International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 13 No. 3 Nov. 2015, pp. 734-741
© 2015 Innovative Space of Scientific Research Journals
http://www.ijias.issr-journals.org/

Linear Decoding of QO-STBC under Imperfect Channel Estimation Conditions

Bhupinder Singh and Amit Kumar Kohli

Electronics and Communication Engineering Department,
Thapar University,
Patiala, Punjab, India

Copyright © 2015 ISSR Journals. This is an open access article distributed under the ***Creative Commons Attribution License***, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

ABSTRACT: This paper presents a comparison between Eigen value decomposition (EVD) method and Givens rotation (GR) method for the quasi-orthogonal space-time block coded (QO-STBC) wireless communication systems, working under the multipath Rayleigh fading channels, under the condition of imperfect channel state information (CSI) at the receiver side, and also presents the multi-input and multi-output (MIMO) extension of both the QO-STBC methods. The matrix manipulation in both schemes leads to the removal of interference terms in the detection matrix, which in turn results in reduced computational complexity as compared to the conventional linear decoding technique using maximum likelihood criterion. Under realistic conditions the channel estimator cannot provide perfect/ideal channel state information (CSI). In this correspondence, we describe the impact of imperfect channel state information on the diversity gain. The simulation results are presented to demonstrate the adverse effects of imperfect channel state information (CSI) on the bit error rate performance of QO-STBC systems.

KEYWORDS: Space-time block code, channel state information (CSI), Eigen value decomposition (EVD), Givens rotation (GR), signal-to-noise ratio (SNR).

1 INTRODUCTION

In the field of wireless communication, the multipath fading is one of the major factors that affect the performance of wireless communication systems in terms of capacity of the system and the quality of received signal.

Telatar [2] proved that the capacity of a wireless communication system grows at least linearly with the number of transmit antennas under the condition that the number of receiver antennas must be less than or equal to the number of transmit antennas. Now, to achieve an increases in capacity and to overcome the fading impact, new coding schemes and new signals are considered, which are respectively known as space-time codes and space-time signals.

To design a space-time code, two important design criterions are given as (i) It should achieve full diversity. (ii) Decoding should be fast and simple.

First space-time codes was given by Alamouti [1] in 1998, for two transmit antennas and one receiver antenna and this code was developed using orthogonal design criterion and known as orthogonal space-time block code (O-STBC). It achieves full diversity order and rate one transmission with simple linear decoding at receiver. This scheme is quite similar to the receiver diversity scheme known as maximal ratio receiver combining (MRRC) in terms of the bit error rate performance.

For more than two transmit antennas, an orthogonal space-time block coding scheme was given by Tarokh, Jafarkhani, Calderbank [3]. It achieves full diversity order, and the decoding of transmitted symbols can be done separately not jointly, means decoding complexity increases linearly with space-time code size. But, these orthogonal space-time block codes for more than two transmit antennas lack in the transmission rate i.e., maximum transmission rate of these codes for complex signals is 3/4 for four transmit antennas and 1/2 for more than four transmit antennas.

To achieve rate one transmission, space-time block code was given by Jafarkhnai [4] using quasi-orthogonal design criterion i.e., orthogonal condition was relaxed to achieve rate one transmission for communication systems with more than two transmit antennas. Such schemes achieve rate one transmission, but due to quasi-orthogonal characteristics, the decoding complexity get increased at receiver side i.e., maximum likelihood (ML) decoder works with a pair of transmitted symbols instead of single symbol and this degrades the performance of QO-STBC codes.

U. Park [8] and Xiao Li-ping [9], proposed new QO-STBC schemes, which achieves rate one transmission and simple linear decoding i.e., ML decoder works with a single symbol at a time, known as Givens rotation QO-STBC (GR-QO-STBC) and Eigen value QO-STBC (EVD-QO-STBC) respectively. These new QO-STBC schemes were proposed by using some manipulations on the QO-STBC scheme discussed earlier.

