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Comparison between solution of POCKLINGTON'S and HALLEN'S integral equations for Thin wire Antennas using Method of Moments and Haar wavelet

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ABSTRACT: In this paper, it is attempted to approach a fast efficient algorithm for solving the famous Hallen's and Pocklington's integral equations, regarding the current distribution on a finite-length linear thin wire antenna. Here, the conventional moment method in conjunction with wavelet basis functions was used to obtain the current distribution of the antenna. The aim of this work is first to introduce the application of wavelet in electromagnetic scattering, secondly a comparison of the two method of analysis the thin wire antenna. By using the wavelet expansion, wavelets as basis and testing functions, a sparse matrix is generated from the previous moment method dense matrix. A sparsely filled matrix is easier to store and invert. The result extracted from Pocklington's integral equation gives better convergence at the feeding point, though it takes more time to be computed because of the complexity in Pocklington's equation. Results are compared to the previous work done and published, excellent results are obtained.

KEYWORDS: Thin wire Antennas, Hallen's integral equations, Pocklington's integral equations, Moments Method, Haar wavelet, Multiresolution.

1 INTRODUCTION

The problem of time harmonic radiation of thin-wire antennas involves the solution of the Electric Field Integral Equation (EFIE) [2]. To solve numerical the EFIE for its unknown current distribution along the antenna, one can employ the method of moments (MoM), in this method, the unknown current can be expanded by various basis functions, including the conventional triangular, sinusoidal, and piecewise constant functions [3-4]. The application of appropriate boundary conditions reduces the problem to a matrix equation. The matrix, known as the Impedance Matrix (IM), is generally dense [2-6].

Recently, wavelets found their application in solving integral equations, resulting in sparse impedance matrices [1-2]. This is due to features of vanishing moments, orthogonality and multiresolution analysis in wavelets. Despite the above nice properties, standard wavelets are defined on the whole real line, while practical electromagnetic problems are often restricted to a finite interval or domain [6-8].

The paper follows by a brief description of two particular electromagnetic problems (Hallen's and Pocklington's), we determine the current distribution on a dipole antenna of finite length, we solve numerical the integral equations Hallén's and Pocklington's using the method of moments and wavelet, and then compare the effectiveness of each model using several examples.

2 FORMULATION OF THE PROBLEM

Let us consider the scattering from a thin wire, shown in Figure 1. Consider a perfectly conducting long, z-oriented thin wire of length L whose radius a is much less than L and ʎ. An incident electric field Er excites on this wire a surface current Jr [4].

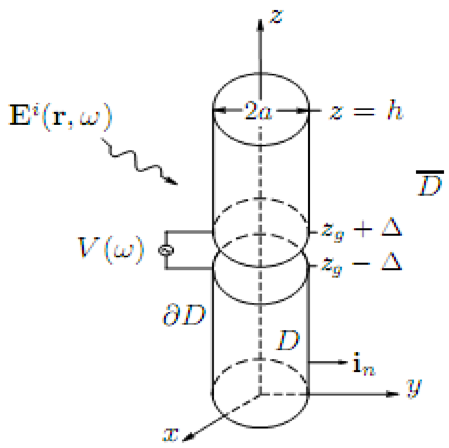


Fig1: Geometry of Thin Wire Antenna [9]

Since the wire is very thin, we will assume that $J(r)$ can be written in terms of a z-oriented filamentary current $I_Z(z)$ as

$$J(r) = \frac{I_Z(z)}{2\pi a}\vec{z} \qquad (1)$$

Where there is no dependence on wire azimuthal angle Φ.

In cylindrical coordinates, we can write the corresponding magnetic vector potential Az in terms of the surface integral

$$Az(\rho,\varphi,z) = \mu \int_{-L/2}^{L/2}\int_{0}^{2\pi} \frac{Iz(z')}{2\pi}\frac{e^{-ikr}}{4\pi r}d\varphi' dz' \qquad (2)$$

If we assume be a very small, r can be approximated as $r = \sqrt{(z-z')^2+\rho^2}$.

The expression becomes Az:

$$Az(\rho,\varphi,z) = \mu \int_{-L/2}^{L/2} Iz(z')\frac{e^{-ikr}}{4\pi r}dz' \qquad (3)$$

The radiated field can be given by the following formula:

$$E_z^s = -j\omega Az - \frac{j}{\omega\mu\varepsilon}\frac{\partial^2}{\partial z^2}Az \qquad (4)$$

By enforcing the boundary condition of the zero tangential electric field on the surface of the wire, we can now a write a thin wire EFIE in terms of the incident field $E_z^s = -E_z^i$.

$$E_z^i = j\omega Az + \frac{j}{\omega\mu\varepsilon}\frac{\partial^2}{\partial z^2}Az = \frac{j}{\omega\varepsilon\mu}\left[\frac{\partial^2}{\partial z^2}+k^2\right]Az \qquad (5)$$

There are two common forms by which (5) are commonly written.

- The first retains the differential operator outside the integral in (5), which is

$$E_z^i = \frac{j}{\omega\varepsilon\mu}\left[\frac{\partial^2}{\partial z^2}+k^2\right]Az = \frac{j}{\omega\varepsilon}\left[\frac{\partial^2}{\partial z^2}+k^2\right]\int_{-L/2}^{L/2} Iz(z')\frac{e^{-ikr}}{4\pi r}dz' \qquad (6)$$

- This is called Hallen's integral equation.
- We may also move the differential operator under the integral sign,

$$E_z^i = \frac{j}{\omega\varepsilon}\int_{-L/2}^{L/2} Iz(z')\left[\frac{\partial^2}{\partial z^2}+k^2\right]\frac{e^{-ikr}}{4\pi r}dz' \qquad (7)$$

This is called Pocklington's integral equation.

