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Mixed convection heat transfer of Cu-water nanofluid in a lid driven square cavity with several heated triangular cylinders

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ABSTRACT: The problem of mixed convection heat transfer of nanofluid in a lid driven square cavity containing several heated triangular cylinders is studied numerically using the finite volume discretization method. The upper and bottom walls are thermally insulated while the left and right walls are cooled at constant temperature, T_c. The present investigation considered the effects of pertinent parameters such as; size and number of the heated triangular cylinders on the flow and Nusselt number. The other parameters governing the problem are the Richardson number $(0.1 \leq Ri \leq 100)$, the Prandtl number of the pure water $(Pr = 6.2)$ and the volume fraction of nanoparticles $(0 \leq \varphi \leq 0.05)$. Results show that increasing size and number of the heated triangular cylinders leads to increase the heat transfer rate. It is also found that by reducing Richardson number and increasing the volume fraction of nanoparticles, the average Nusselt number increases.

KEYWORDS: Mixed convection, Cavity, Triangular cylinder, Nanofluid.

1 INTRODUCTION

Mixed convection heat transfer with lid-driven cavity flow is frequently encountered in numerous engineering applications such as heating and cooling nuclear systems of reactors, lubrication technologies, cooling of electronic devices and heat exchangers. Islam et al. [1] performed a numerical study on a lid-driven cavity with a heated square blockage. They found that size, location and Richardson number of the heater eccentricities affect the average Nusselt number of heater. Work of Khanafer and Aithal. [2] was concentrated on the effect of mixed convection flow and heat transfer characteristics in a lid-driven cavity with a circular body inside. The results showed that the average Nusselt number increases with an increase in the radius of the cylinder for various Richardson numbers and the optimal heat transfer results are obtained when placing the cylinder near the bottom wall. Kalteh et al. [3] considered laminar mixed convection of nanofluid in a lid-driven square cavity with a triangular heat source. Their simulations indicate that increasing the volume fraction, the nanoparticles diameter and Reynolds leads to an increase in average Nusselt number. Boulahia et al. [18] studied mixed convection of the nanofluids in two-sided lid-driven square cavity with a pair of triangular heating cylinders. Work of Oztop et al. [4] can be mentioned as example of such studies, in which they numerically studied mixed convection in square cavities with two moving walls. Their results suggest that when the vertical walls move upwards in the same direction, the heat transfer decreases significantly compared to when the vertical walls move in opposite directions. Sheikholeslami et al. [5] studied the problem of magnetohydrodynamic natural convection heat transfer of nanofluid in a horizontal cylindrical enclosure with an inner triangular cylinder. They found that the results indicate that Nusselt number increase with increasing Rayleigh number and augmentation of the Hartmann number causes the Nusselt number to decrease. El Abdallaoui et al. [6] performed a numerical simulation of natural convection between a decentered triangular heating cylinder and centered triangular heating cylinder [7] in a square outer cylinder filled with a pure fluid or a nanofluid using the lattice Boltzmann method. The results indicate that the horizontal displacement from the centered position to decentered position leads to a considerable increase of heat transfer at weak Rayleigh and the vertical displacement has most important effect on heat transfer at high values of Rayleigh. Boulahia and Sehaqui [16] studied a natural convection of nanofluid in a square cavity including a square heater. They found that by increasing size of the heater, the heat transfer rate enhances.

Oztop et al. [8] investigated mixed convection in a lid driven cavity with a cylindrical blockage inside. The vertical left sidewall was moving up or down. The computation was carried out for wide ranges of Richardson number, inner cylinder diameter, center and location of the inner cylinder. They concluded that the most effective parameter on flow field and temperature distribution was the direction of the moving lid. Pishkar et al. [9] studied mixed convection of nanofluid in a horizontal channel. They found that at low Richardson numbers the heat transfer is increased by enlarging the volume fraction of nanoparticles. In addition, their results showed that at low values of Reynolds number, the effects of Richardson number rising on the heat transfer rate for both the nanofluid and pure fluid are negligible. Chen et al. [14] investigated the natural and mixed convection of Al_2O_3-water nanofluids in a square enclosure. They found that the force convection cause better heat transfer than natural convection. Moumni et al. [15] studied the nanofluid mixed convection in two-sided lid driven cavity including discrete heat Sources by using the finite volume method. The results showed that by increasing Richardson and Reynolds numbers and the solid volume fraction, the heat transfer rate increases. Muthtamilselvan and Doh [17] presented a steady state two-dimensional mixed convection in a lid-driven square cavity filled with Cu–water nanofluid. They found that Richardson number and solid volume fraction affect the fluid flow and heat transfer in the cavity.

This research intends to explore the heat transfer rate of mixed convection in a lid driven cavity with several heated triangular cylinders. The first case under investigation is characterized the numerical models used in our study. The computational procedure elaborated in this study is validated against the numerical results of other investigations. We studied the effects of size and number of the heated triangular cylinders on the heat transfer fluid flow. Wide range of parameters such as Richardson number ($0.1 \leq Ri \leq 100$), and volume fraction ($0 \leq \varphi \leq 0.05$) have been used. The new models of the thermal conductivity and effective viscosity investigated by Corcione et al. [10] are used to estimate themophysical proprieties of the nanofluid. Our numerical results are presented in the form of plots of isotherms, streamlines and average Nusselt numbers to show the influence of nanofluid and pertinent parameters.

