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Empirical Tests of Capital Asset Pricing Model and its Testability for Validity Verses Invalidity

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ABSTRACT: This paper reviews an advanced literature on capital asset pricing model. It starts by a brief introduction in welcoming scholars into the model background and its relevant assumptions and implications. It then explains the model in its real form, both the conceptual and the analytical part of it. The CAPM and the Index Model is then clearly looked at and explained in the dimensions of the Index Model and Realized Returns and also the Index Model and the Expected Return-Beta Relationship. The researchers penultimately look at a number of empirical tests for CAPM to explain the validity of the model. Some of the Empirical tests looked at by this paper are the tests by Lintner, which is reproduced in Douglas (1968), Fama and MacBeth (1973), tests by Black, Jensen and Scholes (1972), tests by Stambaugh (1982), tests by Gibbons (1982), Miller and Scholes (1972) tests and the Roll (1977) Critique. The paper finds that, there is strong empirical evidence invalidating the CAPM and on the other hand it is clear that the empirical findings themselves are not sufficient to discard the CAPM. The paper found out further that CAPM cannot be used for estimating the cost of capital, to evaluate the performance of fund managers or as an aid in event-study analysis. For practical purposes, Merton's (1973) intertemporal CAPM or some form of the APT would have to be resorted to for the purpose of explaining expected stock returns.

KEYWORDS: Mean Return, testable, beta, CAPM, Expected stock returns.

1 INTRODUCTION

The CAPM is based on Markowitz's (1959) mean variance analysis. Markowitz demonstrated that rational investors would hold assets, which offer the highest possible return for a given level of risk, or conversely assets with the minimum level of risk for a specific level of return. Sharpe (1964) and Lintner (1965) build on Markowitz's work after making a number of assumptions. They developed an equilibrium model of exchange showing the return of each asset as a function of the return on the market portfolio. This model known as the Capital Asset Pricing Model has since been the focuses of a number of empirical tests, majority of them deny the validity of the model. This paper discusses some of the empirical tests of the CAPM.

1.1 THE CAPM AND ITS ASSUMPTIONS

Sharpe (1964) and Lintner (1965) assumed that there are no transaction costs and no income taxes. Further, they assumed that assets are infinitely divisible and there are no restrictions to short selling and that investors can lend and borrow unlimited amounts at the risk free rate of interest. More importantly they assumed the homogeneity of expectations and that individuals hold mean variance efficient portfolios. Another implicit assumption of the CAPM is that all assets including human capital are marketable. Moreover the CAPM is essentially a single period model.

It is clear that these assumptions do not hold in the real world and thus, not surprisingly, the model's validity has been suspect from the outset. Closer examination of the assumptions underlying the CAPM shows that they are not as stringent as they first appear to be. Exactly the same results would obtain if short sales were disallowed. Since in equilibrium no investor sells any security short, prohibiting short selling will not change the equilibrium. More formally the derivative of the

Langrangian with respect to each security will have a Kuhn-Tucker multiplier added to it, but since each security is contained in the market portfolio, the value of the multiplier will be zero and hence the solution will remain unaffected.

Further, Fama (1970) and Elton and Gruber (1975) give a set of conditions under which the multi-period problem reduces to a single period CAPM, where all individuals maximise a single period utility function. The conditions are that firstly consumers act as if the one-period returns are not state dependent, i.e. the distribution of one-period returns on all the assets are known at the beginning of the period. Secondly, the consumption opportunities are not state dependent and lastly consumers' tastes are independent of future events. Fama further shows that given these conditions the derived one period utility is equivalent to a multi-period utility function given non-satiation and risk aversion. However, it is argued by many that the above conditions are rather restrictive.

Merton (1973) has shown that a necessary and sufficient condition for individuals to behave as if they were single-period maximisers and for the equilibrium return relationship of CAPM to hold is that the investment opportunity set is constant. Furthermore, the main results of the model hold if income tax and capital gains taxes are of equal sizes. If the assumption of riskless lending or borrowing is violated, then Black (1972) has shown that we still obtain a linear relationship between an asset's returns and its risk as measured by the covariance of the assets returns with the market. This model as distinct from the standard CAPM is known as the zero-beta CAPM.

Thus, even though the assumptions underlying the Capital Asset Pricing Model are demanding and have been the basis for much of the criticism against the model, nevertheless these assumptions are not altogether inflexible. More importantly, the final test of the model is not how reasonable the assumptions underlying it seem to be, but rather how well the model conforms to reality. Indeed, many proponents of the CAPM argue that due to technological advances, capital markets operate as if these assumptions are satisfied.

Sharpe and Lintner, thus making a number of assumptions, extended Markowitz's mean variance framework to develop a relationship for expected returns, which more precisely is:

$$E[R_i] = R_f + \beta_{im}(E[R_m] - R_f)$$

$$\beta_{im} = \frac{Cov[R_i, R_m]}{Var[R_m]}$$

Where R_i = return on asset i; R_m = return on market portfolio; R_f = return on the riskless asset.

Thus the return on an asset depends linearly on beta, β_{im} which is a measure of the covariance of the asset's return with that of the market. Intuitively, in rational and competitive market, investors diversify all systematic risk away and thus price assets according to their systematic or non-diversifiable risk. Thus the model invalidates the traditional role of standard deviation as a measure of risk. This is a natural result of the rational expectations hypothesis (applied to asset markets) because if, on the contrary, investors also took into account diversifiable risks, then over time competition will force them out of the market. If, on the contrary, the CAPM does not hold, then the rationality of the asset's markets will have to be reconsidered.

Black (1972) has derived a more general version of the CAPM, which holds in the absence of a riskless asset. For this *zero beta CAPM*, we have:

$$E[R_i] = E[R_{om}] + \beta_{im}(E[R_m] - E[R_{om}])$$

$$\beta_{im} = \frac{Cov[R_i, R_m]}{Var[R_m]}$$

where R_{om} = return on the zero beta portfolio, i.e. the portfolio (lying on the portfolio frontier) which has a zero correlation with the market portfolio.

