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Elaboration of two stochastic models of EURO/MAD exchange rate and measure of their forecast accuracy

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ABSTRACT: Exchange rate in Moroccan economy has been considered a critical push-forward force for domestic inflation which leads to the depreciation of currency value. Exchange rate is the price for which the currency of a country can be exchanged for another country's currency in the foreign exchange market. This article seeks to adopt two stochastic models for estimation of exchange rate EURO/MAD. Firstly, it aims at the investigation of stochastic models (two models) to show the variation of exchange rate, and, secondly, try to draw a comparison between these models in terms of error estimation performances and, as a result, to recommend a stochastic model for exchange rate EURO/MAD.
In this paper, the geometric Brownian motion (stochastic process without mean reversion propriety) and Vasicek process (stochastic process with mean reversion speed), are used to model the exchange rate EURO / MAD, then they are compared in terms of average estimation error.
In order to calculate models parameters daily close price of the Euro/MAD from 01/12/2008 to 01/03/2016 (2242 values) can be taken from Casablanca stock exchange and ,hence, two stochastic models for exchange rate is to be derived, and compared. According to simulation results, we can finally recommend one of the two models.

KEYWORDS: Exchange rate, Vasicek, Brownian motion, Euromad, Stochastic process.

1 INTRODUCTION

Exchange rates are of great importance for the economy of a country and particularly its foreign trade, in respect of goods or services. The volume of products imported or exported to another country depends on the exchange rate of these countries. This may be an inflation factor in the country whose rate drops, or a factor of commercial, financial and political instability in some cases. For this reason it's necessary to anticipate the future exchange rates. To do mathematical modeling assumes that the exchange rate is a stochastic process (random variable time-dependent). In this work we are interested in the price in which a euro traded against the Moroccan currency MAD. To achieve this, two stochastic models belonging to different families (with and mean reversion property) are used and compared in terms of error estimation: Geometric Brownian motion and the process of Vasicek.

2 THEORETICAL PRINCIPLES OF RESEARCH

2.1 THE GEOMETRIC BROWNIAN MOTION (GBM)

The Geometric Brownian Motion (GBM) is a fundamental example of a stochastic process without mean reversion properties. The GBM is the underlying process from which is derived to form the Black and Scholes formula for pricing European options **[1].** Let the exchange rate be assigned as x_t where $\ln(x_t)$ obeys the following defined equation.

$$dln(x_t) = \mu dt + \sigma dw_t$$

Here **μ** and **σ** are constants and W_t is a standard Brownian motion.

2.2 VASICEK MODEL

The objective behind adopting the Vasicek model in this research is to model the variation of exchange rates as a stochastic process with a mean reversion. Vasicek model was the first to capture the value of mean reversion. In a linear equation, the dynamics of exchange rate is being described by this model, as it can be explicitly solved **[2]**.

$$dx_t = \alpha(\mu - x_t)dt + \sigma dw_t$$

Where **α, μ** and x_0 constants and dW_t represent an increment to a standard Brownian motion W_t. The exchange rate x_t will fluctuate randomly, but, over the long run, tends to revert to some level **μ**. The speed of reversion is known as **α** and the short-term standard deviation is **σ** where both influence the reversion.

This paper brings into play accurate data from 01/12/2008 to 01/03/2016 (2243 values) taken from DirectFN (provider of financial information) **[3]**, and Maximum likelihood function is used to calculate parameters of both GBM and Vasicek model.

3 METHODOLOGY

3.1 NO MEAN REVERSION – GEOMETRIC BROWNIAN MOTION

Let the continuous-time exchange rate be assigned as x_t where $\ln(x_t)$ obeys the following defined equation:

$$dln(x_t) = \mu dt + \sigma dw_t \quad (1)$$

Here, **μ** and **σ** are constants and dw_t is a standard Brownian motion. In ordinary calculating, one can derive that:

$$dln(x_t) = \frac{dx_t}{x_t} \qquad \text{So} \qquad \frac{dx_t}{x_t} = \mu dt + \sigma dw_t$$

If we adopt Ito's Lemma as mentioned in J.C. Hull **[1]**, the equation will be as follows:

$$dln(x_t) = (\mu - \tfrac{1}{2}\sigma^2)dt + \sigma dw_t \qquad \text{with} \qquad \gamma = \mu - \tfrac{1}{2}\sigma^2$$