2 PROBLEM STATEMENT

The system of interest is a lid driven square cavity with several heated triangular cylinders inside the cavity. A schematic diagram of the cavity with triangular cylinders is shown in Fig. 1. The cavity is a square with the height of H, while the size of triangular block is W. The Triangular heat source is maintained at a temperature $T_h = 310\ K$. The vertical walls are cooled at constant temperatures $T_c = 290\ K$, while the horizontal walls are adiabatic. It is assumed that the nanofluid is newtonian, incompressible and laminar and the base fluid and the nanoparticles are in a thermal equilibrium state. The thermo-physical properties of the nanofluid used in this study are evaluated at the average fluid temperature $(T_c + T_h)/2$ as listed in Table 1.

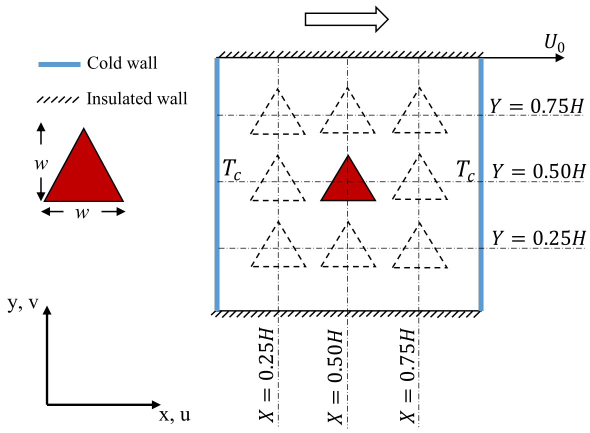


Fig. 1. Schematic of the cavity with triangular heat sources and boundary conditions

Table 1. Thermo-physical properties of water and nanoparticles at T = 300 K [19]

	Copper (Cu)	Water (H_2O)
C_p(J/Kg K)	385	4179
ρ(Kg/m^3)	8933	997.1
k(W/mK)	401	0.6
β(K^{-1})	1.67 10^{-5}	27.6 10^{-5}
μ(kg m^{-1} s^{-1})	–	0.855 10^{-3}

3 MATHEMATICAL FORMULATION

The governing equations including the two-dimensional transient equations of the continuity, momentum and energy for an incompressible flow are expressed in the following format:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{(\rho\beta)_{nf}}{\rho_{nf}}(T-T_c) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where the nanofluid effective density, heat capacity, thermal expansion coefficient and thermal diffusivity are calculated from the following equations [12-11]:

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s \tag{5}$$

$$\left(\rho C_p\right)_{nf}=(1-\varphi)\left(\rho C_p\right)_f+\varphi\left(\rho C_p\right)_s \tag{6}$$

$$(\rho\beta)_{nf}=(1-\varphi)(\rho\beta)_f+\varphi(\rho\beta)_s \tag{7}$$

$$\alpha_{nf}=k_{nf}/\left(\rho C_p\right)_{nf} \tag{8}$$

Corcione model [12-10] for dynamic viscosity and thermal conductivity of the nanofluid are given by:

$$\mu_{nf}=\mu_f/\left(1-34.87\left(d_p/d_f\right)^{-0.3}\varphi^{1.03}\right) \tag{9}$$

$$\frac{k_{nf}}{k_f}=1+4.4Re_B^{0.4}Pr^{0.66}\left(\frac{T}{T_{fr}}\right)^{10}\left(\frac{k_p}{k_f}\right)^{0.03}\varphi^{0.66} \tag{10}$$

$$Re_B=\frac{\rho_f u_B d_p}{\mu_f} \tag{11}$$

$$u_B=\frac{2k_bT}{\pi\mu_f d_p^2} \tag{12}$$

All terms are defined in the Nomenclature.

The boundary conditions for mixed convection written as:

$$\begin{array}{llll} u=0, & v=0, & \partial T/\partial y=0 & \text{on bottom wall of the cavity} \\ u=U_0, & v=0, & \partial T/\partial y=0 & \text{on upper wall of the cavity} \\ v=0, & v=0, & T=T_c & \text{on right wall of the cavity} \\ u=0, & v=0, & T=T_c & \text{on left wall of the cavity} \end{array} \tag{13}$$

The following dimensionless variables for mixed convection are defined based on properties of pure fluid:

$$\tau = \frac{t}{H/U_{ref}}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_{ref}}, \quad V = \frac{v}{U_{ref}}, \quad P = \frac{p}{\rho_{nf} U_{ref}^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \tag{14}$$

Where U_{ref} is considered to be U_0 for mixed convection. Dimensionless numbers for the system are defined as:

$$Re = \frac{u_{ref} H}{\nu_{nf}}, \quad Ri = \frac{Gr}{Re^2}, \quad Gr = \frac{g\beta_f (T_h - T_c) H^3}{\nu_f^2}, \tag{15}$$

$$Ra = Gr.Pr = \frac{g\beta_f (T_h - T_c) H^3}{\alpha_f \nu_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \tag{16}$$