1.2 Framework For Testing The Validity Of CAPM

The standard CAPM can also be written in terms of excess returns

$$E[Z_i] = \beta_{im}E[Z_m]$$

$$\beta_{im} = \frac{Cov[R_i, R_m]}{Var[R_m]}$$

where: $Z_i = R_i - R_f$

$Z_m = R_m - R_f$

Empirical tests of the standard CAPM have focused on three testable implications, namely the intercept is zero, beta completely captures the cross sectional expected returns and that the market excess return is positive.

The intercept is zero

The excess return market model is:

$$Z_t = \alpha + \beta Z_{mt} + \varepsilon_t$$

$$E[\varepsilon_t] = 0, \qquad E[\varepsilon_t \varepsilon_t'] = \Sigma$$

$$E[Z_{mt}] = \mu_m, \quad E[(Z_{mt} - \mu_m)^2] = \sigma_m^2$$

$$Cov[Z_{mt}, \varepsilon_t] = 0$$

where Z_t = (Nx1) vector of excess returns for N assets

β = (Nx1) vector of betas

Z_{mt} = time period t market portfolio of excess return

α = (Nx1) vector of intercepts

ε_t = (Nx1) vector of disturbances

The implication of the standard CAPM is that the vector of intercepts is zero. If this is true then the market portfolio will be the 'tangency' portfolio. Assuming that the returns are IID and are normally distributed, the maximum likelihood estimation technique can be used to estimate the parameters α and β. The probability density function (pdf) of excess returns conditional on the market excess return is given by

$$f(Z_t \mid Z_{mt}) = (2\pi)^{-N/2} \mid \Sigma \mid^{-1/2} \times \exp[-(1/2)(Z_t - \alpha - \beta Z_{mt})' \Sigma^{-1} (Z_t - \alpha - \beta Z_{mt})]$$

and given that the returns are IID the joint pdf is

$$f(Z_1, \ldots, Z_T \mid Z_{m1}, \ldots, Z_{mT}) = \prod f(Z_t \mid Z_{mt})$$

Thus the log-likelihood function is

$$\Lambda(\alpha, \beta, \Sigma) = -(NT/2)\log(2\pi) - (T/2)\log|\Sigma| - (1/2)\sum_t (Z_t - \alpha - \beta Z_{mt})' \Sigma^{-1} (Z_t - \alpha - \beta Z_{mt})$$

The first order conditions are:

$$\partial \Lambda / \partial \alpha = \Sigma^{-1}[\sum_t (Z_t - \alpha - \beta Z_{mt})] = 0$$

$$\partial \Lambda / \partial \beta = \Sigma^{-1}[\sum_t (Z_t - \alpha - \beta Z_{mt}) Z_{mt}] = 0$$

$$\partial \Lambda / \partial \Sigma = -(T/2)\Sigma^{-1} + (1/2)\Sigma^{-1}[\sum_t (Z_t - \alpha - \beta Z_{mt})(Z_t - \alpha - \beta Z_{mt})']\Sigma^{-1} = 0$$

Solving the above First Order Conditions we get estimates for α, β and Σ, and it should be noted that these estimates are the same as the ones obtained using Ordinary Least Squares. However, compared to the OLS they have better large sample properties.

The Wald test can then be used to check whether the intercept is zero. The Wald test statistic is given by:

$$W = \Lambda\, [Var(\Lambda)]^{-1} \Lambda$$

which has a chi-square distribution with N degrees of freedom

1.3 VARIABLES OTHER THAN THE MARKET FACTOR AFFECTING STOCK RETURNS

According to the CAPM, only market risk is priced, i.e. only beta affects returns and all other variables are irrelevant. However, there have been a number of empirical studies, which find that non-market factors have a significant effect on average returns. Basu (1983) provides evidence that shares with high earnings yield (low price to earnings ratio) experience on average higher subsequent returns than shares with low earnings yield. Banz (1981) was the first to provide evidence of the 'size effect'; i.e. low market capitalisation firms have higher average returns compared to larger firms.

Merton (1973) has constructed a generalised intertemporal capital asset pricing model in which factors other than market uncertainty are priced. He models individuals as solving lifetime consumption decisions in a multi-period setting. After making a number of assumptions, Merton show that the returns on assets depend not only on the covariance of the asset with the market but also on its covariance with changes in the investment opportunity set. Hence, changes in interest rates, future income and relative prices will all influence returns. Intuitively, individuals will form portfolios to hedge themselves away from these risks. These actions of investor's will affect returns. According to this 'multi-beta CAPM', the return on securities will be affected by a number of indices apart from the market factor. Hence excess returns will be of the following form

$$E[R_i] – R_f = \beta_{im}(E[R_m] – R_f) + \beta_{iI1}(E[R_{I1}] – R_f) + \beta_{iI2}(E[R_{I2}] – R_f) + \ldots$$

This can also be interpreted as another form of the Arbitrage Pricing Theory (APT).

Elton and Gruber (1993 find that a five – factor APT model better explains expected returns compared to the classic CAPM model in the Japanese market. They find that in Japan, smaller stocks are associated with smaller betas and thus according to the CAPM they should give smaller mean returns. Yet, smaller stocks have higher expected returns than their larger counterparts. They also find that a multi-factor model is more useful in constructing hedge portfolios for futures and option trading.

Fama and French (1992) in an influential paper entitled, "The cross-section of expected stock returns", show that two easily measured variables, size and book-to-market equity combine to capture the cross-sectional variation in average stock returns associated with market beta, size, leverage, book-to-market, and earnings-to-price ratios. Fama and French using data for non-financial firms conduct its asset-pricing tests using the Fama and MacBeth regressional approach. Their results from applying the FM regressions show that market beta clearly does not explain the average stock returns. The average slope from the regression of returns on beta alone is 0.15 per cent per month and it is only 0.46 standard errors from zero.