GR-QO-STBC and EVD-QO-STBC schemes discussed above have been analyzed for perfect channel state information (CSI) at the receiver side [8, 9]. In this paper, we analyze both these schemes under the condition of imperfect channel state information (CSI) at the receiver side. Simulation results show that it degrades the performance of both the QO-STBC schemes. We analyze the performance for the Rayleigh fading channel at 25 dB signal-to-noise ratio (SNR).

In the last section of this paper, we present the multi-input and multi-output (MIMO) representation of both GR-QO-STBC and EVD-QO-STBC schemes.

In section II, we briefly discuss conventional QO-STBC (C-QO-STBC) as well as GR-QO-STBC and EVD-QO-STBC schemes along with system model. In section III, we next analyze GR-QO-STBC and EVD-QO-STBC under imperfect channel state information (CSI) at the receiver side. Section IV introduces the MIMO extension for GR-QO-STBC and EVD-QO-STBC schemes. Section V presents the simulation results for section II, section III and section IV. In section VI, we give the conclusion of this paper.

2 QO-STBC Schemes And System Model

Consider a multi-input and single-output (MISO) wireless communication system under quasi-static Rayleigh flat fading channel condition i.e., channel remains constant during the transmission of a code word. In this paper, we are going to consider 4×1 and 3×1 MISO systems.

After the transmission of symbols, received signal at the receiver is given as

$$\mathbf{R} = \mathbf{HX+N} \tag{1}$$

where **R** is the vector of received signals, **H** is the channel coefficient matrix, **X** represents the vector of transmitted symbols i.e., encoding matrix, and **N** is the noise vector of complex white Gaussian noise coefficients with zero mean and unit variance.

Now, equation (1) can be written as

$$\mathbf{R} = [\mathbf{r_1} \quad \mathbf{r_2^*} \quad \mathbf{r_3} \quad \mathbf{r_4^*}]^{\mathbf{T}} = \mathbf{H}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3 \\ n_4^* \end{bmatrix} \tag{2}$$

In C-QO-STBC scheme as discussed in [4], the channel matrix **H** and the encoding matrix **X** used for four and three transmit antennas are given as

$$\mathbf{H_4} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix}$$

$$\mathbf{H_3} = \begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2^* & -h_1^* & 0 & -h_3^* \\ h_3 & 0 & h_1 & h_2 \\ 0 & -h_3^* & h_2^* & -h_1^* \end{bmatrix} \tag{3}$$

and

$$\mathbf{X} = \begin{bmatrix} \mathbf{X_{12}} & \mathbf{X_{34}} \\ \mathbf{X_{34}} & \mathbf{X_{12}} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & -x_3^* & -x_2^* & x_1^* \end{bmatrix}$$

In GR-QO-STBC (Givens rotation-QO-STBC) [8], the channel matrix **H** and the encoding matrix **X** for four and three transmit antenna systems are given as

$$\mathbf{H_4} = \begin{bmatrix} h_1 - h_3 & h_2 - h_4 & h_1 + h_3 & h_2 + h_4 \\ h_2^* - h_4^* & h_3^* - h_1^* & h_2^* + h_4^* & -h_1^* - h_3^* \\ h_3 - h_1 & h_4 - h_2 & h_3 + h_1 & h_2 + h_4 \\ h_4^* - h_2^* & h_1^* - h_3^* & h_4^* + h_2^* & -h_1^* - h_3^* \end{bmatrix} \tag{4}$$

$$\mathbf{H_3} = \begin{bmatrix} h_1 - h_3 & h_2 & h_1 + h_3 & h_2 \\ h_2^* & h_3^* - h_1^* & h_2^* & -h_1^* - h_3^* \\ h_3 - h_1 & -h_2 & h_3 + h_1 & h_2 \\ -h_2^* & h_1^* - h_3^* & h_2^* & -h_1^* - h_3^* \end{bmatrix}$$