The governing equations (1)–(4) are written in the following dimensionless form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{17}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{\mu_{nf}}{\mu_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{18}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial p}{\partial Y} + \frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{\mu_{nf}}{\mu_f}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\theta \tag{19}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re.Pr}\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{20}$$

Dimensionless form of the boundary conditions can be written as:

$$\begin{array}{llll} U = 0, & V = 0, & \partial\theta/\partial y = 0 & \text{on bottom wall of the cavity} \\ U = 1, & V = 0, & \partial\theta/\partial y = 0 & \text{on upper wall of the cavity} \\ V = 0, & V = 0, & \theta = 0 & \text{on right wall of the cavity} \\ U = 0, & V = 0, & \theta = 0 & \text{on left wall of the cavity} \end{array} \tag{21}$$

The total mean Nusselt number of all cavity's wall is defined as:

$$\overline{Nu}_{tot} = \frac{1}{H}\int_0^H \frac{k_{nf}(\varphi)}{k}\left\{\left|\frac{\partial \theta}{\partial X}\right|_{left} + \left|\frac{\partial \theta}{\partial X}\right|_{right}\right\} dY \tag{22}$$

4 NUMERICAL DETAILS

A finite volume formulation, given by Patankar [20] on a staggered grid, is applied for discretization of the governing equations (Eqs. (17)–(20)) and boundary conditions described by Eq. (21). SIMPLE (Semi-Implicit Method for Pressure Linked Equations) is used to solve the coupled pressure–velocity equation while Hybrid Differencing Scheme (HDS) of Spalding [21] is used for the convective terms. Line by line application of TDMA (Tri-Diagonal Matrix Algorithm) method [20] is applied on equation systems until sum of the residuals became less than 10^{-6}. The developed algorithm was implemented in FORTRAN program.

4.1 GRID INDEPENDENCE STUDY

In order to determine a proper grid for the numerical simulation, a lid driven square cavity filled with Cu–water nanofluid $(\varphi = 0.05)$ having a central triangular heat source with size $w = 0.2H$ is analyzed in two extreme Richardson numbers

($Ri = 0.1$ and 100). The mean Nusselt number obtained using different grid numbers for particular cases is presented in Table 2. As can be observed from the table, a uniform 103×103 grid is sufficiently fine for the numerical calculation.

Table 2. Effect of the grid size on $\overline{Nu}_{tot}$ for the cavity filled with the Cu–water nanofluid ($\varphi = 0.05$) having a central triangular heat source with size $W = w/H = 0.2$ (see Fig. 1)

Ri	63×63	83×83	103×103	123×123
0.1	7,411	7,631	7,794	7,794
100	5,107	5,542	5,608	5,610

4.2 VALIDATIONS

The present numerical scheme was validated against various numerical results available in the literature, three different heat convection problems are chosen. The first case is the numerical results of Iwatsu et al. [13] and Oztop et al. [8] for a top heated moving lid and bottom cooled square cavity filled with air ($Pr = 0.71$). A 100×100 mesh was used and the computations were done for three different Richardson numbers. Table 3 demonstrates an excellent comparison of the average Nusselt number between the present results and the numerical results found in the literature [13-8] with a maximum discrepancy of about 1.6 %. The second case is a mixed convection flow and heat transfer characteristics in a lid-driven cavity with a circular body inside. The third case is a mixed convection flow and heat transfer in a lid-driven cavity with a square heater inside. Fig. 2 illustrates a comparison of the isotherms and streamlines with the results reported by Islam et al [1] and the results reported by Khanafer and Aithal. [2] at Reynolds number (Re = 100) .

Table 3. Comparison of $\overline{Nu}$ at the hot lid between the present results and those reported in the literature

Ri	Average Nusselt number at the hot lid		
	Iwatsu et al. [13]	Oztop et al. [8]	Present
1.0	1.34	1.30	1.36
0.0625	3.62	3.63	3.68
0.01	6.29	6.34	6.29

