$ times in the interval $E_0 \le E \le E_n$; consequently, by the repeated application of Rolle's Theorem [15] [16], $f'(E)$ must vanish $(n+1)$ times, $f''(E)$ must vanish n times etc in the interval $E_0 \le E \le E_n$.

Particularly, $f^{(n+1)}(E)$ must vanish once in the interval $E_0 \le E \le E_n$. Let this point be $E = R$, $E_0 \le E \le E_n$.

Now differentiating equation (53) $(n+1)$ times with respect to E and putting $E = R$, , we got:

$$f^{(n+1)}(R) - K(n+1)! = 0$$

Or
$$K = \frac{f^{(n+1)}(R)}{(n+1)!} \tag{54}$$

Putting this value of K in equation (52), we got:

$$\frac{f^{(n+1)}(R)}{(n+1)!} = \frac{f(E') - P_n(E')}{\varphi(E')}$$

Or
$$f(E') - P_n(E') = \frac{f^{(n+1)}(R)}{(n+1)!}\varphi(E'), \qquad E_0 < E < E_n$$

Since E' is arbitrary therefore on dropping the prime on E' we got:

$$f(E) - P_n(E) = \frac{f^{(n+1)}(R)}{(n+1)!}\varphi(E), \quad E_0 < E < E_n \tag{55}$$

Now we use Taylor's theorem [17] [18]:

$$f(E+h) = f(R) + hf'(R) + \frac{h^2}{2!}f''(R) + ... + \frac{h^n}{n!}f^n(R) + ... \tag{56}$$

Neglecting the terms containing second and higher powers of h in equation (56), we got:

$$f(R+h) = f(R) + hf'(R)$$

Or
$$f'(R) = \frac{f(R+h) - f(R)}{h} \tag{57}$$

Or
$$f'(R) = \frac{1}{h}\Delta f(R) \qquad [\therefore \Delta f(E+h)f(E)]$$

$$Df(R) = \frac{1}{h}\Delta f(R) \qquad [\therefore D = \frac{d}{dR}]$$

$$D = \frac{1}{h}\Delta \qquad \text{[Because } f(R) \text{ is arbitrary]}$$

$$\therefore D^{n+1} = \frac{1}{h^{n+1}}\Delta^{n+1}$$

From equation (57), we got:

$$f^{(n+1)}(R) = \frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)$$

Putting the values of $f^{(n+1)}(R)$ in equation (57), we got

$$f(E) - P_n(E) = \left[\frac{\varphi(E)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)\right]$$

$$f(E)-P_n(E)=\left[\frac{(E-E_0)(E-E_1)(E-E_2)................(E-E_0)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)\right] \quad (58)$$

If $\frac{E-E_0}{h}=X$

Then

$$E-E_0=hX$$
$$E-E_1=E-(E_0+E)=(E-E_0)-h=(hX-h)=h(X-1)$$

Similarly $E-E_2=h(X-2)$

:

:

:

Similarly $E-E_n=h(X-n)$

Putting these values in equation (58), we got:

$$f(E)-P_n(E)=\left[\frac{(hX)\{h(X-1)\}\{h(X-2)\}\{h(X-3)\}.............\{(X-n)\}}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)\right]$$

$$f(E)-P_n(E)=\left[\frac{X(X-1)(e-2)(X-3).........(X-n)}{(n+1)!}\right]\left[\Delta^{(n+1)}f(R)\right]$$

This is mathematical expression for estimation of error, if the point lies in the upper half.

4.2 IF THE POINT LIES IN THE LOWER HALF

In this condition we used following formula [1]:

$$\mathrm{V(E)}=\mathrm{V_n}+\nabla\mathrm{V_n(e)}+\frac{\nabla^2\mathrm{V_n}}{2!}[\{\mathrm{e(e+1)}\}]+.....+\frac{\nabla^n\mathrm{V_n}}{n!}[(\mathrm{e})\{(\mathrm{e}+1)\}..\{\mathrm{e}+(\mathrm{n}-1)\}]$$

[Where ∇ is backward difference operator and $\frac{E-E_n}{h}=e$]

4.2.1 ESTIMATION OF ERROR

Let $V=f(E)$ be a function defined by $(n+1)$ points $(E_0,V_0),(E_1,V_1)..........(E_n,V_n)$. When $E_1,E_2,E_3..............E_n$ are equally spaced with interval h and this function is continuous and differentiable $(n+1)$ times.