and

$$\mathbf{X_4} = \begin{bmatrix} x_1 + x_3 & x_2 + x_4 & x_3 - x_1 & x_4 - x_2 \\ -x_2^* - x_4^* & x_1^* + x_3^* & x_2^* - x_4^* & x_3^* - x_1^* \\ x_3 - x_1 & x_4 - x_2 & x_3 + x_1 & x_2 + x_4 \\ x_2^* - x_4^* & x_3^* - x_1^* & -x_2^* - x_4^* & x_1^* + x_3^* \end{bmatrix}$$

$$\mathbf{X_3} = \begin{bmatrix} x_1 + x_3 & x_2 + x_4 & x_3 - x_1 \\ -x_2^* - x_4^* & x_1^* + x_3^* & x_2^* - x_4^* \\ x_3 - x_1 & x_4 - x_2 & x_3 + x_1 \\ x_2^* - x_4^* & x_3^* - x_1^* & -x_2^* - x_4^* \end{bmatrix}$$

In EVD-QO-STBC (Eigen value decomposition QO-STBC) [9], the channel matrix **H** and the encoding matrix **X** used for four and three transmit antenna systems are given as

$$\mathbf{H_4} = \begin{bmatrix} \mathbf{G_1} & \mathbf{W_1} \\ \mathbf{C_1} & \mathbf{K_1} \end{bmatrix}$$

where

$$\mathbf{G_1} = \begin{bmatrix} h_1 + h_2 + h_3 + h_4 & h_1 - h_2 + h_3 - h_4 \\ h_2^* - h_1^* + h_4^* - h_3^* & h_2^* + h_1^* + h_4^* + h_3^* \end{bmatrix}$$

$$\mathbf{W_1} = \begin{bmatrix} h_1 + h_2 - h_3 - h_4 & h_1 - h_2 - h_3 + h_4 \\ h_2^* - h_1^* - h_4^* + h_3^* & h_2^* + h_1^* - h_4^* - h_3^* \end{bmatrix}$$

$$\mathbf{C_1} = \begin{bmatrix} h_1 + h_2 + h_3 + h_4 & h_1 - h_2 + h_3 - h_4 \\ h_4^* - h_3^* + h_2^* - h_1^* & h_4^* + h_3^* + h_2^* + h_1^* \end{bmatrix}$$

$$\mathbf{K_1} = \begin{bmatrix} -h_1 - h_2 + h_3 + h_4 & h_1 - h_2 + h_3 - h_4 \\ h_4^* - h_3^* - h_2^* + h_1^* & h_4^* + h_3^* - h_2^* - h_1^* \end{bmatrix} \tag{5}$$

$$\mathbf{H_3} = \begin{bmatrix} \mathbf{G_2} & \mathbf{W_2} \\ \mathbf{C_2} & \mathbf{K_2} \end{bmatrix}$$

where

$$\mathbf{G_2} = \begin{bmatrix} h_1 + h_2 + h_3 & h_1 - h_2 + h_3 \\ h_2^* - h_1^* - h_3^* & h_2^* + h_1^* + h_3^* \end{bmatrix}$$

$$\mathbf{W_2} = \begin{bmatrix} h_1 + h_2 - h_3 & h_1 - h_2 - h_3 \\ h_2^* - h_1^* + h_3^* & h_2^* + h_1^* - h_3^* \end{bmatrix}$$

$$\mathbf{C_2} = \begin{bmatrix} h_1 + h_2 + h_3 & h_1 - h_2 + h_3 \\ h_3^* + h_2^* - h_1^* & h_3^* + h_2^* + h_1^* \end{bmatrix}$$

$$\mathbf{K_2} = \begin{bmatrix} -h_1 - h_2 + h_3 & h_1 - h_2 + h_3 \\ h_3^* - h_2^* + h_1^* & h_3^* - h_2^* - h_1^* \end{bmatrix}$$

and

$$\mathbf{X_4} = \begin{bmatrix} \mathbf{E_1} & \mathbf{F_1} \\ \mathbf{F_1} & \mathbf{E_1} \end{bmatrix}$$