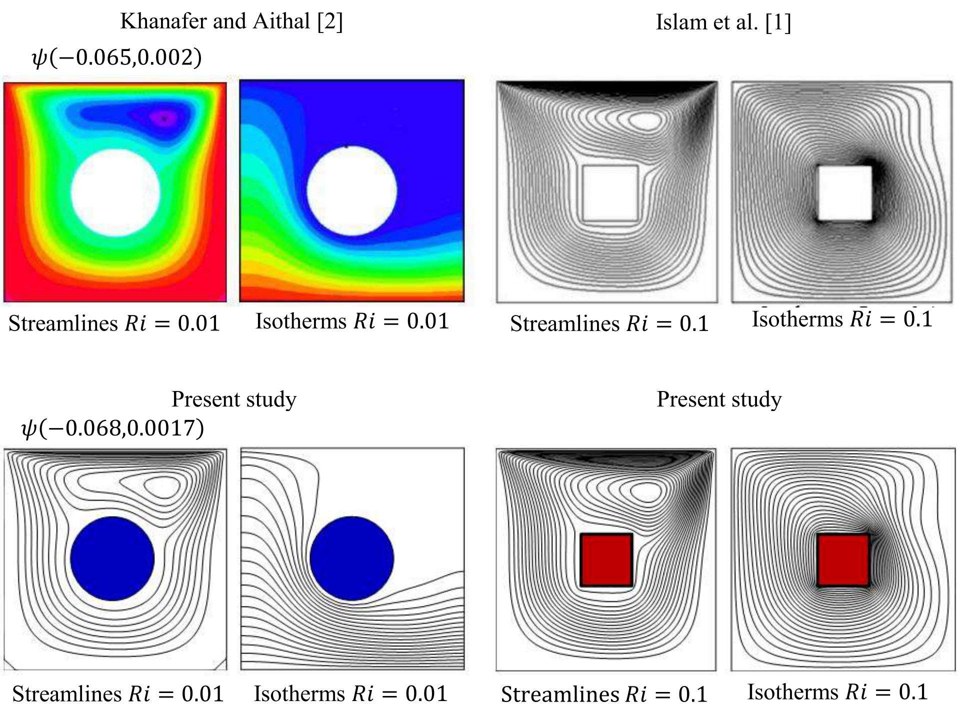
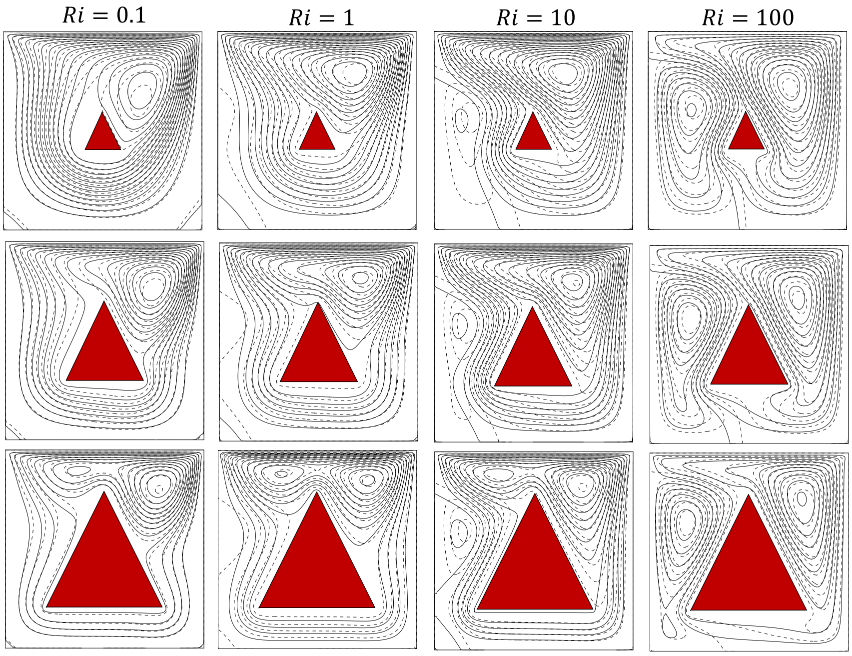


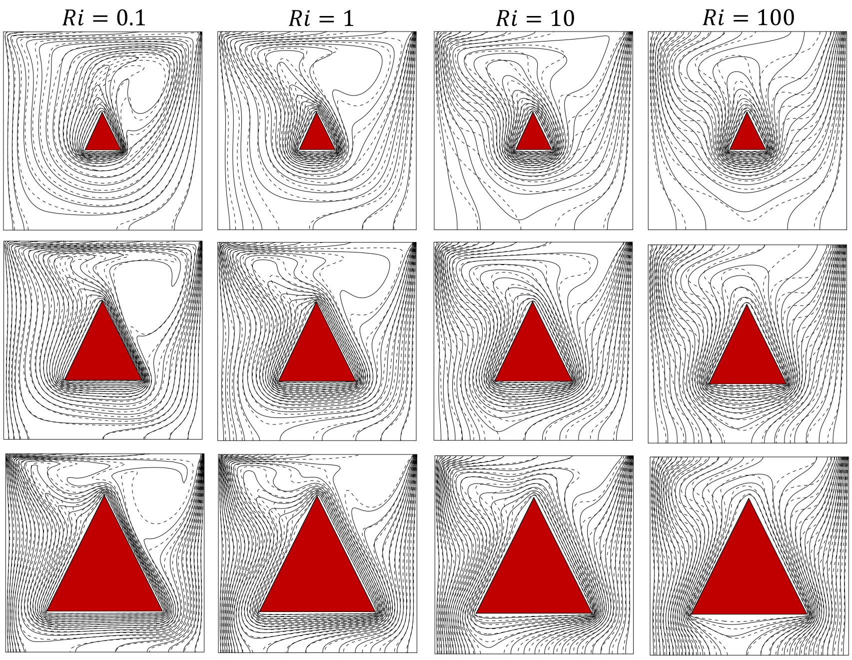
Fig. 2. Comparison of present results with the numerical results of Islam et al. [1] ($h/H = 0.25, Pr = 0.7, Re = 100$) and with results of Khanafer and Aithal [2] ($Re = 100, Pr = 0.7, r/H = 0.2$).

