Printed Eastern Arabic Noisy Numerals Recognition Using Hidden Markov Model and Support Vectors Machine

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ABSTRACT: This research presents a comparison between two methods of learning-classification, the first one is a probabilistic and unsupervised which is the Hidden Model Markov (HMM), while the second is a statistic and supervised that is the Support Vectors Machine (SVM). These techniques are used for printed Eastern Arabic numerals recognition, in different situation: rotated and translated or resized and noisy. In the pre-processing phase we have used the thresholding technique while in the features extraction we exploited the Krawtchouk Invariant Moment (KIM). In fact, in order to make a precise comparison between these two methods, we have introduced two new concepts which are the threshold and the interval of stability of each numeral and for each of these two methods. The simulation results that we obtained demonstrates that SVM is more performing than the HMM technique in this recognition system.

KEYWORDS: The noisy printed Eastern Arabic numerals, the thresholding technique, The Krawtchouk invariant moment, the hidden Markov models, the support vectors machines.

1 INTRODUCTION

Character recognition is one of classical problems in Optical Character Recognition (OCR). Its goal is to predict the class of the input character using its features. There are in fact a several approaches for solving these problems such as that we have opted in this study which are both methods, the first one is a stochastic and unsupervised which is the Hidden Model Markov (HMM) while the second is a statistic and supervised which is the Support Vector Machine (SVM).

On the other hand, an OCR system is a very active and dynamic field of researches and developments in terms of innovation and creativity. It covers really a broad scope of activities like medicine, marketing, machine vision, robotics, remote sensing, and so on. In fact it is formed in general from tree principal phases which are the pre-processing, in this context different operations scan be used such as segmentation, binarization, and noise removal would be done to enhance the quality of input character, then the second phase is the features extraction that serves to extract the primitives from the input patterns such a way that having a great discrimination between characters which will enable realize efficiently the third phase which is the classification that is used to assign a unknown character in a given class.

In this study, we have pre-processed all numerals by the thresholding technique. In the features extraction phase, we have used the Krawtchouk Invariant Moment (KIM) that are used to convert each numeral image to a vector that will used as an input vector of HMM and of SVM which are exploited to train the images of the training database and then to classify those of the test database.

In another context, many studies have been carried on Latin, Arabic numerals and characters by using the hidden models Markov [1-4], the support vectors machines [5-10] or the invariant moments [11-14]. However, this study is focused on printed Eastern Arabic degraded numerals recognition.
Anyway, this paper is organized as follows: In first section the introduction is given then the pre-processing technique is presented in second section then in third section the features extraction explained. While the fourth section describes the recognition process. In fifth section the experimental results are presented, finally this work is ended by a conclusion.

Moreover, the recognition system that we have used is presented in the following figure:

![Proposed System Recognition Diagram](image)

**2 Pre-Processing**

The first phase of Latin numerals recognition system is the pre-processing that serves to noise removal and to produce a best quality of numeral image which will enable to use it efficiently by the features extraction phase. In this study, we have pre-processed each numeral image by a thresholding technique in order to construct all images contains a black and white colors no more according a preset threshold.

**3 Features Extraction**

The extraction of image features is the fundamental step for image classification. In fact selecting appropriate features extraction method is one of the most important factors to archive high classification performances in characters recognition systems, in order to achieve this phase, we have opted the Krawtchouk invariant moments [15].

**3.1 The KRAWTCHOUK INVARIANT MOMENT**

**3.1.1 The Krawtchouk Polynomial**

By definition, the Krawtchouk polynomial of order \( n \) is given by:

\[
K_n(x; p, N) = \sum_{k=0}^{N} a_k(p,x) \frac{x^k}{(c)_k} = _2F_1(-n,-N,-N; 1, -p)
\]  

\( x, n = 0, 1, 2, \ldots \) \( N \), \( N > 0 \), \( p \in [0, 1] \).