where

$$\mathbf{E_1} = \begin{bmatrix} x_1 + x_2 + x_3 + x_4 & x_1 - x_2 + x_3 - x_4 \\ x_2^* - x_1^* + x_3^* + x_4^* & x_2^* + x_1^* + x_3^* + x_4^* \end{bmatrix}$$

$$\mathbf{F_1} = \begin{bmatrix} x_1 + x_2 - x_3 - x_4 & x_1 - x_2 - x_3 + x_4 \\ x_2^* - x_1^* + x_3^* - x_4^* & x_2^* + x_1^* - x_3^* - x_4^* \end{bmatrix}$$

$$\mathbf{X_3} = \begin{bmatrix} \mathbf{E_1} & \mathbf{F_2} \\ \mathbf{F_1} & \mathbf{E_2} \end{bmatrix}$$

where

$$\mathbf{E_2} = \begin{bmatrix} x_1 + x_2 + x_3 + x_4 \\ x_2^* - x_1^* - x_3^* + x_4^* \end{bmatrix}$$

$$\mathbf{F_2} = \begin{bmatrix} x_1 + x_2 - x_3 - x_4 \\ x_2^* - x_1^* + x_3^* - x_4^* \end{bmatrix}$$

$\mathbf{H_4}$ and $\mathbf{H_3}$ represents the channel coefficient matrices for four and three transmit antennas and similarly, $\mathbf{X_4}$ and $\mathbf{X_3}$ represents the encoding matrices for four and three transmit antennas.

As we discussed above about the QO-STBC schemes with their channel and encoding matrices, at the receiver side, for all these schemes, detection of the transmitted symbols are given as

$$\hat{\mathbf{X}} = \mathbf{H^H R} = \mathbf{H^H H X} + \mathbf{H^H N} \tag{6}$$

where $\hat{\mathbf{X}}$ represents the vector of detected transmitted symbols.

Simulation results for all these QO-STBC schemes are discussed in section **5** by using the equation (6).

3 Analysis Of GR-QO-STBC and EVD-QO-STBC Under Imperfect Channel State Information (CSI) Condition

In this section, we are going to analyze the performance of GR-QO-STBC and EVD-QO-STBC schemes under the condition that the channel is not perfectly known at the receiver side i.e., we are assuming imperfect channel condition at the receiver side. Mathematically, received signal at the receiver is given as

$$\mathbf{R} = \mathbf{HX} + \mathbf{N}$$

where **R** is the received signal vector, **X** is the transmitted signal vector and **N** is the vector of complex Gaussian noise coefficients with zero mean and unit variance.

Under the perfect channel condition, i.e., when the receiver has perfect knowledge of the channel state information (CSI), transmitted symbols are detected as

$$\hat{\mathbf{X}} = \mathbf{H}^{\mathbf{H}}\mathbf{R} = \mathbf{H}^{\mathbf{H}}\,\mathbf{HX}+\mathbf{H}^{\mathbf{H}}\mathbf{N} \quad (7)$$

But in the case of imperfect channel condition at the receiver side, i.e., when the receiver does not have perfect channel state information (CSI) then the channel matrix **H,** which is known to the receiver, is given as

$$\hat{\mathbf{H}} = \mathbf{H} + \boldsymbol{\delta}\mathbf{h} \quad (8)$$

where $\hat{\mathbf{H}}$ represents the channel matrix estimated by receiver and $\mathbf{H}$ is the original channel matrix and $\boldsymbol{\delta}\mathbf{h}$ is the channel coefficient error matrix which represents the errors made by the receiver during the estimation of channel coefficients.

Each element of $\boldsymbol{\delta}\mathbf{h}$ matrix is complex Gaussian random variable with zero mean, such that

$$\delta h_k = i_k + j q_k \text{ , } k = 1, 2, \ldots\ldots, n$$

where n represents the total number of channel coefficients.