5 RESULTS AND DISCUSSION

Mixed convection heat transfer of nanofluid in a lid driven square cavity containing several heated triangular cylinders is numerically simulated. The calculations are carried out in two phases. First, the effect of size of triangular heat source, three different sizes are considered as: w = 0.2H, w = 0.4H and w = 0.6H. In the second phase, calculations are performed for different number of triangular block while the Richardson number (Ri) is ranging from 0.1 to 100 and the volume fraction of the nanoparticles is varying from 0 to 0.05. Results of these simulations are presented and discussed in the following two subsections.



(a) streamlines



(b) isotherms

Fig. 3. ***(a) Streamlines and (b) isotherms inside the cavity corresponding to different sizes of triangular heat source, i.e.*** $W = w/H = 0.2, 0.4$ ***and*** 0.6 ***filled with Cu–water the pure fluid (dashed line) and Cu–water nanofluid (solid line) with*** $\varphi = 5\%$ ***and at different*** Ris. $Gr = 10^4$.

5.1 EFFECTS OF TRIANGULAR BLOCK SIZE

In this section, a square cavity filled with Cu–water containing a central triangular heated block with three different sizes (w = 0.2H, w = 0.4H and w = 0.6H) is considered for heat transfer analysis. Fig.3 (a-b) shows the effects of the block size, the forced convection due to the lid motion and the natural convection due to the buoyancy driven flow, on the streamlines and isotherms for various values of Ri ranging from 0.1 to 100 while Grashof number is fixed at $Gr = 10^4$. For comparisons, the streamlines and the isotherms for pure fluid and nanofluid are shown by dashed line and solid line respectively.

As shown in Fig. 3, some differences are observed in streamlines and isotherms of pure fluid and nanofluid. We can explain this by the higher viscosity of nanofluid compared to that of the pure fluid which increases the diffusion of momentum in the nanofluid. It can be seen from Fig. 3(a) that at $Ri =$ 0.1, 1 and 10 the general flow trend for the rightward lid motion alone is a clockwise rotating vortex with its center slightly shifted towards the top right corner. This observation shows the higher forced convection effects in the right upper corner of the cavity. As it is clear, the fluid flow under the triangular heated block is very low compare to the upper which is affected by the lid motion. By increasing size of triangular heated block from $w = 0.2H$ to $0.6H$, we can see clearly that the center core of rotating eddy became with two inner vortices. At these Richardson numbers ($Ri =$ 0.1, 1 and 10) , we can see in Fig. 3(b) that the concentration of the isotherms, close to the triangular heated block increases, which indicates that the heat transfer is through forced convection.

According to Fig. 3(b), when the Richardson number decreases, the isotherms are deformed. The plume that forms above the block is seen to be slightly tilted with respect to the vertical and changes dramatically. This important change undergone by the isotherms and characterized by a complexity of their patterns, results from the flow intensification engendered by the increase of the upper lid's speed (decrease of Ri). At $Ri = 10$, we can see in Fig. 3(a), a small counterclockwise rotating eddy started to appear close to the left wall of the cavity. When Ri increases to 100, the effects of moving wall decreases due to the enhancement in the buoyancy force. At this Ri, we can see in Fig. 3(a) that the flow structure is organized into two convection cells, one clockwise vortex on the right side and one counterclockwise vortex on the left side. The uniformly distributed isotherms and the convection cells appear at high Ri show that the main heat transfer mechanism is through the natural convection.

