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Evaluating the effect of crystal field and transverse field on a 3/2 spin system using the Ising model

M. Zoubir and A. El Hajbi

Laboratory of Physical Chemistry,
Department of Chemistry, Faculty of Science,
Chouaib Doukkali University,
BP 20, 24000 El Jadida, Morocco

ABSTRACT: In the work reported here we looked at the magnetic behaviour of a 3/2 spin system with crystalline anisotropy and transverse field. The spin Hamiltonian considered here contains terms which cannot be commuted further, giving rise to important quantum effects: quantum fluctuations and thermal fluctuations. We used the mean field approximation method to determine the expressions of magnetization along the z (longitudinal) axis and the x (transverse) axis, as well as the partition function. The thermal variations mz and mx for different values of the transverse field show that the latter inhibits order along the z axis and encourages spin confinement in the transverse plane.

KEYWORDS: crystal anisotropy, Ising model, magnetization, spin system, transverse field.

1 INTRODUCTION

Solid state chemists and physicists have long been interested in exchange interactions between transition ions within materials, because of their significance at a fundamental level. When these interactions affect all the magnetic sites of a crystal lattice, quantum models have to be used and numerous approximations are necessary. These quantum models have given theoretical results which are in good agreement with experimental results.

Application of altered spin chain models to αFe_2PO_5 compounds (ignoring interchain reactions) has shown that the behavior of this compound is ferromagnetic [1]. We have previously used the dimer and chain models to determine the magnetic sensitivity, specific heat and entropy of the compound $MnFePO_5$ [2]. The Fisher, dimer and Ising models have been used to determine the magnetic behaviour and certain thermodynamic properties of low dimensionality systems [3] [4]. Several authors have studied the critical behavior of the Ising Hamiltonian in the presence of the transverse field exactly in one dimension [5]. This model shows a quantum transition at T=0 when the transverse field tends towards a specific value (critical value $\Omega=\Omega c$). This transition is due essentially to quantum fluctuations [6]. This Hamiltonian has also been studied at higher dimensions using various approximation methods, e.g. development in series, development in $1/p$ where p is the coordination number [7], effective field methods [8].

In the work presented here, we have used the Ising model to study the effects of crystalline anisotropy and of the transverse field on the magnetic properties of a quantum system of spin 3/2.

2 EFFECT OF CRYSTAL FIELD AND TRANSVERSE FIELD ON A SYSTEM OF SPIN 3/2

2.1 SPIN HAMILTONIAN

The Hamiltonian of a given spin system, under the action of the transverse field and in the presence of the crystal field, is given by the following equation:

$$H = -\sum_{\langle ij\rangle} J_{ij} S_i^z S_i^z - \Omega \sum S_i^x - D \sum_i (S_i^z)^2 \quad (1)$$

where Ω is the transverse field, J_{ij} the exchange coupling constant, which is equal to J for the first near neighbours and to 0 for non-neighbours, and D is the crystalline anisotropy constant.

2.2 Partition function

Let S_i^z and S_i^x be the matrices of the system of spin S = 3/2. They are written in the eigenvector base S_i^z as:

$$S_i^z = \begin{pmatrix} 3/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & -3/2 \end{pmatrix} \quad (2)$$

$$S_i^x = \begin{pmatrix} 0 & \sqrt{3/2} & 0 & 0 \\ \sqrt{3/2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \sqrt{3/2} \\ 0 & 0 & \sqrt{3/2} & 0 \end{pmatrix} \quad (3)$$

$$(S_i^z)^2 = \begin{pmatrix} 9/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 9/4 \end{pmatrix} \quad (4)$$

Using the mean field approximation method, the Hamiltonian (1) is reduced for a given spin system to the following equation:

$$-\beta H_1 = \lambda' S_1^z + \Gamma S_1^z + \Delta (S_1^z)^2 \quad (5)$$

Replacing the matrices S_i^x, S_j^z and $\left(S_j^z\right)^2$ in equation (5), the Hamiltonian (-βH1) can be written out as the following matrix:

$$-\beta H_1 = \begin{pmatrix} \left(\frac{3\lambda'}{2} + \frac{9\Delta}{4}\right) & \frac{\sqrt{3}\Gamma}{2} & 0 & 0 \\ \frac{\sqrt{3}\Gamma}{2} & \left(\frac{\lambda'}{2} + \frac{\Delta}{4}\right) & \Gamma & 0 \\ 0 & \Gamma & \left(-\frac{\lambda'}{2} + \frac{\Delta}{4}\right) & \frac{\sqrt{3}\Gamma}{2} \\ 0 & 0 & \frac{\sqrt{3}\Gamma}{2} & \left(-\frac{3\lambda'}{2} + \frac{9\Delta}{4}\right) \end{pmatrix} \quad (6)$$