\( \_2F_1 \) is the hyper geometric function defined by:

\[
\_2F_1(a, b; c; x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!}
\]

And \((a)_k\) is the pochhammer symbol (rising factorial) defined by:

\[
(a)_k = a(a+1)(a+2)\ldots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)}
\]

The \( \Gamma \) function is defined by:

\[
\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt
\]
The set of Krawtchouk polynomial \( \{ kn(x; p, N) \} \) forms a complete set of discrete basis functions with the weight function:

\[
w(x; p, N) = \binom{N}{x} p^x (1 - p)^{N-x}
\]

(6)

And satisfies the orthogonally condition:

\[
\sum_{x=0}^{N} w(x;p,N)K_{n}(x;p,N)K_{m}(x;p,N) = \rho(n,p,N)\delta_{nm}
\]

(7)

\(\rho(n; p, N)\) is the squared norm defined by:

\[
\rho(n; p, N) = (-1)^n \left( \frac{1 - p}{p} \right)^n \frac{n!}{(-N)_n}
\]

(8)

And \(\delta_{nm}\) is the Kronecker symbol defined by:

\[
\delta_{nm} = \begin{cases} 
1 & \text{if } n = m \\
0 & \text{else}
\end{cases}
\]

(9)

### 3.1.2 The Krawtchouk Moment

The Krawtchouk moment have the interesting property of being able to efficiently extract local features of an image this moment of order \((n+m)\) of an image \(f(x,y)\) is given by:

\[
Q_{nm} = \sum_{x=0}^{N} \sum_{y=0}^{N} K_n(x;p,N-1)K_m(y;p,M-1)f(x,y)
\]

(9)

The \(N \times M\) is the number of pixels of an image \(f(x,y)\). The set of weighted Krawtchouk polynomial \(\tilde{K}_n(x;p,N)\) is:

\[
\tilde{K}_n(x; p, N) = K_n(x; p, N) \sqrt{\frac{w(x; p, N)}{\rho(x; p, N)}}
\]

(10)

### 3.1.3 The Krawtchouk Invariant Moment

The geometric moment of an image \(f(x,y)\) is given by:

\[
M_{pq} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^p y^q f(x,y)
\]

(11)

The standard set of the geometric invariant moments that’s independent to rotation, scaling and translation is:

\[
V_{nm} = M_{pq}^{\frac{1}{2}} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} [(x-x_0)\cos\theta+(y-y_0)\sin\theta]^n [(y-y_0)\cos\theta-(x-x_0)\sin\theta]^m f(x,y)
\]

(12)

The Krawtchouk invariant moment is:

\[
\Omega_{nm} = \Omega_{nm} \sum_{i=0}^{n} \sum_{j=0}^{m} a_{i,n;p_1} a_{j,m;p_2} \tilde{V}_{ij}
\]

(13)

\[
\Omega_{nm} = [\rho(n; p_1, N-1),\rho(m; p_2, M-1)]^{-1/2}
\]

(14)

\[
\tilde{V}_{ij} = \sum_{p=0}^{i} \sum_{q=0}^{j} \binom{i}{p} \binom{j}{q} \left( \frac{N^2}{2} \right)^{p+q} \left( \frac{N}{2} \right)^{i+p} \frac{1}{p!} V_{pq} \]

(15)

\[
\binom{i}{j} = \frac{x!}{y!(x-y)!}
\]

(16)
4 RECOGNITION

4.1 THE HIDDEN MARKOV MODEL

The hidden Markov model is a stochastic process [16] that, after a learning phase, estimates the probability of an observation being generated by this model. The hidden Markov model can be seen as a set of discrete states and transitions between these states; it can be defined by $\lambda = (A, B, \pi)$, where $A$ is the matrix of the probabilities of transitions, $B$ is the matrix of the probabilities of observations, and $\pi$ is the vector probability of initial states.

$N$ : The number of states $s_1, s_2, \ldots, s_N$.

$T$ : The number of observations.

$q_t$ : The state of the process at the time $t$ ($q_t = \{s_1, s_2, \ldots, s_N\}$).

$o_t$ : The observation at the time $t$ ($o_t = \{v_1, v_2, \ldots, v_M\}$).

$M$ : The size of observations $v_1, v_2, \ldots, v_M$.

\[
A = \{a_{ij} = \text{Prob}(s_j / s_i)\} ; \sum_{j=1}^{N} a_{ij} = 1
\] (17)

\[
\pi = \{\pi_i = \text{Prob}(s_i)\} ; \sum_{i=1}^{N} \pi_i = 1
\] (18)

\[
B = \{b_j(k) = \text{Prob}(o_k = v_j / o_i = s_j)\} ; \sum_{k=1}^{M} b_j(k) = 1
\] (19)

In the case where observation is continuous, the HMM is defined by $\lambda = (A, \pi, \mu_i, \sigma_i)$, where $\mu_i$ and $\sigma_i$ are respectively the mean and the standard deviation of the state $i$ of the Gaussian function:

\[
b_j(k) = \text{Prob}(o_k = v_j / o_i = s_j) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(o_k - \mu_i)^2}{2\sigma_i^2}}
\] (20)