Pocklington's equation is not as well behaved as Hallén's, since the differential operator acts directly on the Green's function. As we will see, the results obtained by it typically exhibit slower convergence and less accuracy than those obtained from Hallén's [3].

3 MOMENT METHOD FORMULATION

3.1 SOLVING THE EQUATION OF HALLEN'S

Hallén's equation $-j\omega\varepsilon\mu E_z^i = \left[\frac{\partial^2}{\partial z^2} + k^2\right] Az$ is an inhomogeneous scalar Helmholtz equation for (z) , which we can solve by the Green's function method [4-10].

The general and particular solution to the homogeneous equation is

$$Az(z) = C_1 e^{jkz} + C_2 e^{-jkz} - \frac{j\mu}{2\eta}\int_{-\frac{l}{2}}^{\frac{l}{2}} \sin(k|z - z'| E_z^i(z')dz' \qquad (8)$$

Substituting the expression of Az on the left side are:

$$\int_{-L/2}^{L/2} Iz(z')\frac{e^{-ikr}}{4\pi r}dz' = C_1 e^{jkz} + C_2 e^{-jkz} - \frac{j\mu}{2\eta}\int_{-\frac{l}{2}}^{\frac{l}{2}} \sin(k|z - z'| E_z^i(z')dz' \qquad (9)$$

Note also that the two homogeneous terms in the above can also be written as

$$C_1 e^{jkz} + C_2 e^{-jkz} = D_1 \cos(kz) + D_2 \sin(kz) \qquad (10)$$

Since we expect the solution to be symmetric, we set D2 in the above to zero.

Method of Moments is to transform an integral equation in a linear matrix equation can be solved numerically. To do so, let us divide the wire uniformly to N equal segments each by height Δ=l/N as shown in figure (2).

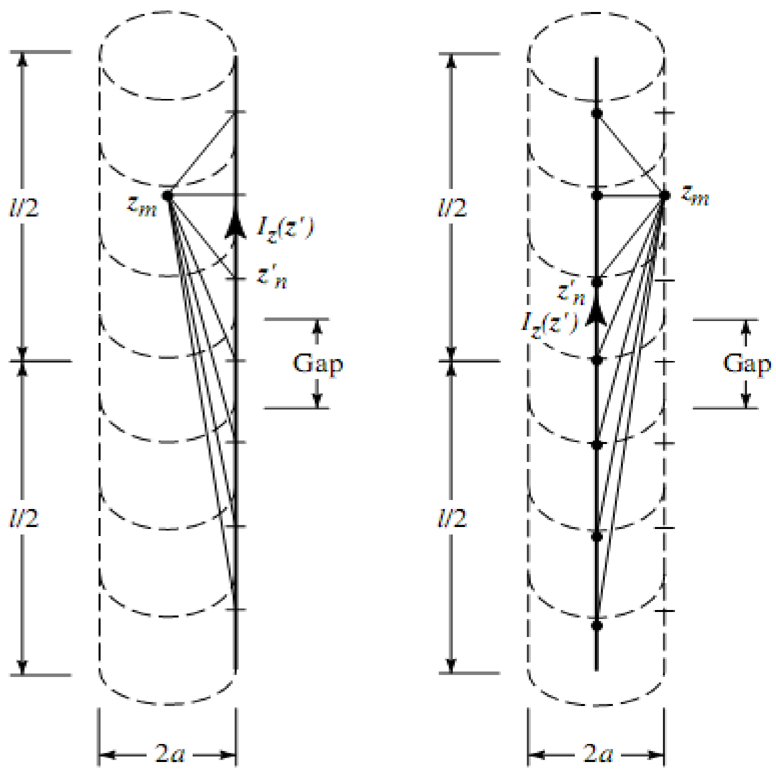


Fig. 2. Dipole segmentation and its equivalent current. [3]

We first expand the left-hand side of (9) using N weighted subdomain basis functions yielding and Testing by N functions f_m, giving

$$\sum_{n=1}^{N}\int f_m(z)\int a_n f_n \frac{e^{-jkR}}{4\pi R}dzdz' = D_1\int f_m(z)\cos(kz)\,dz - \frac{j}{2\eta}\int f_m(z)\int_{-\frac{l}{2}}^{\frac{l}{2}}\sin(k|z-z'|)\,E_z^i(z')dz'dz \qquad (11)$$

The above expressions result in the following matrix equation:

$$Za = D_1 s + b \qquad (12)$$

Matrix elements Z_{mn} given by:

$$Z_{mn} = \int f_m(z)\int f_n \frac{e^{-jkR}}{4\pi R}dzdz' \quad (13)$$

To obtain the solution vector a we must determine the constant D1, which can be done by enforcing the boundary conditions at the ends of the wire, i.e., Iz (l/2)=Iz(-l/2)=0. We will do this in our discretized version by forcing the basis function coefficient at each end to be zero. This can be expressed vectorially as $u^T a=0$, where u^T[1,0,…,0,1]. Solving (12) for a we obtain

$$a = D_1 Z^{-1}s + Z^{-1}b \qquad (14)$$

And solving for D_1 yields $D_1 = \frac{u^T Z^{-1} b}{u^T Z^{-1} s}$

To solve the symmetric Hallén's equation with pulse basis functions and point matching.