Let $P_{3/2}$ be the characteristic polynomial of the matrix (6):

$$P_{3/2} = \det((-\beta H_1) - xI_4) = x^4 - Px^3 + Qx^2 + Rx + S \quad (7)$$

This polynomial is of degree 4 and allows four roots as solutions (x_1, x_2, x_3 and x_4). These roots are the eigenvalues of matrix (6) and the parameters P, Q, R and S of the polynomial $P_{3/2}$ are given by the following:

$$P = 5\Delta$$

$$Q = \frac{59}{8}\Delta^2 - \frac{5}{2}\lambda'^2 - \frac{5}{2}\Gamma^2$$

$$R = \frac{33}{3}\Delta\Gamma^2 - \frac{45}{16}\Delta^3 + \frac{9}{4}\Delta\lambda'^2$$

$$S = \frac{9}{8}\left(\frac{9}{32}\Delta^4 + \frac{1}{2}\lambda'^4 + \frac{1}{2}\Gamma^4 - \frac{21}{1}\Delta^2\Gamma^2 - \frac{5}{4}\Delta^2\lambda'^2 + \Gamma^2\lambda'^2\right)$$

To find the eigenvalues of matrix (6), we need to solve the equation $P_{3/2} = 0$, i.e.:

$$x^4 - Px^3 + Qx^2 + Rx + S = 0 \tag{8}$$

The variable $x = Y + P/4$ is exchanged in order to eliminate the order 3 term. Equation (8) then becomes:

$$Y^4 + aY^2 + bY + c = 0 \tag{9}$$

where:

$$a = Q^2 - \frac{3}{8}p^2$$

$$b = R + \frac{1}{2}QP - \frac{1}{8}P^3$$

$$c = \frac{1}{4}PR + \frac{1}{16}QP^2 - \frac{3}{256}P^4 + S$$

Equation (9) can also be written as follows:

$$(Y^2 + tY + \mu)(Y^2 - tY + \eta) = 0 \tag{10}$$

where *t*, *μ* and *η* are coefficients other than zero.

Equation (10) is equal to zero, which implies that:

$$Y^2 + tY + \mu = 0 \tag{11}$$

where

$$Y^2 - tY + \eta = 0 \tag{12}$$

Equations (11) and (12) both admit two square roots as solution, and these can be written as:

$$Y_{1,2} = \frac{1}{2}t \pm \frac{1}{2}\sqrt{t^2 - 4\mu} \tag{13}$$

and:

$$Y_{3,4} = \frac{1}{2}t \pm \frac{1}{2}\sqrt{t^2 - 4\eta} \tag{14}$$

On the basis of equations (13) and (14), we can deduce the four square roots (x_1, x_2, x_3 and x_4) of equation (7).

$$x_1 = \frac{P}{4} - \frac{2t}{4} + \frac{1}{4}\sqrt{4t^2 - 16\mu} \tag{15}$$

$$x_2 = \frac{P}{4} - \frac{2t}{4} - \frac{1}{4}\sqrt{4t^2 - 16\mu} \tag{16}$$

$$x_3 = \frac{P}{4} + \frac{2t}{4} + \frac{1}{4}\sqrt{4t^2 - 16\eta} \tag{17}$$

$$x_4 = \frac{P}{4} + \frac{2t}{4} - \frac{1}{4}\sqrt{4t^2 - 16\eta} \tag{18}$$

We need to find *t*, *η* and *μ* in order to determine x_1, x_2, x_3 and x_4.

$$\mu = \frac{1}{2}\left(a + t^2 - \frac{b}{t}\right) \tag{19}$$

Combining equations (9) and (10) we find:

$$\eta = \frac{1}{2}\left(a + t^2 + \frac{b}{t}\right) \tag{20}$$

$$\eta\mu = c \tag{21}$$

To calculate μ and η, we need to find t. Equation (21) is equivalent to:

$$t^6 + 2at^4 + (a^2 - 4c)t^2 - b^2 = 0 \tag{22}$$

In order to solve equation (22), we replace the variable with $t^2 = Z$, and equation (22) becomes:

$$Z^3 + AZ^2 + BZ + C = 0 \tag{23}$$

where:

$$A = 2a$$
$$B = a^2 - 4c$$
$$C = -b^2$$

When the variable is replaced with $X = Z + \frac{A}{3}\alpha\exp$, equation (23) becomes:

$$X^3 + p_0X + q_0 = 0 \tag{24}$$

where:

$$p_0 = B - \frac{A^2}{3}$$

$$q_0 = C - \frac{BA}{3} + \frac{2}{27}A^3$$

Let $\Delta_0 = \left(\frac{q_0}{2}\right)^2 + \left(\frac{p_0}{3}\right)^3$ be the discriminant of equation (24):