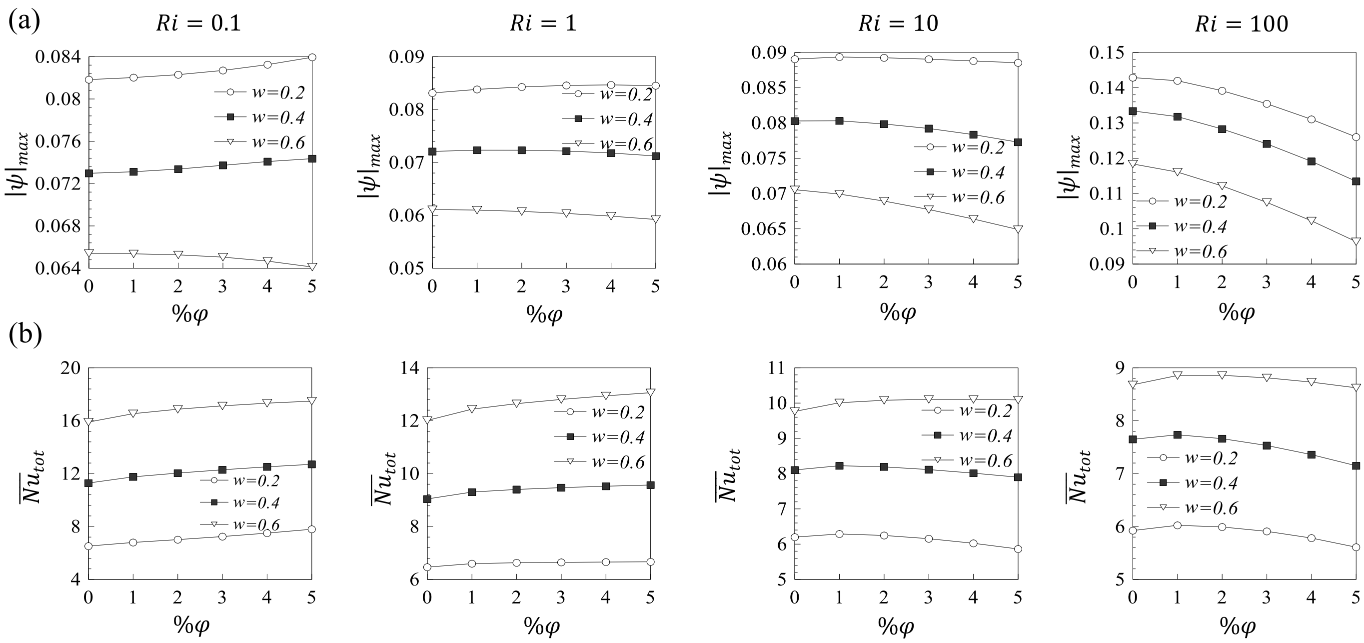
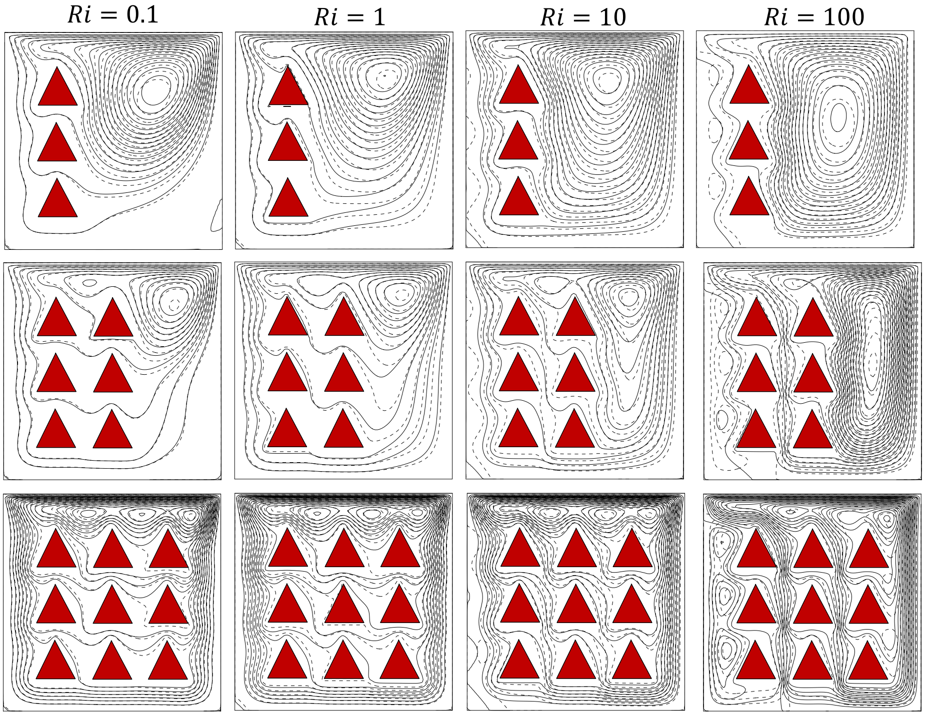
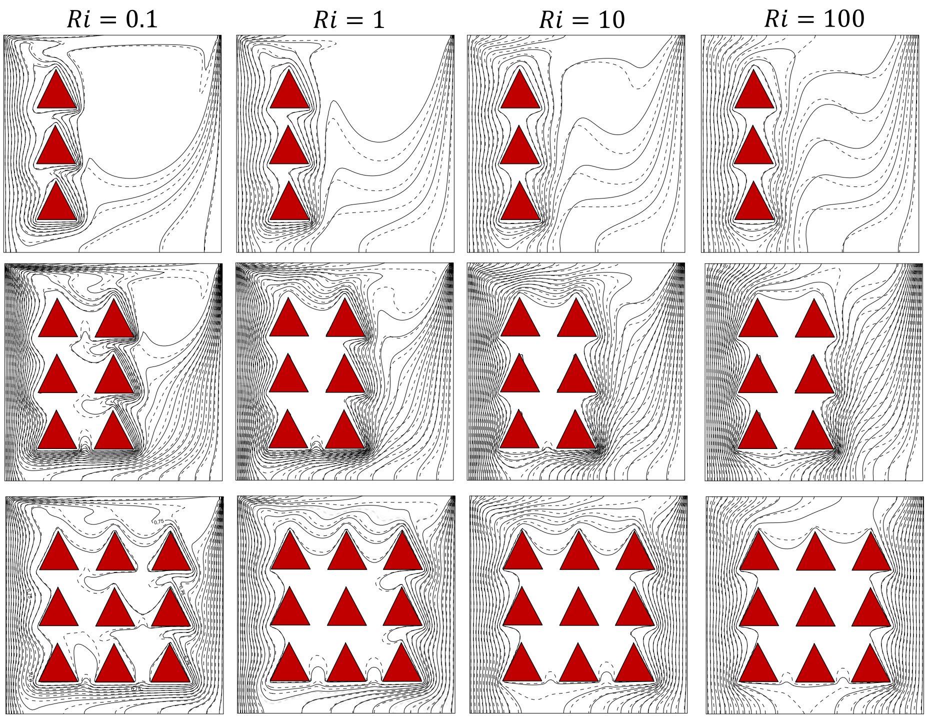