The matrix elements of (13) become

$$Z_{mn} = \int_{zn-\Delta z/2}^{zn+\Delta z/2}\frac{e^{-jkR}}{4\pi R}dz' \qquad (15)$$

The right-hand side vector elements are

$s_m = \cos(kz_m),\ b_m = -\frac{j}{2\eta}\sin(k|z_m|)$, and $R = \sqrt{(zm-z')^2+a^2}$

The major advantage of Hallén's equation is the ease with which a converged solution may be obtained, while its major drawback lies in the additional work required in finding the integration constants C1 and C2 [13].

3.2 SOLVING POCKLINGTON'S EQUATION

Pocklington's equation can be solved by a straightforward application of the moment method, since the differential operator is inside the integral and acts on the Green's function only [10].

$$-j\omega\varepsilon E_z^i = \int_{-L/2}^{L/2} Iz(z')\left[\frac{\partial^2}{\partial z^2}+k^2\right]\frac{e^{-ikr}}{4\pi r}dz' \qquad (16)$$

Expanding the current into a sum of N weighted basis functions and applying N testing functions we obtain a linear system with matrix elements.

$$Zmn = \int f_m(z)\int f_n(z')\left[\frac{\partial^2}{\partial z^2}+k^2\right]\frac{e^{-ikr}}{4\pi r}dz'dz \qquad (17)$$

And excitation vector elements bm given by

$$bm = -j\omega\varepsilon\int f_m(z)\,E_z^i(z)dz \qquad (18)$$

Using pulse basis functions for the current and point matching for testing, we obtain the following excitation vector elements:

$$bm = -j\omega\varepsilon E_z^i(zm) \qquad (19)$$

The impedance matrix elements can be written as:

$$z_{mn} = \frac{k^2}{4\pi}\int_{zn-\Delta z/2}^{zn+\Delta z/2} \frac{e^{-jkR}}{R} dz' + \left[(zm - z')\frac{1+jkR}{R^3} e^{-jkR}\right]_{z'=zn-\Delta z/2}^{z'=zn+\Delta z/2} \qquad (20)$$

For example, to solve a matrix equation in our problems, we first generate the impedance matrix, then calculate its inverse and multiply this inverse matrix by both sides of the equation which is extremely expensive considering time and memory. In the next, we are going to introduce one of the most popular fast algorithms called "Fast Wavelet Transform (FWT)" to achieve the recent solutions, using less time and memory allocations.

4 WAVELET EXPANSION

The analysis through the wavelets has been a good alternative in replacement of the classical analyses that utilize the Fourier series, chiefly when treating acoustic signals, interpreting seismic signals and in the solution of numerical methods applied to electromagnetism and electrostatics [6].

In this case, the moment method can be directly applied and wavelets are used as expansion functions [7].The wavelets used here are Haar basis an orthogonal type. Its study is useful from theoretical point of view, because it offers an intuitive understanding of many multi-resolution properties. Furthermore, due to its simplicity Haar wavelets are widely employed. The scaling function is φ(x), and the mother wavelet function is ψ(x), these are defined as [2-17]:

$$\phi(x) = \begin{cases} 1, & for\ 0 \le x \le 1 \\ 0, & Otherwise \end{cases} \qquad \text{And} \qquad \psi(x) = \begin{cases} 1, & for\ 0 \le x \le 1/2 \\ -1, & \text{for } 1/2 \le x \le 1 \\ 0, & Otherwise \end{cases}$$

Also the scaling and the mother wavelets functions are defined by:

$$\phi_{jn}(x) = 2^{\frac{j}{2}}\phi(2^j x - n) \qquad \text{And} \qquad \psi_{mn}(x) = 2^{\frac{m}{2}}\phi(2^m x - n)$$

Where (m or j) are the resolution level and (n) is the translation factor. The vector space Vj Linear span of ϕ_{jn} for j=0,1…. And n=0,1……..2j-1. The property of Wj $\in$ Vj $+ 1$ holds the relation between the vector spaces of different functions by:

$$V^{j+1} = V^j \oplus W^j$$

The above equation states that the sub-space W^j in a larger subspace V^{j+1} which means for a given function fϵR^N with N samples or N Dimensional vector, the projection of this function into the orthogonal basis is as fellow:

$$V^k = V^0 \oplus W^0 \oplus W^1 \ldots\ldots.. \oplus W^{k-1} \qquad \text{For k=0, 1….N-1}$$

And can be expressed in an inner product as:

$$f = < f, v_0 > v_0 + < f, v_1 > v_1 + \cdots + < f, v_{N-1} > v_{N-1} \qquad (21)$$

The wavelets are applied directly upon the integral equation. The density of current will be represented as a linear combination of the set wavelets functions and scaling functions as fellow:

$$J_Z(x) = \sum_n a_n \emptyset_{j.n}(x) + \sum_{m=j}^{2^j-1}\sum_n C_{m.n}\psi_{m.n}(x) \qquad (22)$$