$$\begin{aligned}\Delta_0 = {} & \frac{16}{3}\lambda'^8\Gamma^2\Delta^2 - \frac{196}{3}\lambda'^6\Gamma^4\Delta^2 - \frac{392}{3}\lambda'^2\Gamma^6\Delta^4 - 72\lambda'^6\Gamma^6\Delta^2 - \frac{284}{3}\lambda'^4\Gamma^6\Delta^2 - 160\lambda'^4\Gamma^2\Delta^6 - \frac{256}{3}\lambda'^2\Gamma^2\Delta^8 - \frac{116}{3}\lambda'^2\Gamma^8\Delta^2 \\ & - \frac{500}{3}\lambda'^4\Gamma^4\Delta^4 - \frac{352}{3}\lambda'^2\Gamma^4\Delta^6 - \frac{64}{3}\lambda'^4\Delta^8 - 44\lambda'^8\Delta^8 - \frac{160}{3}\lambda'^6\Delta^6 + \frac{40}{3}\lambda'^{10}\Delta^2 - \frac{4}{3}\Gamma^8\Delta^4 - \frac{4}{3}\Gamma^{10}\Delta^2 \\ & - 8\Gamma^{10}\lambda'^2 - 20\lambda'^4\Gamma^8 - \frac{80}{3}\lambda'^6\Gamma^6 - 20\lambda'^8\Gamma^4 - 8\lambda'^{10}\Gamma^2 - \frac{4}{3}\Gamma^{12} - \frac{4}{3}\lambda'^{12}\end{aligned}$$

Whatever the values of λ', Γ and Δ, the sign of Δ_0 is negative, which means that the solutions of equation (24) are real, and are expressed as:

$$X = \alpha'\cos(\theta) \tag{25}$$

Parameters α' and ϑ are given by the following:

$$\alpha' = 2\sqrt{-\frac{p_0}{3}}$$

$$\theta = \frac{1}{3}\arccos\left(\frac{q_0}{2\sqrt{\left(-\frac{p_0}{3}\right)^3}}\right) = \frac{1}{3}\arccos(\psi)$$

where:

$$\psi = \frac{q_0}{2\sqrt{\left(-\frac{P_0}{3}\right)^3}} \tag{26}$$

The parameter *t* is given by:

$$t = \sqrt{Z} = \sqrt{X - \frac{A}{3}} = \sqrt{\alpha' \cos(\theta) - \frac{A}{3}} \tag{27}$$

Using equations (19) and (20) the eigenvalues (x_1, x_2, x_3 and x_4) of matrix (6) become:

$$x_{1,2} = \frac{P}{4} - \frac{2t}{4} \pm \frac{1}{4}\sqrt{-4t^2 - 8a + \frac{16}{2t}b} \tag{28}$$

and:

$$x_{3,4} = \frac{P}{4} + \frac{2t}{4} \pm \frac{1}{4}\sqrt{-4t^2 - 8a - \frac{16}{2t}b} \tag{29}$$

When *t*, *a* and *b* are replaced by their expressions, equations (28) and (29) become:

$$x_{1,2} = \frac{P}{4} - \frac{W}{4} \pm \frac{1}{4}\sqrt{\frac{(W-P)^2 W - 16(W-P)y - 16R}{W}} \tag{30}$$

$$x_{3,4} = \frac{P}{4} + \frac{W}{4} \pm \frac{1}{4}\sqrt{\frac{(W-P)^2 W - 16(W-P)y + 16R}{W}} \tag{31}$$

where *W* is given by:

$$W = 2t$$

$$= \sqrt{\frac{8}{3}\sqrt{3}(Q^2 + 3RP + 12S)^{1/2}\cos(\theta) - \frac{8}{3}Q + P^2} = \sqrt{8\left(\frac{\sqrt{3}}{3}(Q^2 + 3RP + 12S)^{1/2}\cos(\theta) + \frac{1}{6}Q\right) - 4Q + P^2}$$

or:

$$W = \sqrt{8y - 4Q + P^2} \tag{32}$$

and *y* and *r* by:

$$y = r\cos(\theta) + \frac{1}{6}Q$$

$$r = \left(\frac{Q^2 + 3RP + 12S}{3}\right)^{1/2} \tag{33}$$

Let:

$$\alpha = \frac{P-W}{4}$$

$$\delta = \frac{P+W}{4}$$

$$v = \frac{1}{4}\sqrt{\frac{(W-P)^2 W - 16(W-P)y + 16R}{W}}$$

and

$$\gamma = \frac{1}{4}\sqrt{\frac{(W-P)^2W-16(W+P)y-16R}{W}}$$

The eigenvalues x_i (i =1 to 4) of matrix (6) become:

$$x_{1,2} = \alpha \pm v \tag{34}$$

$$x_{3,4} = \delta \pm \gamma \tag{35}$$

We can now determine the expression of the partition function $Z_{3/2}$:

$$Z_{3/2} = Tr\,\exp(-\beta H_1) = e^{x_1} + e^{x_2} + e^{x_3} + e^{x_4}$$

Finally, when equations (31) and (32) are taken into account the partition function $Z_{3/2}$ becomes:

$$Z_{3/2} = 2\,\exp(\alpha)\cosh(v) + 2\,\exp(\delta)\cosh(\gamma) \tag{36}$$

2.3 MAGNETIZATION

2.3.1 MAGNETIZATION ALONG THE Z AXIS

According to equation (4), longitudinal magnetization m_z is given by the following expression:

$$m_z = \langle S_i^z \rangle = \frac{TrS_i^z\exp(-\beta H_1)}{Z_{3/2}} = \frac{\partial \ln(\mathrm{Tr}e^{-\beta H_1})}{\partial \lambda\prime} = \frac{\partial \ln(Z_{3/2})}{\partial \lambda\prime} \tag{37}$$

After calculation the final expression of magnetization is given by:

$$m_Z = \frac{1}{\exp(\alpha)\cosh(v)+\exp(\delta)\cosh(\gamma)}[G_{11}\exp(\alpha)\cosh(v) + G_{12}\exp(\alpha)\sinh(v) + G_{11}\exp(\alpha)\cosh(v) + G_{13}\exp(\delta)\cosh(\gamma) + G_{14}\exp(\delta)\sinh(\gamma)] \tag{38}$$

where G_{11}, G_{12}, G_{13} and G_{14} are given by the following expressions.

- For G_{11} we have:

$$G_{11} = \frac{\partial \alpha}{\partial \lambda\prime} = \frac{\partial(P-W)}{4\partial \lambda\prime} = -\frac{\partial W}{4\partial \lambda\prime}$$

(since P is not dependent on λ').

Applying equation (32), we obtain:

$$\frac{\partial W}{\partial \lambda\prime} = \left[\frac{1}{2}(8y + P^2 - 4Q)^{-1/2}\ \frac{\partial(8y+P^2-4Q)}{\partial \lambda\prime}\right] = \frac{4\partial y}{W\partial \lambda\prime} - \frac{2\partial Q}{W\partial \lambda\prime}$$

Similarly:

$$\frac{\partial Q}{\partial \lambda\prime} = -5\lambda'$$

Thus the final expression of $G_{11}$is:

$$G_{11} = -\frac{1}{4}\left[\frac{4\partial y}{W\partial\lambda\prime} + \frac{10}{W}\lambda'\right]$$

- For G_{12} we have:

$$G_{12} = \frac{\partial y}{\partial\lambda\prime} = \frac{1}{32v}\left[2(W-P)\frac{\partial W}{\partial\lambda\prime} - 16\frac{\partial y}{\partial\lambda\prime} + \frac{16P\partial y}{W\partial\lambda\prime} + \frac{16\partial R}{W\partial\lambda\prime} - \frac{16R\partial W}{W^2\partial\lambda\prime} - \frac{16Py\partial W}{W^2\partial\lambda\prime}\right]$$

Also:

$$\alpha = \frac{P-W}{4} \rightarrow P - W = 4\alpha$$

$$\frac{\partial y}{\partial\lambda\prime} = \frac{9}{2}\Delta\lambda'$$

and

$$\frac{\partial W}{\partial\lambda\prime} = \frac{4\partial y}{W\partial\lambda\prime} + \frac{10}{W}\lambda'$$

So the final expression of G_{12} is:

$$G_{12} = \frac{1}{4vW^3}\left\{4[\alpha W^2 - 2(R+Py)]\frac{\vartheta y}{\vartheta\lambda'} - 10[\alpha W^2 + 2(R+Py)\lambda' + 9\Delta W^2\lambda']\right\} = \frac{1}{4W^3v}\left[4(M-2N)\frac{\vartheta y}{\vartheta\lambda\prime} - 10(M-2N)\lambda' + P\Delta W^2\lambda'\right] \quad (39)$$

where:

$$M = \alpha W^2$$

and $$N = R + Py$$

- For G_{13} we have:

$$G_{13} = \frac{\partial y}{\partial\lambda'} = \frac{\partial(P+W)}{4\partial\lambda'} = \frac{\partial W}{4\partial\lambda'}$$

The final expression of G_{13} is:

$$G_{13} = \frac{1}{4}\left[\frac{4}{W}\frac{\partial y}{\partial\lambda'} + \frac{10}{W}\lambda'\right] \quad (40)$$

- Finally, for G_{14} we have:

$$G_{14} = \frac{\partial y}{\partial\lambda'} = \frac{1}{4\gamma W^3}\left\{4[\delta W^2 - 2(R+Py)]\frac{\partial y}{\partial\lambda'} + 10[\delta W^2 + 2(R+Py)]\lambda' - 9\Delta W^2\lambda'\right\}$$

The final expression of G_{14} is:

$$G_{14} = \frac{1}{4\lambda W^3}\left[4(M-2N)\frac{\partial y}{\partial\lambda'} + 10(M+2N)\lambda' - 9W^2\lambda'\right] \quad (41)$$