Fig. 4. ***Variation of (a) ψ_{max} and (b) $\overline{Nu}_{tot}$ corresponding to different sizes of triangular heat source, i.e. $W = w/H = 0.2, 0.4$ and 0.6 for different Richardson numbers and volume fraction of the nanoparticles. $Gr = 10^4$.***

Fig. 4 shows values of ψ_{max} and $\overline{Nu}_{tot}$ in the same conditions of Fig. 3 at different block sizes, Richardson number and volume fraction of nanoparticles. According to Fig. 4(b), it is noticed that by increasing volume fraction of nanoparticles and decreasing Ri the heat transfer rate increases. It is clear that the heat transfer rate enhances with increasing block size from $w = 0.2H$ to $0.6H$. At low Richardson number (forced convection are dominant), the maximum stream function does not change significantly (see Fig. 4(a)) therefore enhancement of heat transfer rate is maximum for $\varphi = 5\%$. At high Richardson number (Ri > 10) the effects of natural convection are dominant so there is an optimum volume fraction of nanoparticles which maximize the heat transfer is about 1% in the most cases.



(a) streamlines



(b) isotherms

Fig. 5. ***(a) Streamlines and (b) isotherms inside the cavity corresponding to different number of triangular heated blocks, i.e. $N = 3$, 6 and 9 filled with Cu–water the pure fluid (dashed line) and Cu–water nanofluid (solid line) with $\varphi = 5\%$ and at different Ris. $Gr = 10^4$.***

5.2 EFFECTS OF BLOCK'S NUMBER

In this section results on the effects of increase in the number of triangular heated blocks (three, six and nine of triangular heated blocks) inside the cavity filled with Cu–water nanofluid, on the fluid flow and the heat transfer rate is presented.

Fig.5 (a-b) shows the effects of block's number, on the streamlines and isotherms for various values of Ri ranging from 0.1 to 100 while Grashof number is fixed at $Gr = 10^4$ and the size of triangular heat source is fixed at $w = 0.2H$. As shown in Fig.5 (a), by increasing Richardson number, the effects of moving wall decreases and the vortex intensity becomes stronger. By increasing the number of triangular heated block from $N = 3$ to 9, the passage width between the triangular blocks and the cavity wall decreases. When $Ri \leq 10$ (forced convection), the main recirculation bubble gets gradually trapped between the moving lid and the upper surface of the blockage getting more and more squeezed. We can see in Fig.5 (b), that by decreasing Ri the concentration of the isotherms, close to the triangular heated blocks increases and their patterns are characterized by a complex form. We can see clearly in Fig.5 (a), at $Ri = 100$ a vertical recirculation bubbles develop in the left wall of the cavity. This interesting flow is a direct consequence of dominating natural convection at $Ri = 100$. The uniformly distributed isotherms at this Richardson number show that the main heat transfer mechanism is through the natural convection.