For example for N=2, equation (18) can be expressed as:

$$J_x(x) = a_0\emptyset_{0,0} + c_{0,0}\psi_{0,0} + c_{1,0}\psi_{1,0} + c_{1,1}\psi_{1,1} + c_{2,0}\psi_{2,0} + c_{2,1}\psi_{2,1} + c_{2,2}\psi_{2,2} + c_{2,3}\psi_{2,3}$$

The number of wavelets used here are 8 and the matrix corresponding is of 8x8 = 64 elements. The main mathematical properties which enable sparse matrix generation are the orthogonally and the vanishing moment. A function ψ(x) is said to have vanishing moment of N order if:

$$\int_{-\infty}^{+\infty} x^n \psi(x).\,dx = 0 \qquad \forall\, n = 0,1 \ldots..(N-1) \qquad (23)$$

The fact that the wavelets are orthogonal and the presence of vanishing moment, this is enabling sparse matrix production. When applying equation (18) into (10) we obtain the set of matrix equation as fellow:

$$\begin{bmatrix} [Z_{\emptyset,\emptyset}] & [Z_{\emptyset,\psi}] \\ [Z_{\psi,\emptyset}] & [Z_{\psi,\psi}] \end{bmatrix} \begin{bmatrix} a_n \\ c_{m,n} \end{bmatrix} = \begin{bmatrix} < E_z^{inc}, \emptyset_{j,n} > \\ < E_z^{inc}, \psi_{m,n} > \end{bmatrix} \quad (24)$$

Where:

$$[Z_{\emptyset,\emptyset}] =< \emptyset_{j,n}, \int_{-\frac{L}{2}}^{\frac{L}{2}} \emptyset_{j,n} \left[\frac{\partial^2}{\partial z^2} + k^2\right] \frac{e^{-ikr}}{4\pi r} dz' >$$

$$[Z_{\emptyset,\psi}] =< \emptyset_{j,n}, \int_{-\frac{L}{2}}^{\frac{L}{2}} \psi_{m,n} \left[\frac{\partial^2}{\partial z^2} + k^2\right] \frac{e^{-ikr}}{4\pi r} dz' >$$

$$[Z_{\psi,\emptyset}] =< \psi_{m,n}, \int_{-\frac{L}{2}}^{\frac{L}{2}} \emptyset_{j,n} \left[\frac{\partial^2}{\partial z^2} + k^2\right] \frac{e^{-ikr}}{4\pi r} dz' >$$

$$[Z_{\psi,\psi}] =< \psi_{m,n}, \int_{-\frac{L}{2}}^{\frac{L}{2}} \psi_{m,n} \left[\frac{\partial^2}{\partial z^2} + k^2\right] \frac{e^{-ikr}}{4\pi r} dz' >$$

Since Galerkin Method employs the same testing functions and basis function, in the same manner is the equation for incident field is expressed. After then the unknown constants are determined, and the current density can be found using equation (24).

5 NUMERICAL RESULTS

In this paper, two numerical examples are presented to illustrate the validity and the merits of this technique.

In both examples data are given for two selected wire lengths so that they include special cases of practical interest, e.g, L=ʎ/2 and L=1.5*ʎ.

5.1 RESULTS OBTAINED WITH HALLEN'S AND POCKLINGTON'S INTEGRAL EQUATION USING MOMENT'S METHOD

A computer program has been coded in Matlab language for the technique described above, now for applying the point matching form of MOM we take observation (field) points (or matching points) zm at the midpoint of each subinterval where the positions of zm and zn are:

$$z_m = \frac{(2m-1)l}{2N} \text{ And } z_n = \frac{(n-1)l}{N}, \text{ m=1,2,………..,N}$$

Representative results are shown for a perfectly electrically conducting (PEC) straight thin wire in free space. First, we compute the current distribution of a thin wire placed along the positive z-axis illuminated by a normally incident plane wave |Ei|= 1 V/m polarized in the z-direction.

The parameters of the structure are defined as following: Frequency f=1.5e+8 Hz, applied Voltage V=1v, length of the Wire l=ʎ/2 and l=1.5ʎ and wavelength of the Field ʎ=c/f.

The results obtained for Hallén's integral equation are shown in Figures (3.a), (3.b):