Let $V=f(E)$ be approximated by a polynomial $P_n(E)$ of degree not exceeding a such that

$$P_n(E_i)=V_i \qquad [\text{Where } i=1,2,3..........n] \quad (59)$$

Since the expression $f(E)-P_n(E)$ vanishes for $E_1,E_2,E_3..............E_n$,

We put $f(E)-P_n(E)=K\varphi(E)$ (60)

Where $\varphi(E)=(E-E_0)(E-E_1).............(E-E_n)$ (61)

And K is to be determined in such a way that equation (60) holds for any intermediate values of E, say $E-E'$ [where $E_0\le E'\le E_n$].

Therefore from (60)

$$K=\frac{f(E')-P(E')}{\varphi_1(E')} \tag{62}$$

Now we construct a function $f(E)$ such that

$$f(E_0)=f(E_1)-P_n(E)-K\varphi_1(E)$$

Where K is given by equation (62).

It is clear that

$$f(E_0)=f(E_1)=f(E_2)=f(E_3)=...........f(E_n)=f(E')=0 \tag{63}$$

Let $f(E)$ vanishes $(n+2)$ times in the interval $E_0 \le E \le E_n$; consequently, by the repeated application of Rolle's Theorem [15] [16], $f'(E)$ must vanish $(n+1)$ times, $f''(E)$ must vanish n times etc in the interval $E_0 \le E \le E_n$.

Particularly, $f^{(n+1)}(E)$ must vanish once in the interval $E_0 \le E \le E_n$. Let this point be $E=R,\ E_0 \le E \le E_n$.

Now differentiating equation (5) $(n+1)$ times with respect to E and putting $E=R$, , we got:

$$f^{(n+1)}(R)-K(n+1)!=0$$

Or
$$K=\frac{f^{(n+1)}(R)}{(n+1)!} \tag{64}$$

Putting this value of K in equation (62), we got:

$$\frac{f^{(n+1)}(R)}{(n+1)!}=\frac{f(E')-P_n(E')}{\varphi_1(E')}$$

Or
$$f(E')-P_n(E')=\frac{f^{(n+1)}(R)}{(n+1)!}\varphi_1(E'), \qquad E_0<E<E_n$$

Since E' is arbitrary therefore on dropping the prime on E' we got:

$$f(E)-P_n(E)=\frac{f^{(n+1)}(R)}{(n+1)!}\varphi_1(E), \quad E_0<E<E_n \tag{65}$$

Now we use Taylor's theorem [17] [18]:

$$f(E+h)=f(R)+hf'(R)+\frac{h^2}{2!}f''(R)+.........+\frac{h^n}{n!}f^n(R)+..... \tag{66}$$

Neglecting the terms containing second and higher powers of h in equation (66), we got:

$$f(R+h)=f(R)+hf'(R)$$

Or
$$f'(R)=\frac{f(R+h)-f(R)}{h} \tag{67}$$

Or
$$f'(R)=\frac{1}{h}\Delta f(R) \qquad [\therefore \Delta f(E+h)f(E)]$$

$$Df(R)=\frac{1}{h}\Delta f(R) \qquad [\therefore D=\frac{d}{dR}]$$

$$D=\frac{1}{h}\Delta \qquad \text{[Because } f(R) \text{ is arbitrary]}$$

$$\therefore D^{n+1}=\frac{1}{h^{n+1}}\Delta^{n+1}$$

From equation (67), we got:

$$f^{(n+1)}(R)=\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)$$

Putting the values of $f^{(n+1)}(R)$ in equation (65), we got

$$f(E)-P_n(E)=\left[\frac{\varphi_1(E)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)\right]$$

$$f(E)-P_n(E)=\left[\frac{(E-E_0)(E-E_1)(E-E_2)................(E-E_0)}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)\right] \qquad (68)$$

If $\frac{E-E_n}{h}=e$

Then

$$E-E_n=he$$
$$E-E_{n-1}=E-(E_n-h)=(E-E_n)+h=(he+h)=h(e+1)$$

Similarly $E-E_{n-2}=h(e+2)$

:

:

:

Similarly $E-E_0=h(e+n)$

Putting these values in equation (68), we got:

$$f(E)-P_n(E)=\left[\frac{(he)\{h(e-1)\}\{h(e-2)\}\{h(e-3)\}.............\{(e-n)\}}{(n+1)!}\right]\left[\frac{1}{h^{(n+1)}}\Delta^{(n+1)}f(R)\right]$$

$$f(E)-P_n(E)=\left[\frac{e(e-1)(e-2)(e-3).........(e-n)}{(n+1)!}\right]\left[\Delta^{(n+1)}f(R)\right]$$

This is mathematical expression for estimation of error, if the point lies in the lower half.