This means that $\ln(x_t)$ is an Arithmetic Brownian Motion. By integrating equation between **u** and **t**, and according to Damiano Brigo et al **[4]**, gives:

$$\ln(x_u) - \ln(x_t) = (\mu - \frac{1}{2}\sigma^2)(u-t) + \sigma(w_u - w_t) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)(u-t);\ \sigma^2(u-t)\right)$$

By considering $u = T$, $t = 0$ and taking the exponent on equation above leads to:

$$x_T = x_0 exp\left(\left(\mu - \tfrac{1}{2}\sigma^2\right)T + \sigma w_T\right) \qquad (w_0=0)$$

The mean and the variance of x_T according to Damiano Brigo et al (2007) **[4]** are:

$$E(x_T) = x_0 e^{\mu T} \qquad \text{And} \qquad Var(x_T) = e^{2\mu T}{x_0}^2\left(e^{\sigma^2 T} - 1\right)$$

Therefore, the version of a simulation equation for the GBM, using the fact that is $dW = Z\sqrt{\Delta t}$ **[1]:**

$$\gamma = \mu - \frac{1}{2}\sigma^2$$

$$\ln\left(x_{t_{i+1}}\right) - \ln\left(x_{t_i}\right) = \gamma\Delta t + \sigma Z_i\sqrt{\Delta t} \qquad Z_i \sim N(0,1)$$

By taking the exponent of both sides, it results:

$$\boldsymbol{x_{t_{i+1}} = x_{t_i} exp(\gamma\Delta t + \sigma Z_i\sqrt{\Delta t})} \qquad Z_i \sim N(0,1)$$

3.1.1 MAXIMUM LIKELIHOOD ESTIMATION (MLE) – GEOMETRIC BROWNIAN MOTION

According to *Damiano Brigo et al* (2007) **[4]**, the parameters that must be optimized are $\theta(\mu, \sigma)$ for the GBM. Let the logarithmic return be given as:

$$y_{t_i} = ln(x_{t_i}) - \ln(x_{t_{i-1}})$$

Which is normally distributed for all $y_{t_1}, y_{t_2} \dots\dots.. y_{t_n}$. And these later values assumed independent. The likelihood function will be denoted as:

$$L(\theta) = f_\theta(y_{t_1}, y_{t_2} \dots\dots.. y_{t_n}) = \prod_{i=1}^{n} f_\theta(y_{t_i}) = \prod_{i=1}^{n} f(y_{t_i} | \theta)$$

Here, f_θ is the probability density function. Let $\theta = (\mu, \sigma)$, then the probability density function f_θ is:

$$f_\theta(y_{t_i}) = \frac{1}{x_{t_i}\sigma\sqrt{2\pi t}} exp\left[-\frac{\left(\left(\frac{y_{t_i}}{y_{t_0}}\right)-\left(\mu-\frac{1}{2}\sigma^2\right)t\right)^2}{2\sigma^2 t}\right]$$

The likelihood function needs to be maximized to obtain the optimal estimators $\hat{\theta}(\hat{\mu}, \hat{\sigma})$.

First, we have to determine $\hat{w}$ and $\hat{\gamma}$:

$\hat{w} = \left(\hat{\mu} - \frac{1}{2}\hat{\sigma}^2\right)\Delta t$ *with* $\hat{w} = \sum_{i=1}^{n} \frac{y_{t_i}}{n} = \frac{\ln(x_{t_n})-\ln(x_{t_0})}{n}$

$\hat{\gamma} = \hat{\sigma}^2 \Delta t$ *with* $\hat{\gamma} = \sum_{i=1}^{n} \frac{(y_{t_i}-\hat{w})^2}{n}$

Then the MLE's parameters are:

$\hat{\sigma}^2 = \frac{\hat{\gamma}}{\Delta t}$ *And* $\hat{\mu} = \frac{1}{2}\hat{\sigma}^2 + \frac{\hat{w}}{\Delta t}$

3.1.2 EURO/MAD EXCHANGE RATE: GEOMETRIC BROWNIAN MOTION

In order to calculate $\hat{\mu}$ *and* $\hat{\sigma}$ daily close price of the Euro/MAD from 01/06/2006 to 01/03/2016 can be taken directly from DirectFN **[3]** for Casablanca Stock Exchange. And considering $\Delta t = \frac{1}{365}$ (daily data)

3.1.2.1 SIMULATION RESULTS

Using the daily close price of the Euro/MAD from 01/06/2006 to 01/03/2016 and Microsoft Excel's solver, we obtain:

$\boldsymbol{\hat{\mu} = 0,0009594}$ **and** $\boldsymbol{\hat{\sigma} = 0,01069047}$