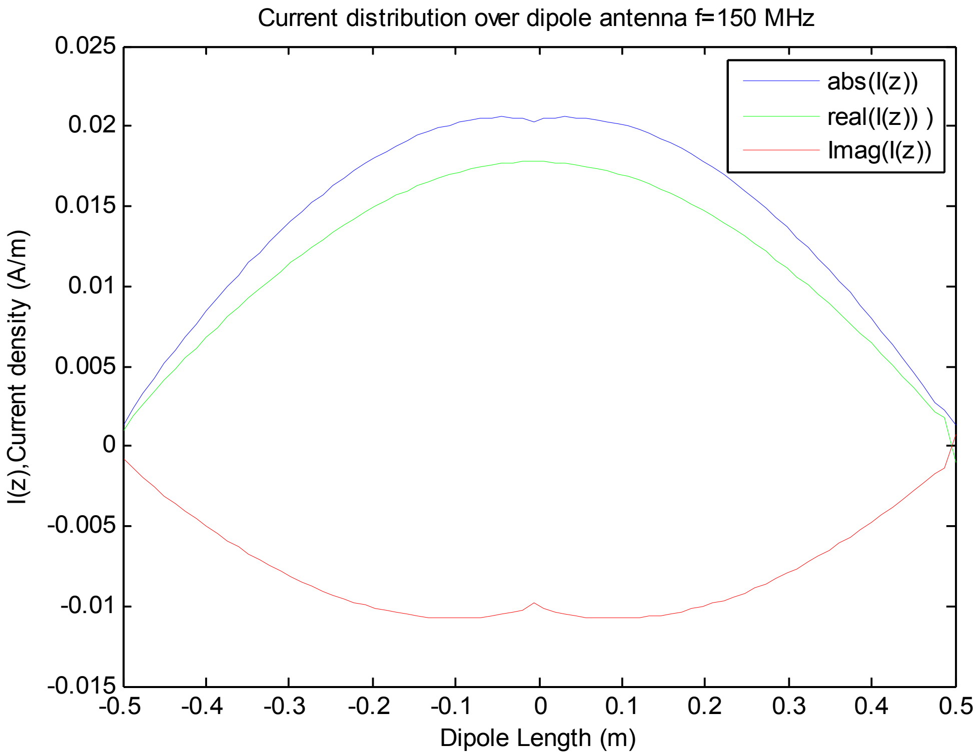


Fig. 3.a. Current distribution versus the position along a Thin wire, l=λ/2 and l/a= 518, using N = 80 basis functions

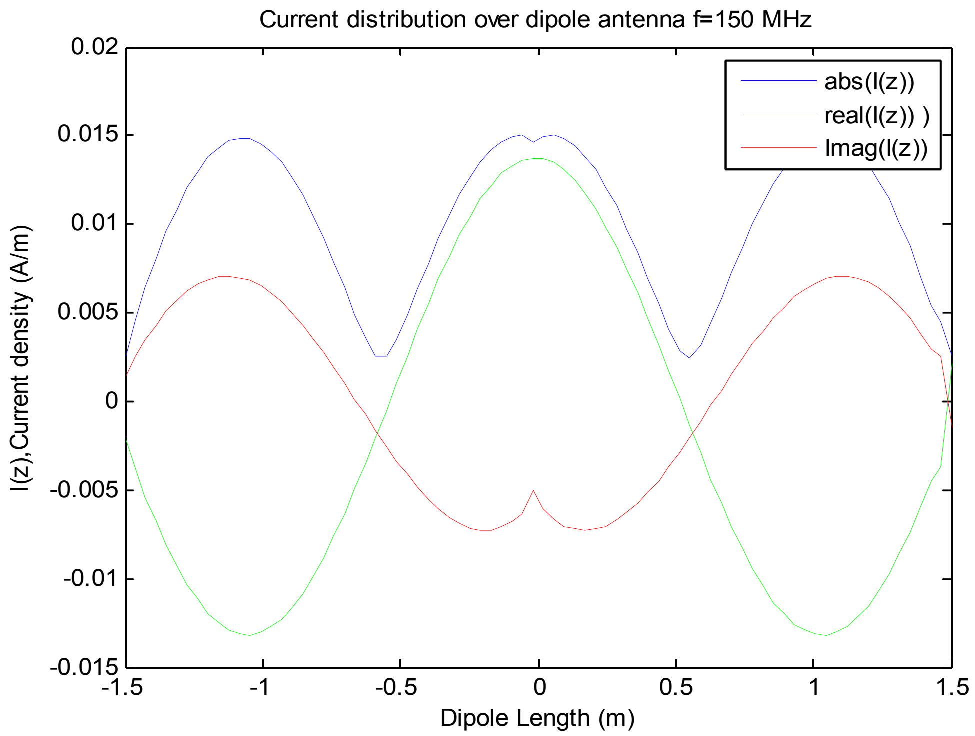


*Fig. 3.b. Current distribution versus the position along a Thin wire, l=1.5*λ and l/a= 518, using N = 80 basis functions*

The results obtained for Pocklington's integral equation are shown in Figures (4.a), (4.b):