Now, the detection of transmitted symbols under imperfect channel state information at the receiver will be given as

$$\begin{aligned}\hat{\mathbf{X}} &= \hat{\mathbf{H}}^{\mathbf{H}}\mathbf{R} = \hat{\mathbf{H}}^{\mathbf{H}}\mathbf{HX} + \hat{\mathbf{H}}^{\mathbf{H}}\mathbf{N} \\ &= (\mathbf{H}+\boldsymbol{\delta}\mathbf{h})^{\mathbf{H}}\mathbf{R} = (\mathbf{H}+\boldsymbol{\delta}\mathbf{h})^{\mathbf{H}}\,\mathbf{HX}+(\mathbf{H}+\boldsymbol{\delta}\mathbf{h})^{\mathbf{H}}\mathbf{N} \\ &= \mathbf{H}^{\mathbf{H}}\,\mathbf{HX}+\boldsymbol{\delta}\mathbf{h}^{\mathbf{H}}\,\mathbf{HX}+\mathbf{H}^{\mathbf{H}}\,\mathbf{N}+\boldsymbol{\delta}\mathbf{h}^{\mathbf{H}}\,\mathbf{N} \\ &= \mathbf{H}^{\mathbf{H}}\,\mathbf{HX}+\mathbf{H}^{\mathbf{H}}\,\mathbf{N} + (\mathbf{HX}+\mathbf{N})\,\boldsymbol{\delta}\mathbf{h}^{\mathbf{H}} \end{aligned} \quad (9)$$

where $\boldsymbol{\delta}\mathbf{h}$ represents the channel coefficient error matrix during the estimation of channel coefficients at the receiver side as given in equation (8); and from equation (9), it is quite clear that the channel estimation error acts as an additive noise along with AWGN noise during the detection of the transmitted symbols and this additional noise impacts the performance of both the QO-STBC schemes.

Simulation results for both the QO-STBC schemes under imperfect CSI at the receiver side are shown in section **5** using the equation (9).

4 MIMO EXTENSION OF GR-QO-STBC AND EVD-QO-STBC SCHEMES

In this section, we are going to extend GR-QO-STBC and EVD-QO-STBC schemes from multi-input and single-output (MISO) systems to multi-input and multi-output (MIMO) systems.

Suppose there are N_R receiver antennas at the receiver side and the received signal at the receiver side can be given as

$$\bar{\mathbf{R}} = \bar{\mathbf{H}}\mathbf{X}+\bar{\mathbf{N}} \quad (10)$$

where

$$\bar{\mathbf{R}} = \begin{bmatrix}\bar{\mathbf{r}}_1\\ \bar{\mathbf{r}}_2\\ \bar{\mathbf{r}}_3\\ \cdot\\ \cdot\\ \cdot\\ \bar{\mathbf{r}}_{\mathbf{N_R}}\end{bmatrix},\ \bar{\mathbf{r}}_p = \begin{bmatrix}r_{1p}\\ r_{2p}\\ r_{3p}\\ r_{4p}\end{bmatrix},\ p = 1, 2, \ldots.., N_R$$

r_{ip} represents the signal received at p^th^ antenna at i^th^ time slot and i = 1,2,3,4 and p = 1, 2,……,N_R.

$\bar{\mathbf{H}}$ is the channel matrix and it is given as –

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{H}_{N_R} \end{bmatrix}$$

where $\mathbf{H}_p$ represents the channel matrix between the p^{th} receiver antenna and all the transmit antennas, in which p = 1, 2,….,N_R.

X and $\bar{\mathbf{N}}$ are the encoding and noise matrices respectively, as discussed in earlier sections.

Simulation results for the QO-STBC schemes for MIMO system are shown in the section **5** using the equation (10).

5 SIMULATION RESULTS

In this section, first, we present the simulation results for the QO-STBC schemes discussed in section II using the equation (6) in the Fig. 1.

For the simulation results, modulation scheme used is 4-PSK and the channel under consideration is Rayleigh flat fading channel; which is assumed to be quasi-static.