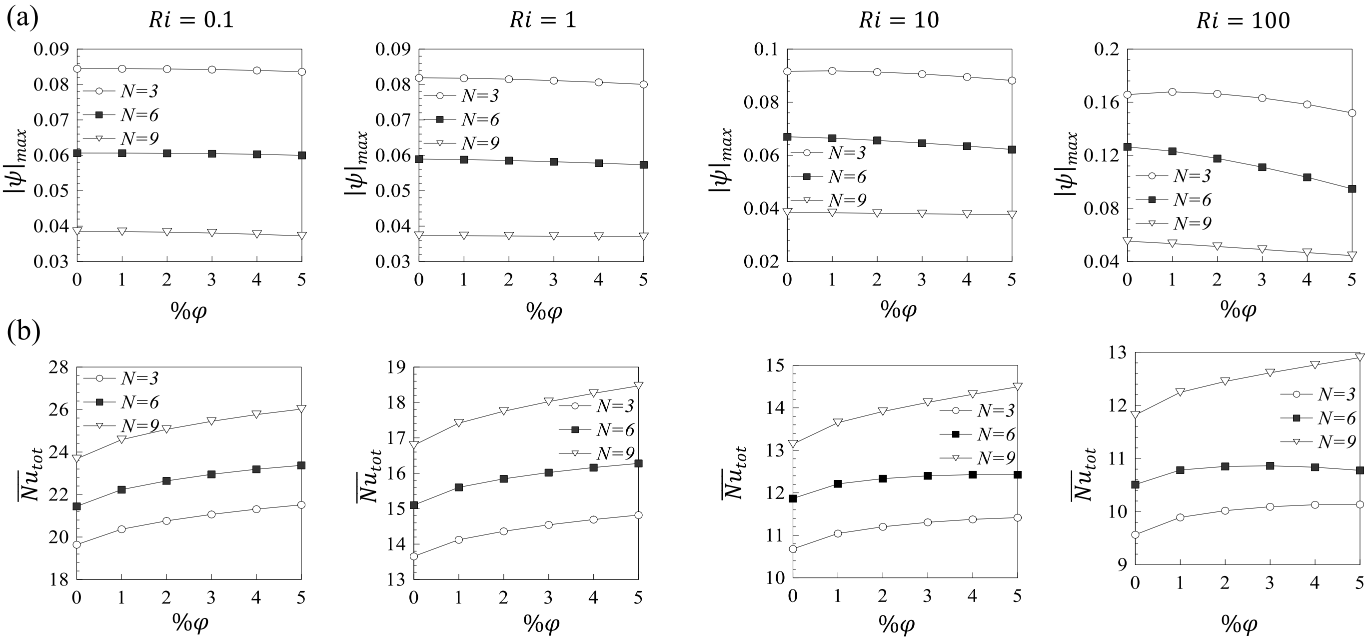


Fig. 6. Variation of (a) ψ_{max} and (b) $\overline{Nu}_{tot}$ corresponding to different number of triangular heated blocks, i.e. $N = 3$, 6 and 9 for different Richardson numbers and volume fraction of the nanoparticles. $Gr = 10^4$.

Fig. 6 shows values of ψ_{max} and $\overline{Nu}_{tot}$ in the same conditions of Fig. 5 at different number of triangular heated blocks, Richardson number and volume fraction of nanoparticles. It is clear that by decreasing the Richardson number and increasing the number of triangular heated blocks, the rate of convective heat transfer is increased (see Fig. 6(b)). On the other hand, in a constant Richardson number, by increasing the number of triangular heated blocks from $N = 3$ to 9, the convection suppresses and therefore flow intensity decreases (see Fig. 6(a)).

6 CONCLUSION

This study investigates mixed convection heat transfer of Cu-water nanofluid in a lid driven square cavity containing several heated triangular cylinders. The calculations are carried out in two phases. First, the effect of size of triangular heat source, three different sizes are considered as: $w = 0.2H$, $w = 0.4H$ and $w = 0.6H$. In the second phase, calculations are performed for different number of triangular block while the Richardson number (Ri) is ranging from 0.1 to 100 and the volume fraction of the nanoparticles is varying from 0 to 0.05. According to the presented results, the following conclusions are drawn:

- By increasing volume fraction of nanoparticles, φ, and decreasing Richardson number, Ri , the heat transfer rate increases.

- At all Richardson numbers, the heat transfer rate enhances with increasing block size from $w = 0.2H$ to $0.6H$.

- In a square cavity filled with Cu–water containing a central triangular heated block, when Ri > 10 the effects of natural convection are dominant so there is an optimum volume fraction of nanoparticles which maximize the heat transfer is about 1% in the most cases.

- At low Richardson number (forced convection are dominant), the maximum stream function does not change significantly, therefore enhancement of heat transfer rate is maximum for $\varphi = 5\%$.

- By decreasing the Richardson number and increasing the number of triangular heated blocks, the rate of convective heat transfer is increased and the flow intensity decreases.

NOMENCLATURE

C_p	specific heat, $Jkg^{-1}K^{-1}$
d_p	diameter of the nanoparticle, m
d_f	diameter of the base fluid molecule, m
g	gravitational acceleration, $m\ s^{-2}$
Gr	Grashof number $(= g\beta\Delta T H^3/\nu^2)$
H	enclosure height, m
k	thermal conductivity, $W\ m^{-1}K^{-1}$
k_b	Boltzmann's constant $= 1.38066 \times\ 10^{-23}$
N	Number of triangular heated blocks
$\overline{Nu}_{tot}$	heat transfer of cavity's wall
p	dimensional pressure, Nm^{-2}
P	dimensionless pressure
Pr	Prandtl number $\left(= \nu_f/\alpha_f\right)$
Ra	Rayleigh number $\left(= g\beta_f(T_h - T_c)H^3/\alpha_f\nu_f\right)$
Re_B	Brownian-motion Reynolds number
Re	Reynolds number $(= U_0H/\nu)$
Ri	Richardson number $(= Gr/Re^2)$
T	temperature, K
t	time, s
τ	dimensionless time $(t/H/U_{ref})$
u, v	velocity components, ms^{-1}
u_B	Brownian velocity of the nanoparticle, ms^{-1}
U, V	dimensionless velocity components
w	dimensional width of the triangular heater
W	dimensionless width of the triangular heater
x, y	Cartesian coordinates, m
X, Y	dimensionless Cartesian coordinates *(x, y)/H*

Greek symbols

α	thermal diffusivity, m^2s^{-1}
β	thermal expansion coefficient, K^{-1}
θ	dimensionless temperature
μ	dynamic viscosity, $kg\ m^{-1}s^{-1}$

ν kinematic viscosity, m^2s^{-1}
ρ density,kgm^{-3}
φ volume fraction of the nanoparticles
ψ stream function $(= -\int_{Y_o}^{Y} U\partial Y + \psi(X, Y_0))$

Subscripts
c cold
f fluid
h hot
nf nanofluid
s solid nanoparticles

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