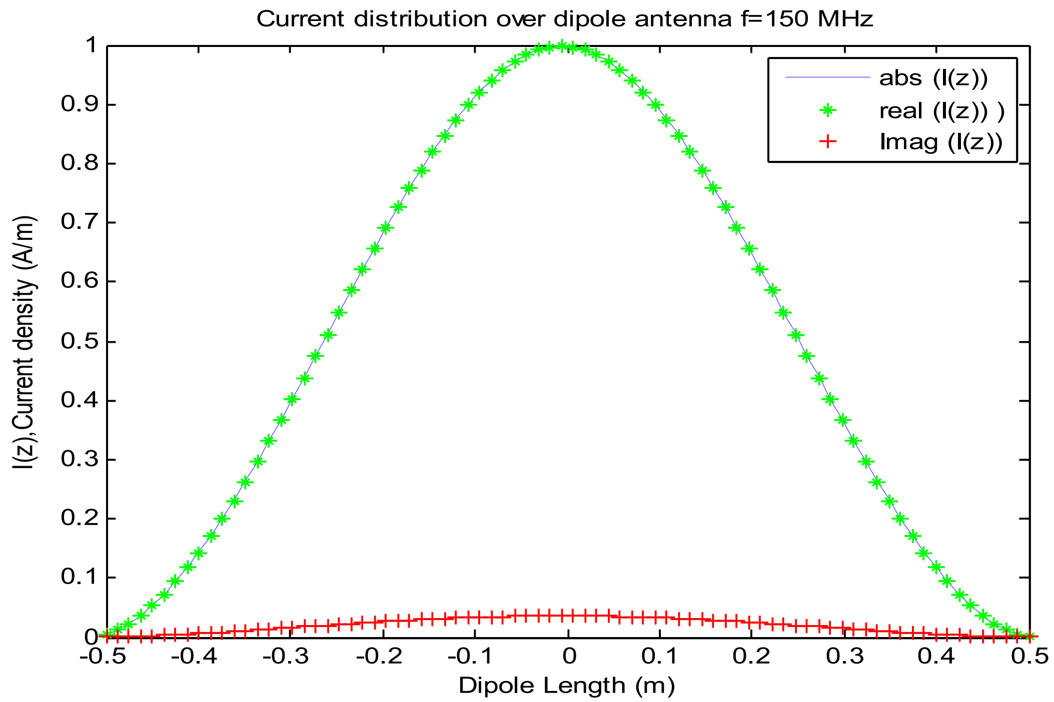


Fig. 4.a. Current distribution versus the position along a Thin wire, l=λ/2 and l/a= 518, using N = 80 basis functions

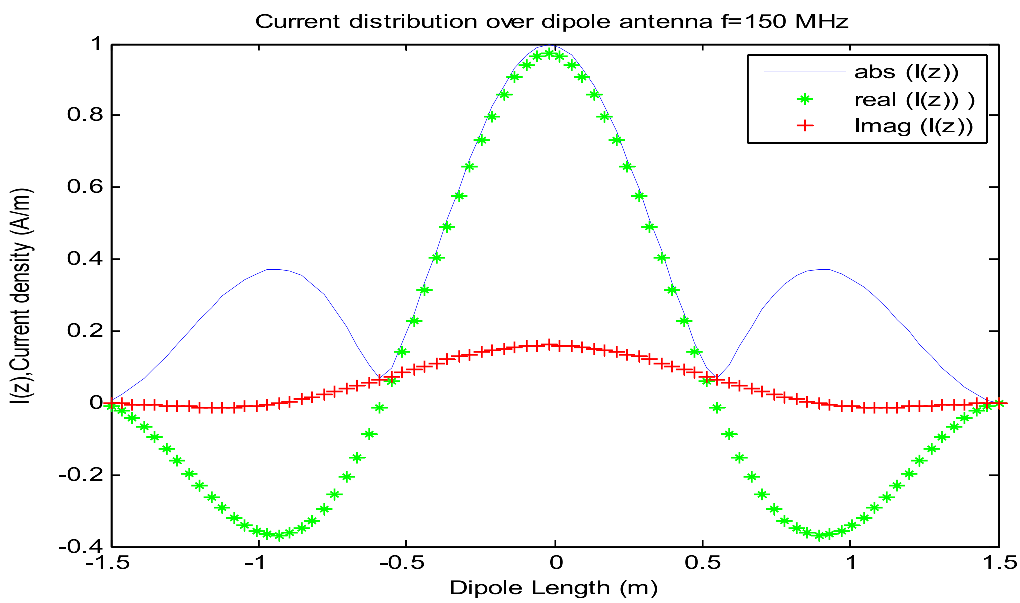


Fig. 4.b. Current distribution versus the position along a Thin wire, l=1.5*λ and l/a= 518, using N = 80 basis functions

Observations are analogous to the results demonstrated in [3]. We can observe that in figure (3.b) the current is distributed sinusoidal (as it was expected) with a good approximation over all parts of the wire except at the end points where it should be zero according to the boundary conditions explained previously.

5.2 Results Obtained With Hallen's And Pocklington's Integral Equation Using Moment's Method And Wavelet

A computer program has been coded in Matlab language for the technique described above, the wavelet employed is constructed from Haar orthogonal wavelet with vanishing moment N=7, the lowest resolution level is chosen 2^j = 27= 128, since 128 wavelets are involved, a system of matrix (of 128 elements) is generated. A transmitting dipole antenna of radius l/518 meter, operating at $1.5e^{+08}$

Hz has been considered. This simulation was made by using the method of MoM and Haar wavelet.

The current distribution along the dipole is shown in Figure 4, we can notice almost good agreement between the two methods, especially the first lobe.