Furthermore, Fama and French point out that variable such as size, earning yield, leverage, and book-to-market are all scaled versions of a firm's stock price and thus some of them are redundant to explain returns. They show that of these variables only size and book-to-market equity explain cross-sectional average returns. Moreover, they find that when allowing for variations in beta that are unrelated to size, the relationship between beta and average return is flat. Hence, they naturally argue that the CAPM is dead.

1.4 CAPM IMPLICATIONS

One of the most important *implications* of the CAPM is that all investors will choose to hold a portfolio of risky assets in proportions that duplicate representation of the assets in the market portfolio (*M*), which includes all traded assets. For simplicity, we generally refer to all risky assets as stocks*.*

Not only the market portfolio be on the efficient frontier but it also will be the tangency portfolio to the optimal capital allocation line (CAL) derived by each and every investor. As a result, the capital market line (CML), the line from the risk-free rate through the market portfolio, *M*, is also the best attainable capital allocation line. All investors hold *M* as their optimal risky portfolio, differing only in the amount invested in it versus in the risk free asset.

The risk premium on the market portfolio will be proportional to its risk (standard deviation) and the degree of risk aversion of the typical investor. Note that because *M* is the optimal portfolio, which is efficiently diversified across all stocks, standard deviation *(σ^2_M)* is the systematic risk of this universe.

$$E(r_M) - r_f = \overline{A}\sigma_M^2$$

The risk premium on individual assets will be proportional to the risk premium on the market portfolio, *M*, and the *beta coefficient* of the security relative to the market portfolio. Beta measures the extent to which returns on stock and the market move together.

$$\beta = \frac{Cov(r_i, r_M)}{\sigma_M^2}$$

2 THE CAPM MODEL

2.1 CONCEPTUAL MODEL

The CAPM is given as follows:

$$R_i = R_F + [E(R_M - R_F)]ß$$

Where R_i is required return of security i

R_F is the risk free rate of return

$E(R_M)$ is the expected market rate of return

ß is Beta.

Note $ß_i = \frac{Cov_{(im)}}{\delta^2_m}$

Where $Cov_{(im)}$ is the covariance between asset i and the market return; and δ^2_m is the variance of the market return.

If we graph $ß_i$ and $E(R_i)$ then we can observe the following relationship

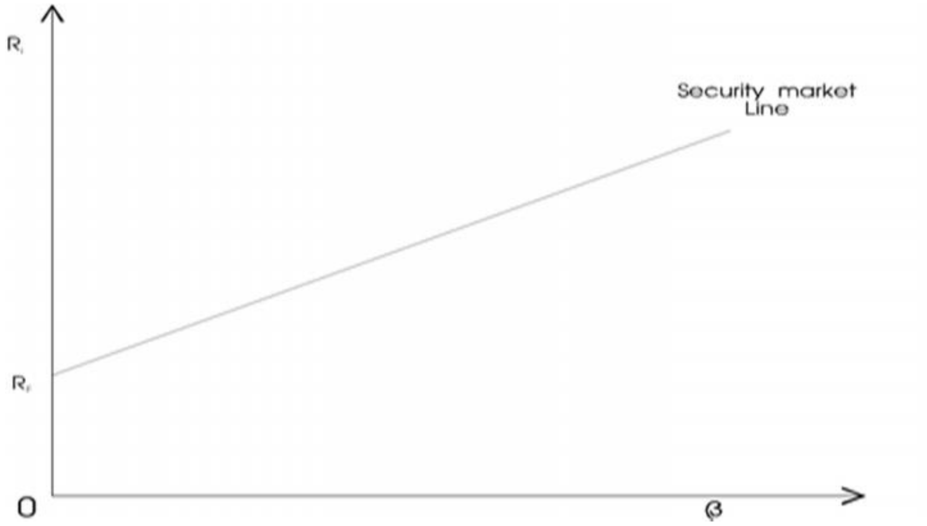


Figure 1

All correctly priced assets will lie on the security market line. Any security off this line will either be overpriced or underpriced.

The security market line therefore shows the pricing of all assets if the market is at equilibrium. It is a measure of the required rate of return if the investor were to undertake a certain amount of risk.

2.1 ANALYTICAL MODEL

2.1.1 THE RISK PREMIUM OF THE MARKET PORTFOLIO

The equilibrium risk premium on the market portfolio will be proportional to the average degree of risk aversion of the investor population and the risk of the market portfolio. Recall that each individual investor chooses a proportion *y*, allocated to the optimal portfolio *M*, such that:

$$y = \frac{E(r_M) - r_f}{A\sigma^2_M}$$

In the simplified CAPM economy, risk-free investments involve borrowing and lending among investors. Any borrowing position must be *offset* by the lending position of the creditor. This means that net borrowing and lending across all investors must be zero, and in consequence, substituting the representative investor's risk aversion $\bar{A}$, for *A*, the average position in the risky portfolio is 100%, or $\bar{y}$ = 1. Setting *y* = 1 in Equation (2.1) and rearranging, we find that the risk premium on the market portfolio is related to its variance by the average degree of risk aversion:

$$E(r_M) - r_f = \overline{A}\sigma_M^2$$

In equilibrium, of course, the risk premium on the market portfolio must be just high enough to induce investors to hold the average supply of stocks. If the risk premium is too high compared to the average degree of risk aversion, there will be excess demand for securities, and prices will rise; if it is too low, investors will not hold enough stocks to absorb the supply, and the prices will fall. The *equilibrium* risk premium of the market portfolio is therefore proportional to both the risk of the market, as measured by the variance of its returns, *and* to the degree of risk aversion of the average investor, *A*, in the above equation.

2.1.2 Expected Returns On Individual Securities

All investors use the same input list, that is, the same estimates of expected returns, variances, and covariances. These covariances can be arranged in a covariance *matrix*, so that the entry in the fifth row and third column for example, would be the covariance between the rates of return on the fifth and third securities. Each diagonal entry of the matrix is the covariance of one security's return with itself, which is simply the variance of that security.