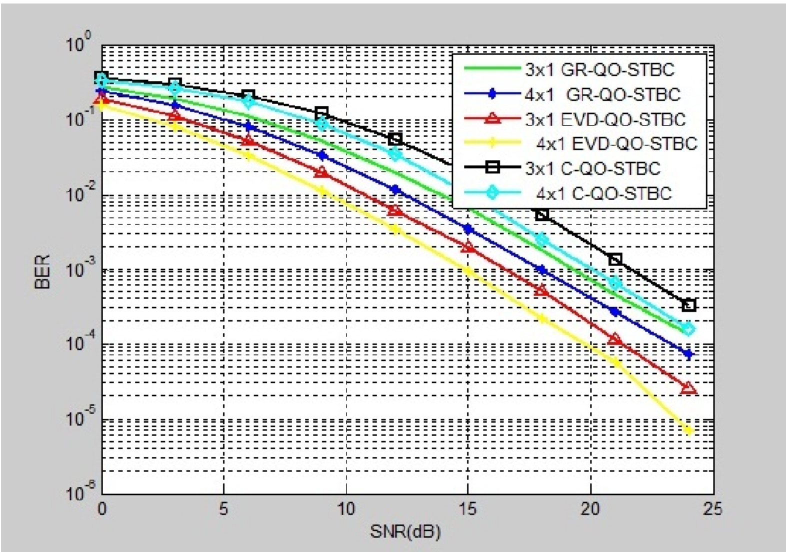

Fig.1. BER performance of C-QO-STBC, GR-QO-STBC and EVD-QO-STBC schemes for four and three transmit antennas under the perfect CSI at the receiver.

Fig. 2 presents the simulation results for the GR-QO-STBC and EVD-QO-STBC schemes under imperfect channel state information (CSI) at the receiver side. In this analysis, modulation scheme used is 4-PSK and the channel is Rayleigh flat fading quasi-static channel. Signal-to-noise ratio (SNR) assumed to be at 25 dB and we vary the channel estimation error from -100 dB to -10 dB.

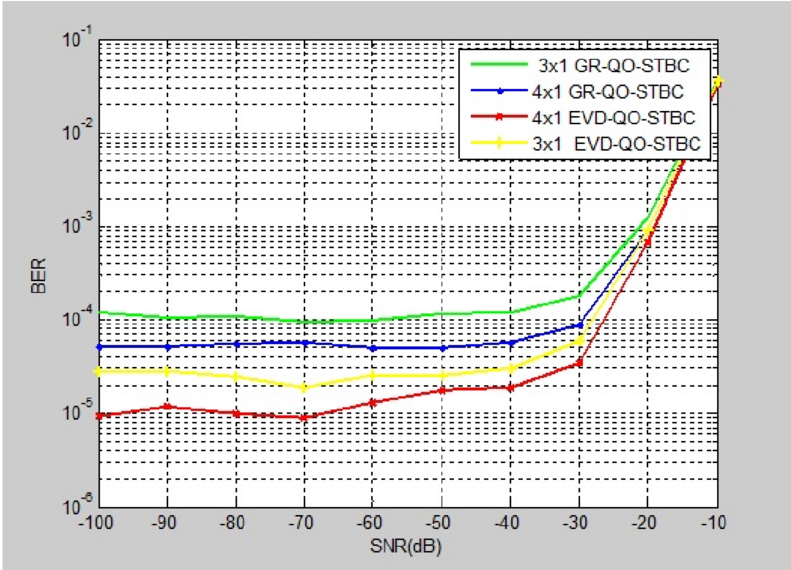

Fig.2. BER performance of GR-QO-STBC and EVD-QO-STBC schemes under imperfect channel state information (CSI) at the receiver for three and four transmit antennas.

From section **4**, simulation results for MIMO systems under the EVD-QO-STBC and GR-QO-STBC schemes are given below in Fig. 3. Modulation used here is 4-PSK and the channel is Rayleigh flat fading quasi-static.