4.3 IF INTERVALS ARE NOT BE EQUALLY SPACED

In this condition we used following formula [1]:

$$\mathrm{V(E)}=\mathrm{V}_1+\Delta_{\mathrm{d}}\mathrm{V}_1(\mathrm{E}-E_1)+\Delta_{\mathrm{d}}^2\mathrm{V}_1(\mathrm{E}-E_1)(\mathrm{E}-E_2)+.....+\Delta_{\mathrm{d}}^{\mathrm{n}}\mathrm{V}_1[(\mathrm{E}-E_1)(\mathrm{E}-E_2)...(\mathrm{E}-E_{\mathrm{n}})]$$

[Where Δ_d is divide difference operator]

4.3.1 ESTIMATION OF ERROR

Let $f(E)$ be a real-valued function define n interval and $(n+1)$ times differentiable on (a,b). If $P_n(E)$ is the polynomial. Which interpolates $f(E)$ at the $(n+1)$ distinct points $E_0, E_1 ..... E_n \in (a,b)$, then for all $\bar{T} \in [a,b]$, there exists $\xi = \xi\left(\bar{E}\right) \in (a,b)$

$$e_n\left(\bar{E}\right) = f\left(\bar{E}\right) - P_n\left(\bar{E}\right)$$
$$= \frac{f^{(n+1)}(\xi)}{(n+1)} \prod_{j=0}^{n}\left(\bar{E} - E_j\right) \tag{69}$$

This is mathematical expression for estimation of error, if intervals are not be equally spaced.

4.4 WHEN THE TABULATED VALUES OF THE FUNCTION ARE NOT EQUIDISTANT

In this condition we used following formula [2]:

$$\mathrm{V(E)} = \sum_{i=1}^{n} \mathrm{V}_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{(\mathrm{E} - \mathrm{E}_j)}{(\mathrm{E}_i - \mathrm{E}_j)}$$

4.4.1 ESTIMATION OF ERROR

Since the approximating polynomial $f(E)$ given by Lagrangian formula has the same values $f(E_0)$ $f(E_1)$ $f(E_2)$ $f(E_3)$ $f(E_4)$ .... $f(E_n)$ as does $V = f(E)$ for the arguments E_0, E_1, E_2, E_3, E_4 ...., E_n the error term must have zeros at these $(n+1)$ points.

Therefore $(E - E_0)$ $(E - E_1)$ $(E - E_2)$ $(E - E_3)$ .... $(E - E_n)$ must be factors of the error and we can write:

$$F(E) = f(E) + \frac{(E - E_0)(E - E_1)(E - E_2)(E - E_3) .... (E - E_n)}{(n+1)!} K(E) \tag{70}$$

Let E to be fixed in value and consider the function

$$W(x) = F(x) - f(x) \frac{(x - E_0)(x - E_1)(x - E_2)(x - E_3) .... (x - E_n)}{(n+1)!} K(E) \tag{71}$$

Then $W(x)$ has zero $x = E_0, E_1, E_2, E_3 .... E_n$ and E.

Since the $(n+1)^{th}$ derivative of the n^{th} degree polynomial $f(E)$ is zero.

$$W^{(n+1)}(x) = F^{(n+1)}(x) - K(E) \tag{72}$$

As a consequence of Rolle's Theorem [15] [16] the $(n+1)^{th}$ derivative of $W(x)$ has at least one real zero $x = \xi$ in the range $E_0 < \xi < E_n$