Since:

$$\frac{\partial y}{\partial \lambda'} = r_{11}\cos(\theta) + r r_{12} - \frac{5}{6}\lambda' \tag{42}$$

with:

$$r_{11} = \frac{\partial y}{\partial \lambda'} = \frac{52{\lambda'}^3 + 52\lambda'\Gamma^2 - 40\lambda'\Delta^2}{6r}$$

and

$$r_{12} = \frac{\partial \cos(\theta)}{\partial \lambda'} = \theta_{11}\sin(\theta)$$

it follows that θ_{11} is given by:

$$\theta_{11} = -\frac{\frac{\partial \Psi}{\partial \lambda'}}{3\sqrt{1-\Psi^2}} = \frac{3\sqrt{3}}{2}\frac{\frac{\frac{3q_0 p_{11}}{2} + p_0 q_{11}}{(-p_0)^{5/2}}}{3\sqrt{1-\Psi^2}}$$

where:

$$q_{11} = \frac{\partial q_0}{\partial \lambda'} = \frac{160}{9}\lambda'\Delta^4 + \frac{16}{9}\lambda^3\Delta^2 + \frac{520}{9}\lambda'\Delta^2\Gamma^2 - \frac{140}{9}\lambda^3\Gamma^2 - \frac{140}{9}\lambda'\Gamma^4$$

$$p_{11} = \frac{\partial(-p_0)}{\partial \lambda'} = \frac{40}{3}\lambda'\Delta^2 + \frac{52}{3}{\lambda'}^3 + \frac{52}{3}\lambda'\Gamma^2$$

And

$$\Psi^2 = \frac{27q_0^2}{4(-p_0)^3}$$

By solving equation (38) numerically we can trace the variation in longitudinal magnetization m_z as a function of temperature for different values of the transverse field (Ω/J) and the crystal field (D/J).

Figure 1 shows thermal variation of longitudinal magnetization for a fixed value of the crystal field (D/J=0.1) and with arbitrary values of the transverse field (Ω/J = 1, 2 and 3). We see that as the value of the transverse field (Ω/J) increases, longitudinal magnetization declines and the system moves to a lower critical temperature, showing the role of the transverse field in confining spins within the x,y plane. Figure 2 shows the thermal variation of longitudinal magnetization of the same system with a fixed value of the transverse field ($\Omega/J = 3$) and for arbitrary values of crystalline anisotropy (D/J=1, 2 and 3). Here, as the value (D/J) increases the transition temperature of the system also increases.

2.3.2 MAGNETIZATION ALONG THE X AXIS

Transverse magnetization is calculated in the same way as longitudinal magnetization. Using equation (5) magnetization m_x is expressed as:

$$m_x = \langle S_i^x \rangle = \frac{Tr S_i^x \exp(\beta H_1)}{Z_{3/2}} = \frac{\partial \ln(Tr e^{-\beta H_1})}{\partial \Gamma} = \frac{\partial \ln(Z_{3/2})}{\partial \Gamma} \tag{43}$$

The final expression of m_x is:

$$m_x = \frac{1}{\exp(\alpha)\cosh(v) + \exp(\delta)\cosh(\gamma)}[G_{21}\exp(\alpha)\cosh(v) + G_{22}\exp(\alpha)\sinh(v) + G_{23}\exp(\delta)\cosh(\gamma) + G_{24}\exp(\delta)\sinh(\gamma)] \tag{44}$$

where G_{21}, G_{22}, G_{23} and G_{24} are given by the following equations:

For G_{21}:

$$G_{21} = \frac{\partial \alpha}{\partial \Gamma} = \frac{\partial (p-W)}{4\partial \Gamma} = \frac{\partial w}{4\partial \Gamma}$$

since p does not depend on Γ.

$$G_{21} = -\frac{1}{4}\left[\frac{4\partial y}{W\partial \Gamma} + \frac{10}{W}\Gamma\right] \tag{45}$$

For G_{22}:

$$G_{22} = \frac{\partial v}{\partial \Gamma} = \frac{1}{32v}\left[2(W-P)\frac{\partial W}{\partial \Gamma} - \frac{16\partial y}{\partial \Gamma} + \frac{16p\partial y}{W\partial \Gamma} - \frac{16py\partial R}{W^2\partial \Gamma} + \frac{16\partial R}{W\partial \Gamma} - \frac{16R\partial W}{W^2\partial \Gamma}\right]$$

Replacing $(W-P)$, $\frac{\partial R}{\partial \Gamma}$ and $\frac{\partial W}{\partial \Gamma}$ by their expression we obtain the final expression of