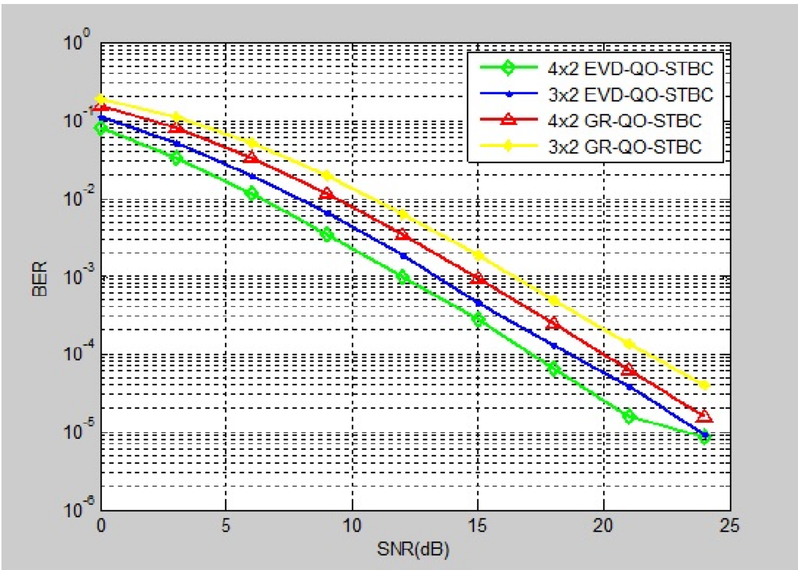

Fig. 3. BER performance of GR-QO-STBC and EVD-QO-STBC schemes under MIMO systems.

6 CONCLUSION

After analyzing the simulation results in section V, we conclude that EVD-QO-STBC scheme performs better than GR-QO-STBC scheme under the conditions of perfect channel state information (CSI) and imperfect channel state information (CSI) at the receiver side. In the case of imperfect CSI at the receiver side, EVD-QO-STBC scheme dominates the performance of GR-QO-STBC scheme in error regime from -100 dB to -40 dB i.e., the performance of EVD-QO-STBC is less affected by the channel estimation error as compared to the GR-QO-STBC scheme in lower error regime. But in case of higher estimation error regime near about -30 dB, performance of both the QO-STBC schemes degrades sharply and either of the QO-STBC schemes can be used based on the complexity level.

In section V, we also presented the simulation results for MIMO systems under both the QO-STBC schemes. Under MIMO systems, both the QO-STBC schemes achieve the diversity gain of 3-4 dB as compared to MISO systems i.e., both the schemes achieve better performance using MIMO systems.

REFERENCES

[1] S.M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J.Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

[2] I.E. Telatar, "Capacity of multi-antenna Gaussian Channels," *AT&T Bell Labs.*, Tech. Rep., 1995.

[3] V. Tarokh and H. Jafarkhani, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, Jul. 1999.

[4] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans Commun*., vol. 49, pp. 1-4, Jan. 2001.

[5] Peng A Y and Kim I, "Low-complexity sphere decoding algorithm for quasi-orthogonal space-time block codes," *IEEE Trans Commun*., vol. 54, pp. 377-382, 2006.

[6] Leuschner J and Yousefi S, "ML decoding of quasi-orthogonal space-time block codes via sphere decoding and exhaustive search," *IEEE Trans Commun*., vol. 7, pp. 4088-4093, 2008.

[7] Le M and Pham V, "Low-complexity maximum-likelihood decoder for four-transmit-antenna quasi-orthogonal space-time block code," *IEEE Trans Commun.,* vol. 53, pp. 1817-1821, 2005.

[8] U. Park and Sooyoung Kim, "A Novel QO-STBC Scheme with Linear Decoding for Three and Four Transmit Antennas," *IEEE Trans Commun.,* vol. 12, pp. 868-870, Dec. 2008.

[9] Xiao Li-ping, "A Novel QO-STBC scheme with linear decoding," *ScienceDirect,* vol. 18, pp. 53-57, Oct. 2011.