To calculate the variance of the market portfolio we use the *bordered* covariance matrix with the market portfolio weights,. Recall that we calculate the variance of the portfolio by summing over all the elements of the covariance matrix, first multiplying each element by the portfolio weights from the row and the column. The contribution of one stock to portfolio variance therefore can be expressed as the sum of all the covariance terms in the column corresponding to the stock, where each covariance is first multiplied by both the stock's weight from its row and the weight from its column. For example, the covariance of GE stock to the variance of the market portfolio is:

$$w_{GE}[w_1Cov(r_1, r_{GE}) + w_2Cov(r_2, r_{GE}) + ... + w_{GE}Cov(r_{GE}, r_{GE}) + ... + w_nCov(r_n, r_{GE})]$$

The *best* measure of the stock's contribution to the risk of the market portfolio is its covariance with that portfolio. Thus, the GE's stock contribution to the market portfolio's variance is:

$$GE\, contribution = w_{GE}Cov(r_{GE}, r_M)$$

To demonstrate this more rigorously, note that the rate of return on the market portfolio may be written as:

$$r_M = \sum_{k=1}^{n} w_k r_k \,,$$

Therefore, the covariance of the return on GE with the market portfolio is:

$$Cov(r_{GE}, r_M) = Cov(r_{GE,} \sum_{k=1}^{n} w_k r_k) = \sum_{k=1}^{n} w_k Cov(r_{GE}, r_k)$$

Notice that the last term in Equation (2.4) is precisely the same as the term in brackets in Equation (2.3). Therefore, Equation (2.3), which is the contribution of GE to the variance of the market portfolio, may be simplified to: $w_{GE}Cov(r_{GE}, r_M)$. We also observed that the contribution of our holding of GE to the risk premium of the market portfolio is $w_{GE}[E(r_{GE}) - r_f]$. Therefore, reward-to-risk ratio for investment in GE can be expressed as:

$$\frac{GE's\, contribution\, to\, Risk\, Premium}{GE's\, contribution\, to\, Variance} = \frac{E(r_{GE}) - r_f}{Cov(r_{GE}, \mathrm{r_M})}$$

Having measured the contribution of GE stock to market portfolio variance, we may determine the appropriate risk premium for GE. We note first that the market portfolio is the tangency (efficient mean-variance) portfolio. The reward-to-risk ratio for investment in the market portfolio is:

$$\frac{Market\ risk\ \ premium}{Market\ variance} = \frac{E(r_M) - r_f}{\sigma_M^2}$$

The ratio in Equation (2.5) is often called the market price of risk, because it quantifies the extra return that investors demand to bear portfolio risk. The ratio of risk premium to variance tells us how much *extra* return must be earned per unit of portfolio risk. (Notice that for components of the efficient portfolio, such as shares of GE, we measure risk as the contribution to portfolio variance (which depends on its *covariance* with the market). In contrast, for the efficient portfolio itself, its variance is the appropriate measure of risk).

Now, consider an average investor who is currently invested 100% in the market portfolio and suppose she were to increase her position in the market portfolio by a *tiny* fraction, δ, financed by borrowing at the risk-free rate of return. The portfolio rate of return will be $r_M + \delta(r_M - r_f)$. Taking expectations and comparing with the original expected return, $E(r_M)$, the incremental rate of return will be:

$$\Delta E(r) = \delta[E(r_M) - r_f]$$

To measure the impact of the portfolio shift on risk, we compute the new value of the portfolio variance. The new portfolio has a weight of (1 +δ) in the market and (-δ) in the free-risk asset. The portfolio variance will increase by:

$$\Delta\sigma^2 = (1+\delta)^2\sigma_M^2 = \sigma_M^2 + (2\delta + \delta^2)\sigma_M^2$$

However, if δ is very small, then δ^2 will be negligible compared to 2δ, so we may ignore the term. Therefore, the portfolio variance has increased by $2\delta\sigma_M^2$. Summarizing these results, the trade-off between the incremental risk premium and incremental risk, referred to as the marginal price of risk*,* and is given by the ratio:

$$\frac{\Delta E(r)}{\Delta\sigma^2} = \frac{E(r_M) - r_f}{2\sigma_M^2} \ ,$$

and equals one-half the market price of risk (see Equation 2.5 of reward-to-risk ratio). Now suppose that, instead, investors were to invest the increment δ in GE stock. The increase in mean excess return is:

$$\Delta E(r) = \delta[E(r_{GE}) - r_f]$$

This portfolio has a weight of 1.0 in the market, δ in GE and (-δ) in the risk-free asset. The *increase* in variance includes the variance of the incremental position in GE plus twice its covariance with the market:

$$\Delta\sigma^2 = \delta^2\sigma_{GE}^2 + 2\delta Cov(r_{GE}, r_M).$$

Dropping the negligible term involving δ^2, the marginal price of risk of GE is:

$$\frac{\Delta E(r)}{\Delta\sigma^2} = \frac{E(r_{GE}) - r_f}{2Cov(r_{GE}, r_M)}$$

A basic principle of equilibrium is that all investments should offer the same reward-to-risk ratio. (If the ratio were better for one investment than another, investors would rearrange their portfolio, tilting toward the alternative with better trade-off and shying away from the other. Such activity would impart pressure on security prices until the ratios were equalized.) Therefore, we may conclude that, in equilibrium, the marginal price of risk of GE stock must equal that of the market portfolio:

To determine the fair risk premium of GE stock, we rearrange the above equation slightly to obtain:

$$E(r_{GE}) - r_f = \frac{Cov(r_{GE}, r_M)}{\sigma_M^2}[E(r_M) - r_f]$$

The ratio $\mathrm{Cov}(r_{GE}, r_M)/\sigma_M^2$ on the right-hand side of Equation (2.8a) measures the *contribution* of GE stock to the variance of the market portfolio as a fraction of the total variance of the market portfolio. The ratio is called beta and is denoted by *β*. Using this measure, we can restate the previous equation as

$$E(r_{GE}) = r_f + \beta_{GE}[E(r_M) - r_f]$$

This expected return-beta relationship is the most familiar expression of the CAPM to practitioners. We will have a lot more to say about the expected return-beta relationship shortly.