G_{22}:

$$\begin{aligned} G_{22} &= \frac{1}{4vw^3}\left\{4[\alpha W^2 - 2(Py+R)]\frac{\partial y}{\partial \Gamma} - 10[\alpha W^2 + 2(Py+R)]\Gamma + 33\Delta W^2\Gamma\right\} \\ &= \frac{1}{4Wv}\left[4(M-2N)\frac{\partial y}{\partial \Gamma} - 10(M+2N)\Gamma + 33\Delta\Gamma W^2\right] \end{aligned} \tag{46}$$

For G_{23}:

$$G_{23} = \frac{\partial \delta}{\partial \Gamma} = \frac{1}{4}\frac{\partial (p+W)}{\partial \Gamma} = \frac{1}{4}\frac{\partial W}{\partial \Gamma} \tag{47}$$

The final expression of $G_{23}$is given by:

$$G_{23} = \frac{1}{4}\left[\frac{4}{W}\frac{\partial Y}{\partial \Gamma} + \frac{10}{W}\Gamma\right]$$

For G_{24}:

$$\begin{aligned} G_{24} &= \frac{\partial y}{\partial \Gamma} = \frac{1}{4\gamma W^3}\left\{-4[\delta W^2 - 2(R+Py)]\frac{\partial y}{\partial \Gamma} + 10[\delta W^2 + 2(R+Py)]\Gamma - 33\Delta\Gamma \mathrm{W}^2\right\} \\ &= \frac{1}{4\gamma W^3}\left[-4(M'-2N)\frac{\partial y}{\partial \Gamma} + 10(M'+2N)\Gamma - 33\Delta W^2\Gamma\right] \end{aligned} \tag{48}$$

where:

$$M' = \delta W^2 \tag{49}$$

By solving equation (44) numerically we can trace the variation in transverse magnetization m_x as a function of temperature, for various values of the crystal field and the transverse field.

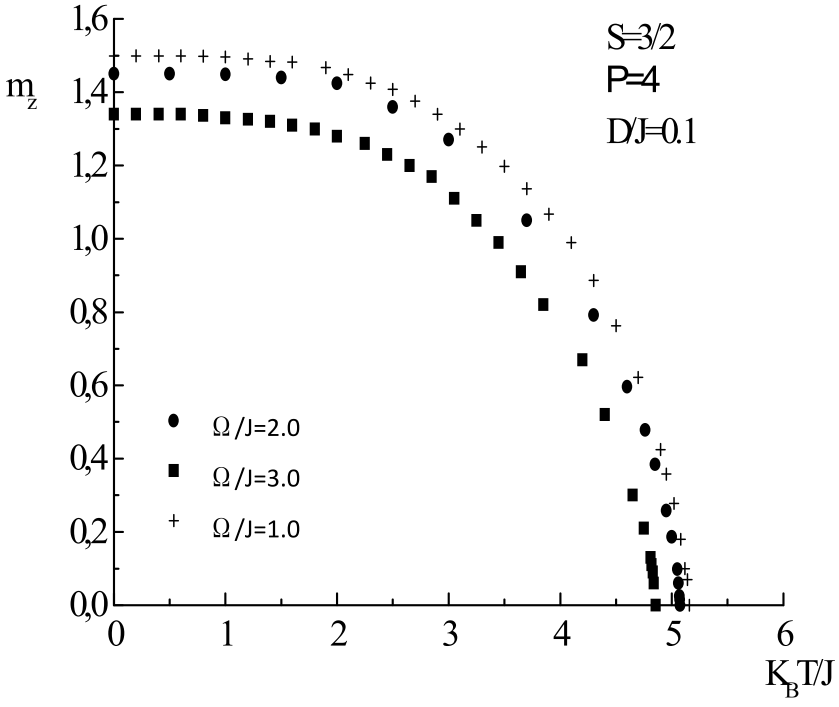

Fig. 1. Thermal variation of longitudinal magnetization of a 3/2 spin system for a fixed value of crystalline anisotropy and various values of the transverse field