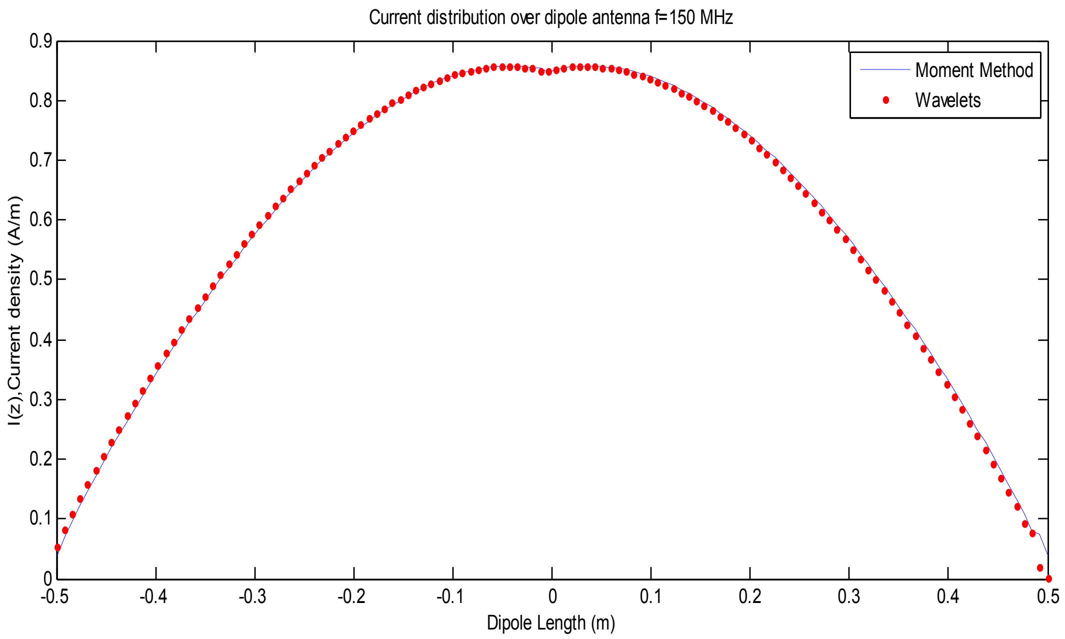


Fig.5.a. Current distribution versus the position along a Thin wire, l=λ/2 and l/a= 518

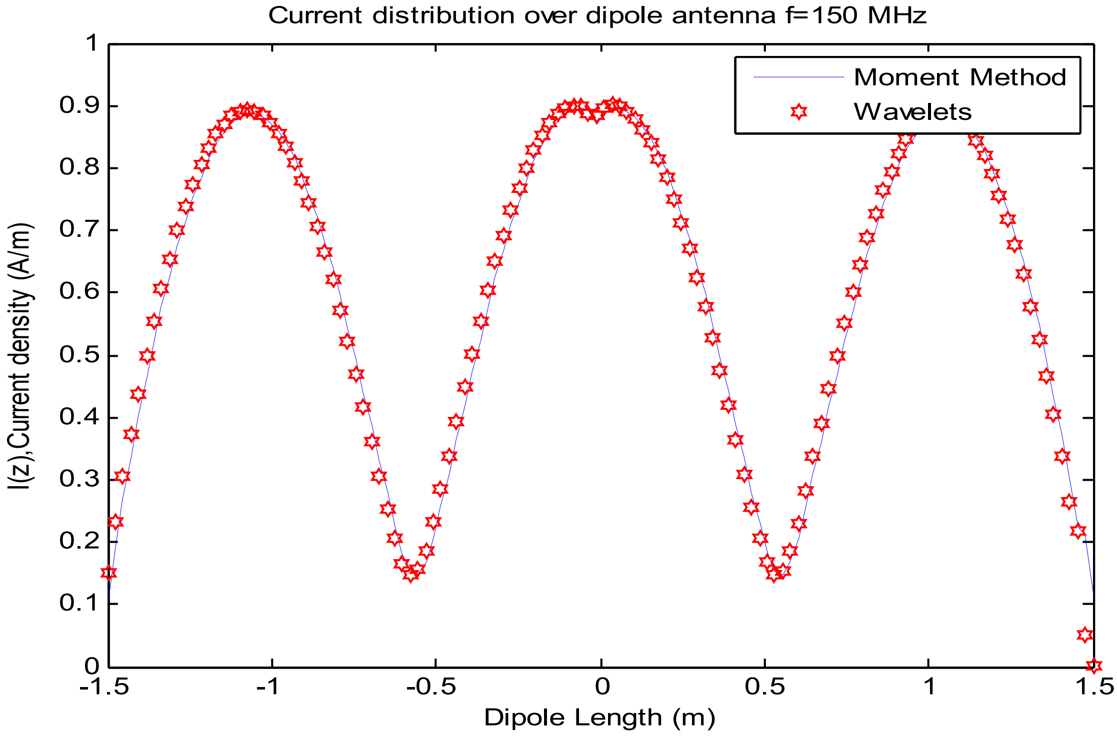


Fig.5.b. Current distribution versus the position along a Thin wire, l=1.5λ and l/a= 518

Table 1: Sparsification with respect to the threshold and the wavelet number [15]

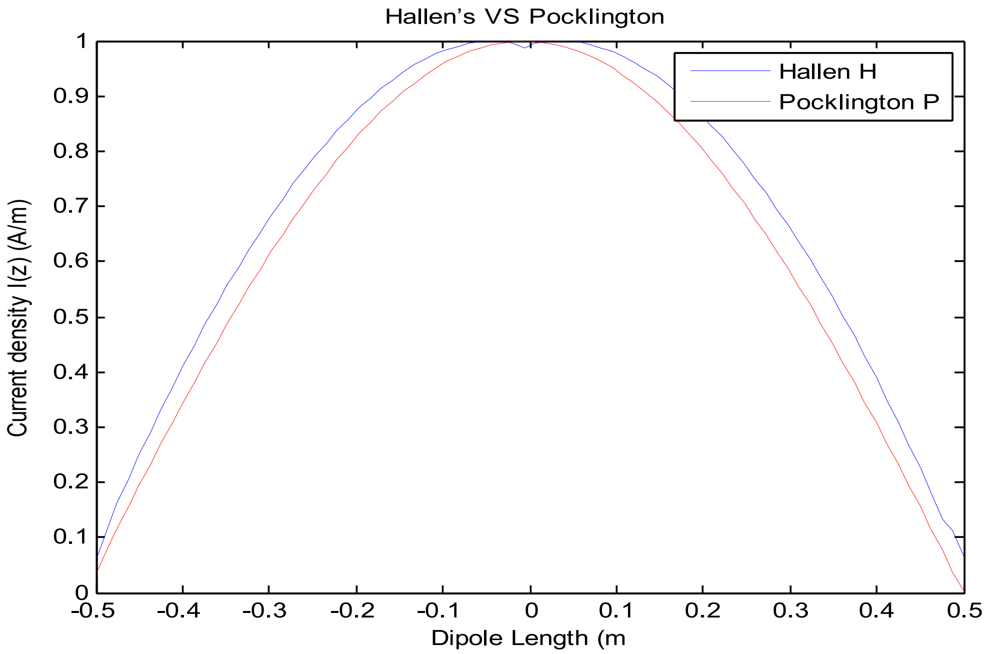
Threshold	Wavelet Number	Sparsity (%)
10^{-2}	32	79.50
	64	58.11
	128	89.34
	512	91.03
10^{-3}	32	36.14
	64	75.33
	128	78.12
	512	80.56
10^{-4}	32	58.22
	64	68.02
	128	71.23
	512	73.05