Does the fact that few real-life investors actually hold the market portfolio imply that the CAPM is of no practical importance? Not necessarily. Recall that reasonably well-diversified portfolios shed firm-specific risk, and are left with mostly systematic or market risk. Even if one doesn't hold the precise market portfolio, a well-diversified portfolio will be so highly correlated with the market that a stock's beta relative to the market will still be a useful risk measure.

Several CAPM authors have shown that modified versions of the CAPM will hold true even if we consider differences among individuals leading them to hold different portfolios. For example, Brennan (1973) examined the impact of differences in investors' personal tax rates on market equilibrium, and Mayers (1972) looked at the impact of non-traded assets such as human capital (earning power). Both found that although the market portfolio is no longer each investor's optimal risky portfolio, the expected return-beta relationship should still hold in a somewhat modified form.

If the expected return-beta relationship holds for any individual asset, it must hold for any combination of assets.

$$E(r_P) = r_f + \beta_P[E(r_M) - r_f],$$

Where $E(r_P) = \sum_k w_k E(r_k)$ is the expected return on the portfolio, and $\beta_P = \sum_k w_k \beta_k$ is the portfolio beta. Incidentally, this result has to be true for the market portfolio itself:

$$E(r_M) = r_f + \beta_M[E(r_M) - r_f]$$

If the market beta is 1, and the market is a portfolio of all assets in the economy, the weighted-average beta of all assets must be 1. Hence, betas greater that 1 are considered *aggressive* in that investment in high-beta stocks entails above-average sensitivity to market swings. Betas below 1 can be described as *defensive*. (A word of caution: we are all accustomed to hearing that well-managed firms will provide high rates of return. We agree with this if one measures the *firm*'s return on investments in plant and equipment. The CAPM, however, predicts returns on investments in the *securities* of the firm.)

2.1.3 THE CAPM AND THE INDEX MODEL

The CAPM is an elegant model. The question is whether it has real-world value – whether its implications are borne out by experience. For starters, one central prediction of CAPM is that the market portfolio is a mean-variance efficient portfolio. Consider that CAPM treats all traded risky assets. To test the efficiency of the CAPM market portfolio, we would need to construct a value-weighted portfolio of a huge size and test its efficiency. So far, this task has not been feasible. An even more difficult problem, however, is that the CAPM implies relationships among *expected* returns, whereas all we can observe are *actual* or realized holding-period returns, and these need not equal prior expectations. Even supposing we could construct a portfolio to represent the CAPM market portfolio satisfactorily, how would we test its mean-variance efficiency? We would have to show that the reward-to-volatility ratio of the market portfolio is higher than that of any other portfolio. However, this ratio is set in terms of expectations, and we have no way to observe these expectations directly.

2.1.3.1 THE INDEX MODEL AND REALIZED RETURNS

We have said that the CAPM is a statement about *ex-ante* or expected returns, whereas in practice all anyone can observe directly are *ex-post* or realized returns. To make the leap from expected to realized returns, we can employ the index model, which we will use in excess return form as

$$R_i = \alpha_i + \beta_i R_M + e_i$$

Let us now see how the standard regression analysis framework for statistically decomposition of the actual stock returns meshes with the CAPM.

We start by deriving the covariance between the returns on stock *i* and the market index. By definition, the firm-specific or non-systematic component is independent of the market-wide or systematic component, that is, Cov(R_M, e_i) = 0. From this relationship it follows that the covariance of the excess rate of return on security *i* with that of the market index is:

$$\begin{aligned} Cov(R_i, R_M) &= Cov(\beta_i R_M + e_i, R_M) = \\ &= \beta_i Cov(R_M, R_M) + Cov(e_i, R_M) \\ &= \beta_i \sigma_M^2 \end{aligned}$$

Note that we drop α_i from the covariance term because α_i is a constant and thus has zero variance with all variables. Because $Cov(R_i, R_M) = \beta_i \sigma_M^2$, the sensitivity coefficient, β_i, in Equation (2.10), which is the slope of the regression line representing the index model, equals:

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma_M^2}$$

The index model beta coefficient turns out to be the same beta as that of the CAPM expected return-beta relationship, except that we replace the (theoretical) market portfolio of the CAPM with the well-specified and observable market index.

2.1.3.2 The Index Model And The Expected Return-Beta Relationship

Recall that, according to CAPM, the excess rate of return is, for any security *i* and the (theoretical) market portfolio:

$$E(r_i) - r_f = \beta_i [E(r_M - r_f)] \qquad \text{where: } \beta_i = Cov(R_i, R_M)/\sigma_M^2 .$$

This is a statement about the mean, or expected excess returns of assets relative to the mean excess return of the (theoretical) market portfolio. If the index *M* in Equation (2.10) represents the *true* market portfolio, we can take the expectations of each side of the equation to show that the index model specification is:

$$E(r_i) - r_f = \alpha_i + \beta_i [E(r_M - r_f)]$$

A comparison of the index model relationship to the CAPM expected return-beta relationship (see above) shows that the CAPM predicts that α_i should be zero for all assets. The *alpha* of a stock is its expected return in excess of (or below) the fair expected return as predicted by the CAPM. If the stock is fairly priced, its alpha must be *zero*.

We emphasise again that this is a statement about expected returns on a security. After the fact, of course, some securities will do better or worse than expected and will have higher or lower than predicted by the CAPM; that is, they will exhibit positive or negative alphas over a sample period. But this superior or inferior performance could not have been forecast in advance. Therefore, if we estimate the index model for several firms, using Equation (2.10) as a regression equation, we should find that the ex-post or realised alphas (the regression intercepts) for the firms in our sample center around zero. If the initial expectation for alpha were zero, as many firms would be expected to have a positive as a negative alpha for some sample period.

The CAPM states that the expected value of alpha is zero for all securities, whereas the index model representation of the CAPM holds that the realized value of alpha should average out to zero for a sample of historical observed returns. Just as important, the sample alphas should be unpredictable, that is, independent from one sample period to the next.