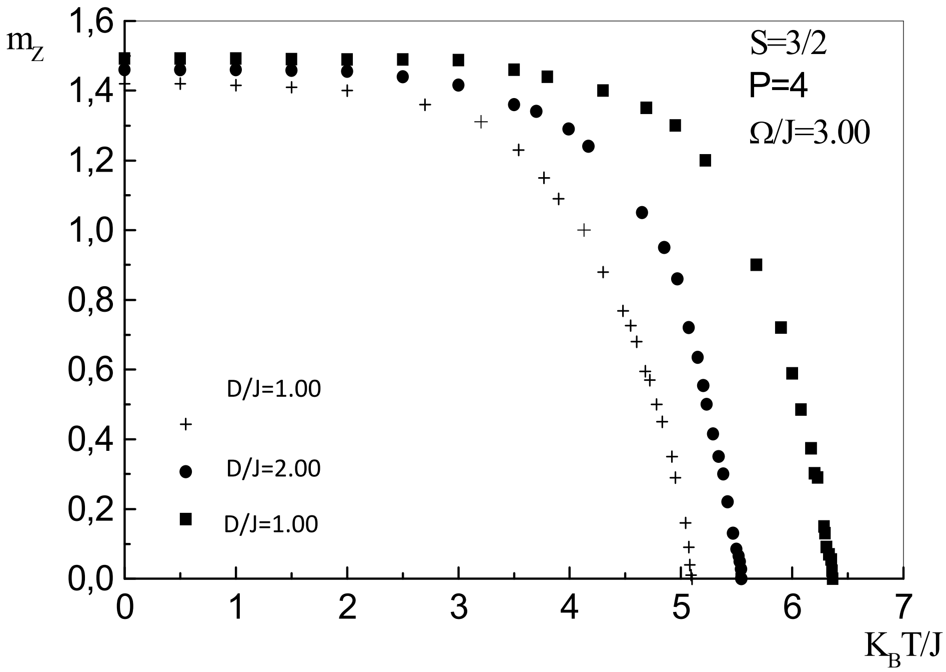

Fig. 2. Thermal variation of longitudinal magnetization of a 3/2 spin system for a fixed value of the transverse field and various values of crystalline anisotropy

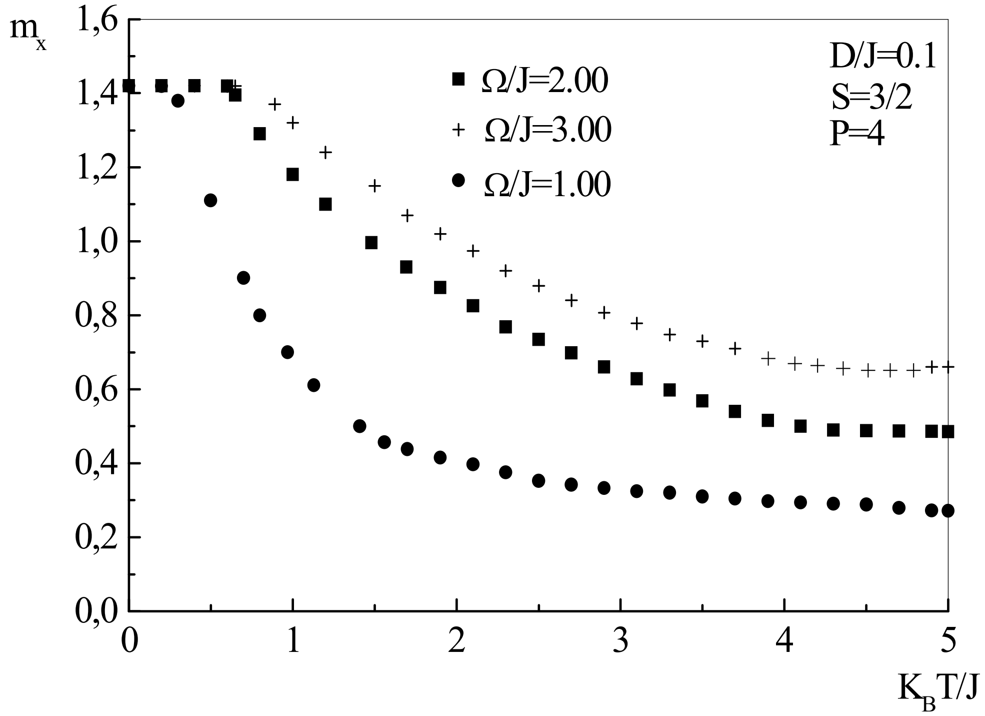

Fig. 3. Thermal variation of transverse magnetization of a 3/2 spin system for a fixed value of crystalline anisotropy and various values of the transverse field.