In our work we used the HMM with the continuous observation. In this context of the learning-classification phase, it carried as follow: in the learning phase, each numeral image that is converted to a vector in the features extraction phase by KIM; this vector is used as an observation vector of an initial own HMM of this numeral just for determining the probability that generated this observation. Then this model is trained for maximizing this probability by using the Baum-Welch algorithm. All these trained models (optimal models) of all numerals are saved for to form a learning base. In the classification phase, an unknown numeral (test numeral) is presented as a vector of observation. Then the probability generated by this observation is calculated by all the optimal models already recorded in the learning base by the forward algorithm. The recognition will be given to the numeral with highest probability.

4.2 THE SUPPORT VECTORS MACHINE

For a set of vectors $x_i \in \mathbb{R}^n$ and both classes which the first contains a party of these vectors that labeled by 1, and the second includes other vectors and bears a label -1. The SVM [17] consists to find a decision function that separates in an optimal manner between these classes that is to say maximizes as much as possible the distance between them.

![Fig. 2. Determination of optimal hyperplane, support vectors, maximum marge and valid hyperplanes.](image-url)
This decision function defined by:

\[ f(x, w, b) : x \rightarrow y \]  

(21)

With \( w \) and \( b \) are the parameters of the classifier \( y \) is the label. Maximization of distance between these classes is equivalent to minimization of \( w \) under constraints (UC) in that called the primal problem:

To minimize \( \frac{1}{2} \|w\|^2 \)  

Subject to \( y_i (wx_i + b) \geq 1, \forall i = 1, 2, ..., n \)  

(22)

The dual problem can be obtained by using the Lagrangian operator:

\[ L(w, b, \alpha) = \frac{1}{2} \|w\|^2 - \sum_{i=1}^{n} \alpha_i [y_i (wx_i + b) - 1] \]  

(23)

Where the variables \( \alpha_i \) are called Lagrange multipliers. The dual problem is therefore:

To maximize \( D(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j K(x_i, x_j) \)  

Subject to \( \sum_{i=1}^{n} \alpha_i y_i = 0, 0 \leq \alpha_i \leq C, \forall i = 1, 2, ..., n \)  

(24)

The parameter \( C \) is a positive constant fixed in advance; it's called the constant of penalty. The decision function is:

\[ f(x) = \sum_{i=1}^{n} \alpha_i^* y_i K(x, x_i) + b \]  

(25)

Where \( \alpha_i^* \) are the nonzero variables, the \( K \) is called a kernel function.

Kernel linear: \( xy \)

Kernel polynomial of degree \( n \): \( (xy + 1)^n \)

Gaussian Radial Basis Function (GRBF): \( e^{-\frac{(x-y)^2}{2\sigma^2}} \)

The method presented above is designed only for a problem of two classes; in fact several studies have generalized of the SVM to \( N \) classes [18]. Among these studies, we have used in this research a strategy called one against all that is based to use \( N \) decision functions allowing to make a discrimination of a class having a label equal to 1 and containing a one vector against all other vectors included in a other class opposite that is labeled by the value -1. In the classification phase, the value image of an unknown vector \( X \) (test numeral) is calculated by all \( N \) decision functions which are already obtained in learning phase. The recognition will be assigned to numeral that the decision function separates its class to another class containing the rest of numerals which gives the highest value.

\[ \text{Class (} X \text{)} = \arg \max_{i=1, 2, ..., N} ( f_i(X) ) \]  

(26)

5 Experiments And Results

We choose the sizes of all images 30x30 pixels.

Firstly, we present the test numeral translated, rotated or resized and not noisy, then we add more and more a quantity of noise of type 'salt & pepper' for to know the effect of noise on the rate recognition of each numeral also to that of all numerals called global rate. We have chosen the KIM parameters: \( p=q=0.96 \). The noise values are \( [0, 0.01, 0.02, ..., 0.30] \).
Fig. 3. Eastern Arabic numerals rotated, translated or resized and noisy by different values of noise ‘salt & pepper’

We group the values of the recognition rate \( \tau_n \) (given in %) for each numeral that we obtained in the following table:

**Table 1. The recognition rate for each numeral using HMM and SVM.**

<table>
<thead>
<tr>
<th>Numeral</th>
<th>( \tau_n ) (HMM)</th>
<th>( \tau_n ) (SVM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>٠</td>
<td>80.64</td>
<td>100.0</td>
</tr>
<tr>
<td>١</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>٢</td>
<td>83.87</td>
<td>90.32</td>
</tr>
<tr>
<td>٣</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>٤</td>
<td>87.09</td>
<td>93.55</td>
</tr>
<tr>
<td>٥</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>٦</td>
<td>93.55</td>
<td>96.77</td>
</tr>
<tr>
<td>٧</td>
<td>100.0</td>
<td>83.87</td>
</tr>
<tr>
<td>٨</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>٩</td>
<td>61.29</td>
<td>67.74</td>
</tr>
</tbody>
</table>

And the associated graphical representation to this table is:

*Fig. 4. The graphic representation of recognition rate for each numeral using HMM and SVM*

We present the evolution of the global recognition rate \( \tau_g \) (given in %):
Table 2: The values of global recognition rate in function of noise added of HMMs and SVMs

<table>
<thead>
<tr>
<th>Noise ($\tau_g$)</th>
<th>$\tau_g$ (HMM)</th>
<th>$\tau_g$ (SVM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.01</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.02</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.03</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.04</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.05</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.06</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.07</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.08</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.09</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.10</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.11</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.12</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.13</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.14</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.15</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.16</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.17</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.18</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.19</td>
<td>90.0</td>
<td>100</td>
</tr>
<tr>
<td>0.20</td>
<td>90.0</td>
<td>100</td>
</tr>
<tr>
<td>0.21</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>0.22</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>0.23</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>0.24</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>0.25</td>
<td>80.0</td>
<td>90.0</td>
</tr>
<tr>
<td>0.26</td>
<td>70.0</td>
<td>80.0</td>
</tr>
<tr>
<td>0.27</td>
<td>60.0</td>
<td>80.0</td>
</tr>
<tr>
<td>0.28</td>
<td>60.0</td>
<td>70.0</td>
</tr>
<tr>
<td>0.29</td>
<td>50.0</td>
<td>60.0</td>
</tr>
<tr>
<td>0.30</td>
<td>40.0</td>
<td>50.0</td>
</tr>
</tbody>
</table>

And the associated graphical representation to table above is:

![Graphical representation](image)

Fig. 5. Graphical representation of the global rate recognition $\tau_g$ in function of noise added for the HMMs and the SVMs.

Analysis and comment:
The global rate recognition $\tau_g$ is a decreasing in function of noise added to each numeral, but the important remark is that the falling of this rate of HMM is greater than the rate of SVM, this shows that the SVM is more efficient than the HMM in recognition of noisy Eastern Arabic numerals.

5.1 RECOGNITION THRESHOLDING AND RECOGNITION INTERVAL

5.1.1 IN THE SVM CASE

Now, we propose to classify the KIM of all numerals according to their degree of robustness against noise presented in them when we will recognize these numerals by HMM then by SVM.

Of this fact, we note $f_i$ the decision function that separates the class which is labeled by 1 of a numeral $N_i$ to other class which is labeled by -1 contains all other numerals $N_j$ for $i,j=\{\text{٠, ١, ٢, ٣, ٤, ٥, ٦, ٧, ٨, ٩}\}$, $i \neq j$.

If a test numeral $N_{\text{test},i}$ translated, rotated, or resized and not much noisy, it is correctly recognized, that is to say its class is:

$$\text{class}(N_{\text{test},i}) = \arg \max_i \left( f_0(N_{\text{test},j}) \ldots f_9(N_{\text{test},j}) \right) = i$$

But this equation is it valid up to what amount of added noise to $N_{\text{test},i}$ ?

Practically, we found that for any test numeral $N_{\text{test},i}$ of the test base and $F_i(O_{\text{test},i}/\lambda_i^*)$ the probability which generate $O_{\text{test},i}$ by $\lambda_i^*$ obtained by the Forward algorithm.

We call this noise value the threshold of recognition or of tolerance or of stability, we symbolize it by $n_{t,i}$ in other word it’s the amount of noise from which the numeral $N_{\text{test},i}$ will be badly recognized.

So, into interval $[0\ n_{t,i})$, we say in this once that the recognition system is a stable system.

For the determination of the threshold of stability $n_{t,i}$ for each numeral $N_{\text{test},i}$, we must follow the evolution of $f_i$ in function of added noise to $N_{\text{test},i}$.