Therefore substituting $x = \xi$ in equation (73):

$$W^{(n+1)}(\xi) = F^{(n+1)}(\xi) - K(E)$$

Or

$$K(E) = F^{(n+1)}(\xi) - W^{(n+1)}(\xi)$$
$$= F^{(n+1)}(\xi)$$

Using this expression for $K(S)$ and writing out $f(S)$

$$f(E) = \frac{(E-E_1)(E-E_2)............(E-E_n)}{(E_0-E_1)(E_0-E_2)............(E_0-E_n)} f(E_0) + \frac{(E-E_0)(E-E_2)............(E-E_n)}{(E_1-E_0)(E_1-E_2)............(E_1-E_n)} f(E_1) + ...$$
$$....... + \frac{(E-E_0)(E-E_1)............(E-E_{n-1})}{(E_n-E_0)(E_n-E_1)............(E_n-E_{n-1})} f(E_n) + \frac{(E-E_0)(E-E_1)............(E-E_n)}{(n+1)!} f^{(n+1)}(\xi)$$

Where $E_0 < \xi < E_n$

This is mathematical expression for estimation of error, if the tabulated values of the function are not equidistant.

5 CONCLUSION

Derived mathematical expressions are useful to estimation of the errors in the formulas for obtaining intermediate values of the rate of enzymatic reaction. All expressions are worked in n limit which is the optimum limit. When we obtain the intermediate values of the rate of enzymatic reaction then these mathematical expressions are useful to estimate the errors in interpolated values of the rate of enzymatic reaction.

REFERENCES

[1] Uddin, N., “Interpolate the Rate of Enzymatic Reaction: Temperature, Substrate Concentration and Enzyme Concentration based Formulas using Newton’s Method,” International Journal of Research in Biochemistry and Biophysics, 2(2), 5-9, 2012.

[2] Uddin, N., “Enzyme Concentration, Substrate Concentration and Temperature based Formulas for obtaining intermediate values of the rate of enzymatic reaction using Lagrangian polynomial,” International Journal of Basic And Applied Sciences, 1(3), 299-302, 2012.

[3] Meijering, Erik (2002), “A chronology of interpolation: from ancient astronomy to modern signal and image processing,” Proceedings of the IEEE 90 (3): 319–342.

[4] Abramowitz, M. and Stegun, I. A. (Eds.). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 9th printing. New York: Dover, p. 880, 1972.

[5] Hildebrand, F. B., “Introduction to Numerical Analysis,” New York: McGraw-Hill, pp. 43-44 and 62-63, 1956.

[6] Whittaker, E. T. and Robinson, G., “Newton's Formula for Unequal Intervals,” §13 in The Calculus of Observations: A Treatise on Numerical Mathematics, 4th ed. New York: Dover, pp. 24-26, 1967.

[7] Abramowitz, M. and Stegun, I. A. (Eds.). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 9th printing. New York: Dover, p. 880, 1972.

[8] Beyer, W. H., “CRC Standard Mathematical Tables,” 28th ed. Boca Raton, FL: CRC Press, p. 432, 1987.

[9] Graham, R. L., Knuth, D. E., and Patashnik, O., “Concrete Mathematics: A Foundation for Computer Science,” 2nd ed. Reading, MA: Addison-Wesley, 1994.

[10] Jordan, C., “Calculus of Finite Differences,” 3rd ed. New York: Chelsea, 1965.

[11] Nörlund, N. E., “Vorlesungen über Differenzenrechnung,” New York: Chelsea, 1954.

[12] Riordan, J., “An Introduction to Combinatorial Analysis,” New York: Wiley, 1980.

[13] Whittaker, E. T. and Robinson, G., “The Gregory-Newton Formula of Interpolation” and “An Alternative Form of the Gregory-Newton Formula,” §8-9 in The Calculus of Observations: A Treatise on Numerical Mathematics, 4th ed. New York: Dover, pp. 10-15, 1967.

[14] Beyer, W. H., “CRC Standard Mathematical Tables,” 28th ed. Boca Raton, FL: CRC Press, p. 433, 1987.

[15] Anton, H., “Rolle's Theorem; Mean Value Theorem,” §4.9 in Calculus: A New Horizon, 6th ed. New York: Wiley, pp. 260-266, 1999.

[16] Apostol, T. M. Calculus, “One-Variable Calculus, with an Introduction to Linear Algebra,” Waltham, MA: Blaisdell, 2nd ed., Vol. 1, p. 184, 1967.

[17] Dehn, M. and Hellinger, D., “Certain Mathematical Achievements of James Gregory,” Amer. Math. Monthly 50, 149-163, 1943.

[18] Jeffreys, H. and Jeffreys, B. S. “Taylor's Theorem,” §1.133 in Methods of Mathematical Physics, 3rd ed. Cambridge, England: Cambridge University Press, pp. 50-51, 1988.