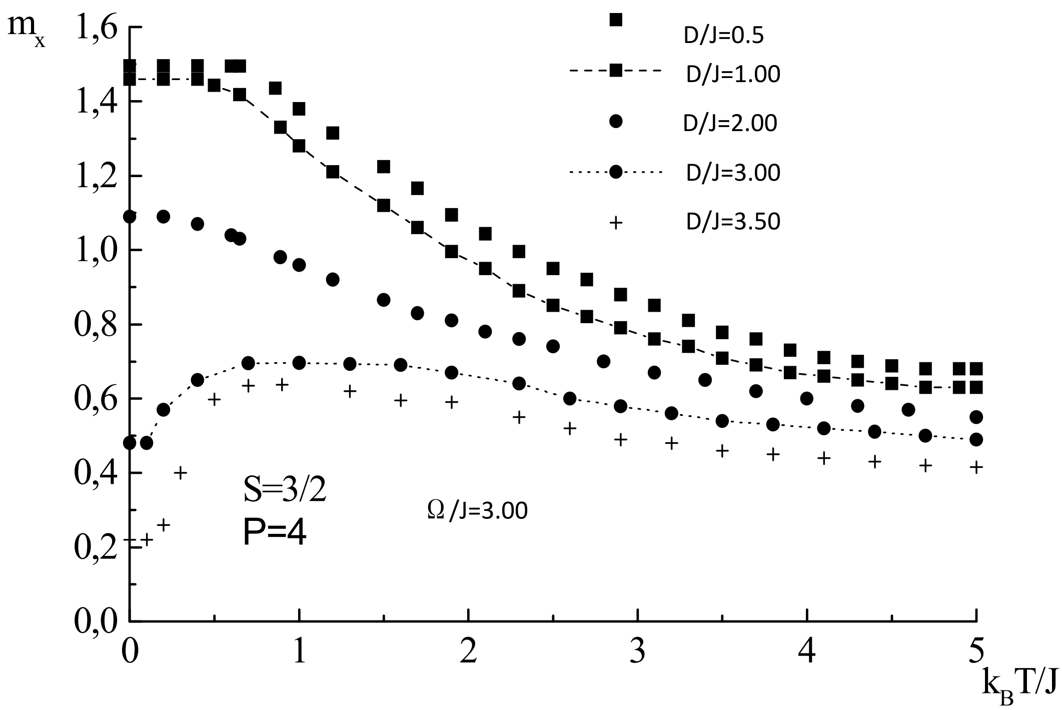

Fig. 4. Thermal variation of transverse magnetization of a 3/2 spin system for a fixed value of the transverse field and various values of crystalline anisotropy.

Figure 3 shows the thermal evolution of transverse magnetization of the 3/2 spin system, for a fixed value of the constant of crystalline anisotropy $(D/J = 0.1)$ and with arbitrary values of the transverse field ($\Omega/J = 1$; 2 and 3). No transition is seen and transverse magnetization continues to decline asymptotically without reaching zero (within the temperature range examined). Figure 4 shows the thermal evolution of transverse magnetization for a fixed value of the transverse field ($\Omega/J = 3$) and arbitrary values for the constant of crystalline anisotropy ($D/J = 0.5$; 1; 2; 3 and 3.5). We see that for high values of crystalline anisotropy, magnetization declines at low temperatures. This behavior indicates competition between the direction of quantification (z axis) and the energy associated with the transverse field.

3 CONCLUSION

Study of longitudinal magnetization of a 3/2 spin system as a function of temperature shows that:

- With a constant crystal field and a transverse field with increasing variable values, magnetization decreases and the system transits at a critical temperature value.
- With a constant transverse field and a crystal field with increasing variable values, the transition temperature increases.

Study of transverse magnetization of the same 3/2 spin system as a function of temperature shows that:

- With a constant crystal field and a transverse field with increasing variable values, no transition is observed.
- With a constant transverse field and a crystal field with increasing variable values, magnetization decreases asymptotically.

REFERENCES

[1] E. Chemseddine and A. El Hajbi, "Application du modèle de chaînes de spins alternés sur le composé αFe_2PO_5," *Ann. Chim. Mat.*, vol. 24, pp 241-244, 1999.

[2] E. Chemseddine and A. El Hajbi, " Theoretical determination of the magnetic susceptibility, magnetisation, specific heat and entropy of $MnFePO_5$," *J. Phys. Chem. News*, vol. 1, pp. 119-123, 2001.

[3] M. Zoubir, F. Erragh, A. Boukhari and A. El Hajbi, "Étude de quelques propriétés magnétiques et thermodynamiques du composé $LiNi_2P_3O_{10}$ par application de modèles théoriques," *J. Phys. Chem. News*, vol. 16, pp. 121-127, 2004.

[4] M. Zoubir, F. Erragh, A. Boukhari and A. El Hajbi, " Study of certain magnetic and thermodynamic properties of low-dimensionality systems: the example of $LiCo_2P_3O_{10}$," *J. of Molecular Structure (Theochem)*, vol. 731, pp. 201-206, 2005.

[5] N. Benayad, A. Dakhama, A. Fathi and R Zerhouni, "The diluted mixed spin-1/2 and spin-1 Ising model in a transverse random field", *J. Phys.: Condens. Matter*, vol. 10, pp. 3141, 1998. DOI: 10.1088/0953-8984/10/14/005

[6] P. Pfeuty and R. J Elliot, "The Ising model with a transverse field. II. Ground state properties," *J. Phys. C: Solid State Phys.,* vol. 4 pp. 2370, 1971. DOI: 10.1088/0022-3719/4/15/024.

[7] F. C. Sa Barreto, I. P. Fittipaladi and B. Zeks, "New Effective Field Theory for the Transverse Ising Model," *Ferroelectrics*, vol. 39, pp 1103, 1981.

[8] R. M. Statt, "Path-integral methods for treating quantal behavior in solids: Mean-field theory and the effects of fluctuations," *Phys. Rev. B*, vol. 33, pp. 1921, 1986. DOI: http://dx.doi.org/10.1103/PhysRevB.33.1921.