The sparsification of the impedance matrix with respect to the chosen threshold and the wavelet number is presented in table 1. One can notice from the table 1, that increasing wavelet number can improve sparsification and lower threshold decrease sparsification. Although accuracy of the results is obtained for very low threshold a compromise is necessary.

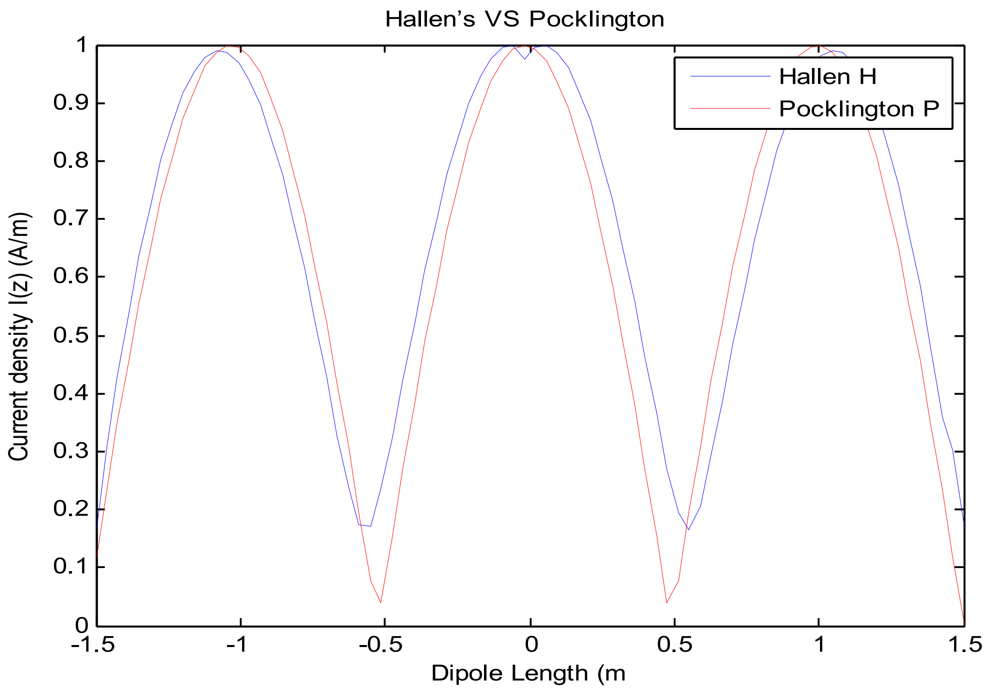
The Haar wavelets are used to solve Hallén's and Pocklington's integral equation for a thine-wire some properties of Haar wavelets are presented and are utilized to reduce the computation of Hallén's and Pocklington's integral equation to some sparse matrix equation. The method is computationally attractive and applications are demonstrated through illustrative examples

5.3 Comparison Between The Solutions Obtained By The Integral Equation Hallen's And Pocklington's

Assume a center-fed linear dipole of l=0.5ʎ, then l=1.5ʎ, and a=0.005ʎ. Determine the normalized current distribution over the length of the dipole using N=64 segments to subdivide the length. Plot the current distribution use Pocklington's and Hallén's equation to solve the problem (figure (6.a) and (6.b)).



(a)



(b)

Fig.7. Current distribution versus the position along a Thin wire, N=64 and l/a= 518, fed by a delta-gap voltage V0 = 1 V in its center
a)-l=ʎ/2; (b)-l=1.5*ʎ

6 CONCLUSIONS

In this paper we have shown that the radiation problem of dipole antennas stated as Hallen's and Pockilington's Equation can be solved using Method of Moment suitably modified with wavelets. A sparse matrix equation is attained from the thin-wire electric-field integral equation (EFIE) by using this technique. The result extracted from Pocklington's integral equation gives better convergence at the feeding point, though it takes more time to be computed because of the complexity in Eq. (16) or (20). Also, the Pocklington's integral equation provides more accurate results in the end points of the wire, as the one obtained of the integral equation Hallen's. The results of half wave dipole antennas and those of full wave antennas agree with those computed directly and those in the literature.

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