Indirect evidence on the efficiency of the market portfolio can be found in a study by Malkiel (1995), who estimates alpha values for a large sample of equity mutual funds. Their results show that the distribution of alphas is roughly bell shaped, with a mean that is slightly negative. On average, it does not appear that mutual funds outperform the market index (S&P 500) on a risk-adjusted basis.

There is yet another applicable variation on the intuition of the index model, the market model. Formally, the market model states that the return "*surprise*" of any security is proportional to the return "*surprise*" of the market, plus a firm-specific surprise:

$$r_i - E(r_i) = \beta_i [r_M - E(r_M)] + e_i$$

This equation divides returns into firm-specific and systematic components somewhat differently from the index model. If the CAPM is valid, however, you can confirm that, substituting for $E(r_i)$ from the CAPM equation, the market model equation becomes identical to the index model. For this reason the terms "index model" and "market model" are used interchangeably.

3 EMPIRICAL TESTS OF THE CAPM

Let us consider for a moment what testability means. A model consists of (i) a set of assumptions, (ii) logical/mathematical development of the model through manipulation of those assumptions, and (iii) a set of predictions. Assuming the logical/mathematical manipulations are free of errors, we can test a model in two ways, normative, and positive. Normative tests examine the assumptions of the model, while positive tests examine the predictions.

The CAPM implications are embedded in two predictions: 1) the market portfolio is efficient, and 2) the security market line (the expected return-beta relationship) accurately describes the risk-return trade-off, that is, alpha values are zero. The central problem in testing these predictions is that the hypothesized market portfolio is unobservable. The “market” portfolio includes *all* risky assets that can be held by investors. This is far more extensive than an equity index. It would include bonds, real estates, foreign assets, privately held businesses and human capital. These assets are often traded thinly or (for example, in case of human capital) not traded at all. It is difficult to test the efficiency of an observable portfolio, let alone an unobservable one. These problems alone make adequate testing of the model infeasible. Moreover, even small departures from efficiency in the market portfolio can lead to large departures from the expected return-beta relationship of the SML, which would negate the practical usefulness of the model.

Most of the early tests of the CAPM employed the methodology of first estimating betas using time series regression and then running a cross sectional regression using the estimated betas as explanatory variables to test the hypothesis implied by the CAPM.

3.1 TESTS BY LINTNER

Using this approach one of the first tests of the CAPM was conducted by Lintner, which is reproduced in Douglas (1968). Using data from 1954-1963, Lintner ran the following regression: $R_t = \alpha + bR_{mt} + e_t$

where R_t = (Nx1) vector of asset returns

R_{mt} = return on the market portfolio

b = (Nx1) vector of estimated betas

Lintner then ran the following second pass regression:

$R = a_1 \cancel{\mp} a_2 b + a_3 S_e^2 + \eta$

where S_e^2 = (NxN) matrix of residual variance (i.e. the variance of e in the first pass regression).

The testable implications of the CAPM are that $\alpha_1 = R_f$; $\alpha_2 = (E[R_m] - R_f)$ and $\alpha_3 = 0$.

However, Lintner found that the actual values did not confirm with the theoretical values. α_1 was found to be much larger than R_f or even R_{om}, α_2 was found to be statistically significant but had a lower value than expected and α_3 was found to be statistically significant as well. Thus Lintner’s results seem to be in contradiction to the Capital Asset Pricing Model.

3.2 TESTS BY FAMA AND MACBETH (1973)

Fama and MacBeth (1973) performed one classic test of the CAPM. They combined the time series and cross-sectional steps to investigate whether the risk premium of the factors in the second pass regression were non-zero. Forming 20 portfolios of securities, they estimated betas from a time-series regression similar to Lintner’s methodology. However, they then performed a cross-sectional regression for each month over the period 1935-1968. Their second pass regression was of the following form: $R_t = \gamma_{0t} + \gamma_{1t}\beta - \gamma_{2t}\beta^2 + \gamma_{3t}S_e + \eta_t$

If the standard CAPM was true then we should have the following:

$E[\gamma_{0t}] = R_f$

$E[\gamma_{1t}] > 0$ as the market risk premium should be positive

$E[\gamma_{2t}] = 0$ as the securities market line (SML) should be linear, i.e. the relationship between return and the relevant risk should be linear.

$E[\gamma_{3t}] = 0$ as the residual risk should not affect asset returns.

All of the above should be true if the standard CAPM is to hold.

Fama and MacBeth (1973) found that γ_3 was statistically insignificant and its value remains very small over several subperiods. Thus, in contrast to Lintner, they find that residual risk has no effect on security returns. Miller and Scholes (1972) showed that residual risk would act as a proxy for risk if beta had a large sampling error. This fact might reconcile Lintner's and Fama and MacBeth's results, as the latter's estimate for beta had much less sampling error due to their use of asset portfolios. Fama and MacBeth further found that γ_2 is not statistically different from zero. Moreover, they found that the estimated mean of γ_1 is positive as predicted by the model. They also find that γ_0 is statistically different from zero. However, their intercept is much greater than the risk free rate and thus this would indicate that the standard CAPM might not hold.

3.3 TESTS BY BLACK, JENSEN AND SCHOLES (1972)

Black, Jensen and Scholes (1972) performed another classic test of the Capital Asset Pricing Model employing time-series regression. They ran the following familiar time series regression:

$$Z_t = \alpha + \beta Z_{mt} + \varepsilon_t$$

As observed before, the intercept should be zero according to the CAPM. Black *et al.* used the return on portfolios of assets rather than individual securities. Time series regression using returns on individual assets may give biased estimates, as it is likely that the covariance between residuals may not equal zero. This is not generally true with portfolios as they utilise more data. The results from the BJS time series regressions show that the intercept term is different from zero and in fact is time varying. They find that when $\beta<1$ the intercept is positive and that it is negative when $\beta>1$. Thus, the findings of Black *et al.* violate the CAPM.