5.1.2 IN THE HMM CASE

Firstly, we denote:

$O_i$ The observation vector that models a numeral $N_i$ of the learning base and $\lambda_i^*$ its optimal model that obtained by the Baum-Welch algorithm.

$O_{\text{test},i}$ The observation vector models a numeral $N_{\text{test},i}$ of the test base and $F_i(O_{\text{test},i}/\lambda_i^*)$ the probability which generate $O_{\text{test},i}$ by $\lambda_i^*$ obtained by the Forward algorithm.

If this numeral is translated, rotated, or resized and not much noisy, it is correctly recognized, that is to mean its class is:

$$\text{class}(O_{\text{test},i}) = \arg \max (F_0 \ldots F_9) = i$$

As we did in SVM, this equation is valid until to a value of added noise to $N_{\text{test},i}$ called the threshold of stability.

Thus for each numeral $O_{\text{test},i}$ to determine the threshold of stability $n_{t,i}$ it must follow the evolution of $F_i$ in function of noise added to it. We want to realize a comparison between the threshold of recognition $n_{t,i}$ of the KIM of each numeral when using the HMM then the SVM (we neglect a very few points of instability).

Also we call $[0\ n_{t,i})$ the stability interval (SI) is that $N_{\text{test},i}$ is correctly recognized.

Seen the results we did, we conclude that we do $n_{t,i}$ is different from one numeral to other and from the HMM to the SVM.
Table 3: The values of the recognition thresholds of each numeral by the HMM and the SVM

<table>
<thead>
<tr>
<th>Numeral</th>
<th>HMM</th>
<th>SVM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_{th}$</td>
<td>$[0\ n_{th}]$</td>
</tr>
<tr>
<td>٠</td>
<td>0.24</td>
<td>[0 0.24]</td>
</tr>
<tr>
<td>١</td>
<td>0.30</td>
<td>[0 0.30]</td>
</tr>
<tr>
<td>٢</td>
<td>0.25</td>
<td>[0 0.25]</td>
</tr>
<tr>
<td>٣</td>
<td>0.30</td>
<td>[0 0.30]</td>
</tr>
<tr>
<td>٤</td>
<td>0.26</td>
<td>[0 0.26]</td>
</tr>
<tr>
<td>٥</td>
<td>0.30</td>
<td>[0 0.30]</td>
</tr>
<tr>
<td>٦</td>
<td>0.28</td>
<td>[0 0.28]</td>
</tr>
<tr>
<td>٧</td>
<td>0.30</td>
<td>[0 0.30]</td>
</tr>
<tr>
<td>٨</td>
<td>0.30</td>
<td>[0 0.30]</td>
</tr>
<tr>
<td>٩</td>
<td>0.18</td>
<td>[0 0.18]</td>
</tr>
</tbody>
</table>

We say into $[0\ n_{th}]$ the recognition system is stable, and out this interval the system is instable.

We group the approximate variations of decision function $f_i$ and the forward probability $F_i$ in function of noise $n \in [0\ 0.3]$ to each numeral in one graph for each numeral $i = \{٠,١,٢,٣,٤,٥,٦,٧,٨,٩\}$ (we neglect a very few points of instability).

Fig. 6. $f_٠$ and $F_٠$ in function of added noise to numeral ٠

Fig. 7. $f_١$ and $F_١$ in function of added noise to numeral ١

Fig. 8. $f_٢$ and $F_٢$ in function of added noise to numeral ٢

Fig. 9. $f_٣$ and $F_٣$ in function of added noise to numeral ٣
Analysis and comment

Taking into account the results obtained after having implemented these recognition systems, we can conclude that:

- For each numeral, the variation of its decision function and its Forward probability are decreasing as a function of noise added to numeral, but the decrease of the Forward probability is greater than the decision function.
- For each numeral, in generally the recognition threshold \( n_{t,i} \) of the SVM is bigger that of the HMM.

All this shows that the SVM is more robust than the HMM in noisy Eastern Arabic numerals recognition.

6 Conclusion

The obtained results in the recognition of noisy Eastern Arabic numerals show that reliable recognition is possible by using the thresholding technique in the preprocessing phase, the KIM in the features extraction phase and the HMM and the SVM in the recognition phase. Our goal in this study is to compare between these both last methods, for well make this comparison; we have introduced a new concepts which are the threshold and interval of recognition or of stability of each numeral when using in first time the HMM then using the SVM in second time. The simulation results demonstrate that the SVM is the more performing than the HMM in this recognition.
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