3.4 TESTS BY STAMBAUGH (1982)

Stambaugh (1982) employs a slightly different methodology. From the market model we have

$$R_t = \alpha + \beta(R_{mt}) + e_t$$

If the CAPM was true then the intercept in the above equation should be constrained and should in fact be:

$$\alpha = \kappa(1 - \beta)$$

where $\kappa = R_f$ (under the Sharpe-Lintner CAPM) or $\kappa = R_{om}$ (under the Black's version of CAPM)

Stambaugh (1982) then estimates the market model and using the Lagrange multiplier test finds evidence in support of Black's version of CAPM but finds no support for the standard CAPM.

3.5 TESTS BY GIBBONS (1982)

Gibbons (1982) uses a similar method as the one used by Stambaugh (1982) but instead of the LM test uses a likelihood ratio test. He uses the fact that if the CAPM is true then the constrained market model should have the same explanatory power as the unconstrained model, but if the CAPM is invalid then the unconstrained model should have significantly more explanatory power than the constrained model. Using this test, Gibbons rejects both the standard and the zero beta CAPM.

3.6 MILLER AND SCHOLES (1972)

Miller and Scholes (1972) in their paper "Rates of return in relation to risk" discuss the statistical problems inherent in all the empirical studies of the CAPM. They point out that the CAPM in time series form is:

$$R_t = R_{ft} + \beta (R_{mt} - R_{ft}) \text{ or } R_t = (1 - \beta)R_{ft} + \beta R_{mt}$$

and thus if the riskless rate is non-stochastic then the CAPM can easily be tested by finding whether the intercept is significantly different from $(1 - \beta)R_{ft}$. However, if R_{ft} varies with time and moreover is correlated with R_{mt}, then we inevitably encounter the problem of omitted variable bias and thus the estimated betas will be biased.

Miller and Scholes (1972) then using historical data find that R_{ft} and R_{mt} are negatively correlated. Intuitively, a rise in the interest rates is conducive to stock market declines. They then prove that if R_{ft} and R_{mt} are negatively correlated then this will lead to an upward bias in the intercept and further the slope will be biased downwards. This is in fact what many empirical studies find and thus the fact that many studies reject the CAPM does not imply that it does not hold.

Another factor that may bias the intercept upward and the slope downwards is the presence of heteroskedasticity. However, Miller and Scholes find no evidence of heteroskedasticity.

Miller and Scholes then go on to show the biases that one may encounter in the two stage regressions used by Lintner and Douglas and by Fama and MacBeth (1973). The problem in this methodology is that estimated betas instead of the true betas are used in the second pass regressions and thus any error in the first stage is carried to the second stage. Miller and Scholes show that this 'errors-in-variables problem' will bias the intercept upward and the slope downwards.

Another possible problem in many tests of the CAPM arises due to it being a single-period model. Most tests of the CAPM use time series regression, which is only appropriate, if the risk premium and betas are stationary, which is unlikely to be true.

3.7 Roll (1977) Critique

In his influential paper "A critique of asset pricing theory's tests", Roll (1977) shows that there has been no single unambiguous test of the CAPM. He points out that tests performed by using any portfolio other than the true market portfolio are not tests of the CAPM but are tests of whether the proxy portfolio is efficient or not. Intuitively, the true market portfolio includes all the risky assets including human capital while the proxy just contains a subset of all assets.

If we choose a portfolio, say *m* from the sample efficient frontier as a proxy for the market, then from efficient set mathematics we know that the mean return on any asset or portfolio *j*, will be a weighted average of the return on *m* and the return on the portfolio which has a zero correlation with *m*,

i.e. $R_j \equiv (1 - \beta_j)R_{om} + \beta_j R_m)$ where $\beta_j = \underline{\text{Cov}(R_j, R_m)}$

More generally, if *A* and *B* are any two sample efficient portfolios, then the mean return on asset *j* is given by:

$$R_j \equiv (1 - \beta_j)R_A + \beta_j R_B \; \forall j$$

Conversely, if R is the mean vector of returns and β is the (Nx1) vector of slope coefficients obtained by regressing asset returns on the returns of some portfolio *m*, then we have

$$R = R_{om}\iota + (R_m - R_{om})\beta \qquad \text{where } \iota = \text{vector of ones}$$

The above relationship will hold if R_m is ex-post efficient. Thus if *m* is not efficient then mean returns will not be linearly related to betas. Using this result from efficient set mathematics, Roll asserted that the only testable implication of the CAPM is that the market portfolio is mean-variance efficient. All other implications of the model, including the linearity of expected returns and beta follow from the efficiency of the market portfolio and thus are not independently testable.

Furthermore, for a given sample of mean returns, there always exist an infinite number of ex-post mean-variance portfolios. For these portfolios there will be an exact linear relationship between sample returns and sample betas. This linearity will hold whether or not the true market portfolio is efficient. Thus, the two-parameter asset pricing theory is not testable unless all assets are included in the sample.

3.8 Conclusive Empirical Reviews And Results

Fama and MacBeth (1973) in their paper incorrectly state that there are three testable implications, namely that the relationship between expected returns and beta is linear; that beta is a complete measure of risk; and that given risk averseness, higher return should be associated with higher risk, i.e. $E[R_m] - E[R_{om}] > 0$.

Roll points out that if *m* is efficient then all the above implications are not independently testable and further asserts that the last inequality follows from the mathematical implication of the assumption about *m* rather than risk averseness per se.

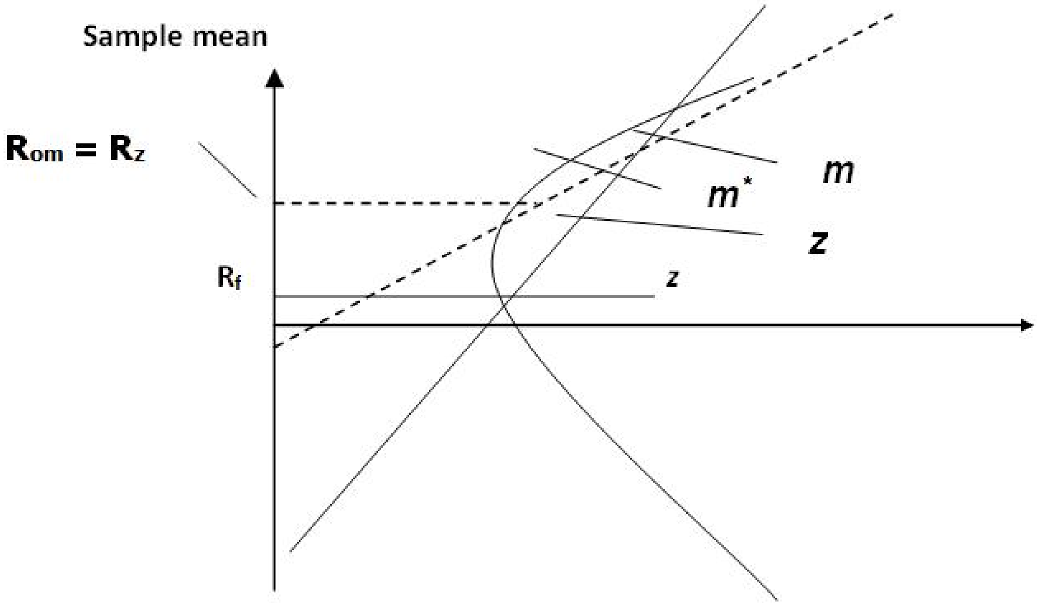
Thus the only testable hypothesis concerning the zero beta CAPM is that the individuals prefer portfolios which are mean-variance efficient and that the market portfolio is ex-ante efficient.

On the contrary, the famous paper of Black, Jensen and Scholes does not even mention the possible efficiency of the market portfolio and concludes that the relationship between expected returns and beta is not linear. This conclusion is enough to prove that the proxy used by BJS does not lie on the sample efficient frontier. If on the other hand, the proxy had been on the efficient part of the frontier than BJS would have found a linear relationship between mean returns and beta. This is all in accordance with efficient set mathematics. The relevant testable implications of the Sharpe-Lintner CAPM can be illustrated by means of the figure below. In the figure, m^* is the tangent portfolio. If m is used as the proxy, then the return on the asset is given by $R_j = R_z + (R_m - R_z)\beta_j$ (a)

On the other hand, if m^* is used as the proxy, then the return on the asset is given by

$R_j = R_f + (R_m^* - R_f)\beta_j^*$

It should be noted that since efficient orthogonal portfolios are unique, β_z^* should be non-zero.



Since each individual will invest partly in the riskless asset and partly in the tangent portfolio m^*, thus the principle testable hypothesis of the Sharpe-Lintner CAPM is that the ex-ante efficient tangent portfolio is the market portfolio. On the other hand, as already mentioned, BJS by using a market proxy estimated the following regression:

$R_j - R_f = \alpha + \gamma\beta_j + \varepsilon_j$

They found that α was not only greater than zero but was also highly variable. Moreover, they found that γ was less than $R_m - R_f$. On the basis of these results, they rejected the standard CAPM.

However, Roll showed that unless BJS were successful in choosing the tangent portfolio m^* as their proxy, their results are actually in accord with the standard CAPM. Suppose, BJS chose m as their proxy, then substituting $j = z$ in (b), we have $R_z = R_f + (R_m^* - R_f)\beta_z^*$

Substituting the above equation in (a) we get:

$$R_j - R_f = \beta_z^*(R_m^* - R_f) + [R_m - R_f - \beta_z^*(R_m^* - R_f)]\beta_j \qquad \text{(d)}$$

Comparing (c) with (d) we see that if the Sharpe-Lintner CAPM is true then α should be equal to $\beta_z^*(R_m^* - R_f)$. Thus, since β_z^* is not equal to 0, α should in fact be not equal to zero and further since the return on the tangent portfolio m^* is a random variable, hence α should also be variable. Thus, it can be seen that the results of BJS are fully compatible with the standard CAPM!

Roll in his paper also shows that the proxy used by BJS was not even close to the tangent portfolio. However, even if BJS had found that the intercept was equal to zero, their result would not have invalidated the CAPM, simply because of the fact that they were not using the true market portfolio.

Fama and MacBeth (1973) in their study use the Fisher's Arithmetic index (an equally weighted portfolio of all the stocks in the NYSE) as their proxy. This portfolio is not even close to the value-weighted portfolio and thus should not have been used as a market proxy. Thus the conclusions of Fama and MacBeth (1973) are also not immune to suspicion. It is clear that there will always exist a portfolio in the tangency position but it is not clear at all whether this portfolio is the value-weighted average of all assets, i.e. the market portfolio.

Furthermore, as shown by Roll, the situation is aggravated by the fact that both the Sharpe-Lintner CAPM and the Black's version of CAPM are liable to a type II error, i.e. likely to be rejected when they are true. This is true even if the proxy is highly correlated with the true market portfolio. Thus the efficiency or the inefficiency of the proxy does not imply anything about the efficiency of the true market portfolio.

4 CONCLUSION

Considering the arguments above, there is no doubt that it is not easy to give an unambiguous conclusion. On the one hand, there is strong empirical evidence invalidating the CAPM and on the other hand it is clear that the empirical findings themselves are not sufficient to discard the CAPM.

Indeed, as noted by many authors including Fama and French in their recent article, "The CAPM is wanted, dead or alive", the empirical tests have been undermined by the inability to observe the true market portfolio. In effect, even though the 'synthetic' CAPM based on the proxy market index can be rejected, nevertheless it is virtually impossible to reject the original CAPM.

Nonetheless, since the true market portfolio cannot be observed it is fair to say that the CAPM is of little use for practical purposes. It cannot be used for estimating the cost of capital, to evaluate the performance of fund managers or as an aid in event-study analysis. This does not imply, however, that its substitute, the synthetic CAPM be used instead because as already seen there is a host of evidence against this form of the CAPM.

Given that we will have to work with the proxy index in the foreseeable future, thus for practical purposes, Merton's intertemporal CAPM or some form of the APT would have to be resorted to for the purpose of explaining expected stock returns.

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