The simulation equation for Euro with Moroccan currency according to GBM is:

$x_{t_{i+1}} = x_{t_i} e^{(0,000029288959.\Delta t + 0,1033947295.Z_i.\sqrt{\Delta t})}$ $Z_i \sim N(0,1)$ And $\Delta t = \frac{1}{365}$

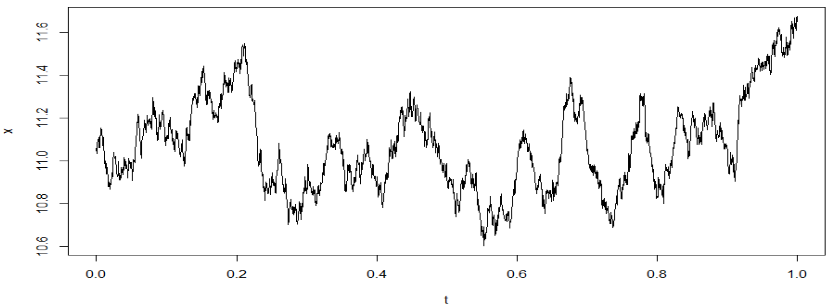

Fig. 1. Exchange rate EURO/MAD simulation using R: GBM model

3.1.2.2 GBM MODEL, ERROR PERFORMANCES

Let $\hat{x}_{t_i}$ be the estimated value of exchange rate euro-mad at time t_i:

The sum of squared errors (SSE).

$$\boldsymbol{SSE} = \sum_{i=1}^{2243} (\boldsymbol{x}_{t_i} - \widehat{\boldsymbol{x}}_{t_i})^2 = \mathbf{16,8722}$$

The mean absolute percentage error (MAPE).

$$\boldsymbol{MAPE} = \frac{\mathbf{1}}{\mathbf{2243}} \sum_{i=1}^{2243} \frac{|\, \boldsymbol{x}_{t_i} - \widehat{\boldsymbol{x}}_{t_i} \,|}{\boldsymbol{x}_{t_i}} * \mathbf{100} = \mathbf{0,5548}\,\%$$

3.2 MEAN REVERSION – THE VASICEK MODEL

The Vasicek model, owing its name to Vasicek (1977), is one of the earliest stochastic models of the short-term interest rate, which is a suitable model to apply the maximum likelihood estimation (MLE) **[5]**.

$$dx_t = \alpha(\mu - x_t)dt + \sigma dw_t \qquad (1)$$

Solving the Ornstein-Uhlenbeck Stochastic Differential Equation includes taking the derivative of $e^{\alpha t}x_t$ and rearranging the order gives:

$$e^{\alpha t}dx_t = d(e^{\alpha t}x_t) - x_t\alpha e^{\alpha t}dt \qquad (2)$$

And multiplying (1) by $e^{\alpha t}$

$$e^{\alpha t}dx_t = e^{\alpha t}\alpha(\mu - x_t)dt - e^{\alpha t}\sigma dw_t \qquad (3)$$

By using equations (2) and (3).

$$d(e^{\alpha t}x_t) = \alpha e^{\alpha t}\mu dt + e^{\alpha t}\sigma dw_t$$

If an integral is taken from time **0** to **t** gives:

$$e^{\alpha t}x_t = x_0 + \int_0^t \alpha e^{\alpha s}\mu ds + \int_0^t e^{\alpha s}\sigma dw_s$$

And this implies

$$x_t = x_0e^{-\alpha t} + \int_0^t \alpha e^{-\alpha t}e^{\alpha s}\mu ds + \int_0^t e^{\alpha s}e^{-\alpha t}\sigma dw_s = x_0e^{-\alpha t} + \int_0^t \alpha e^{-\alpha(t-s)}\mu ds + \int_0^t e^{-\alpha(t-s)}\sigma dw_s$$

$= x_0e^{-\alpha t} + \mu(1 - e^{-\alpha t}) + \int_0^t e^{-\alpha(t-s)}\sigma dw_s$

The solution of the stochastic differential equation between **s** and **t,** if 0<s<t :

$x_t = x_se^{-\alpha(t-s)} + \mu\left(1 - e^{-\alpha(t-s)}\right) + \sigma e^{-\alpha t}\int_s^t e^{-\alpha u}\sigma dw_u \quad (4)$

$\int_0^t e^{-\alpha(t-s)}\sigma dw_s$ follows a normal distribution with a mean of zero and a variance such that:

$E\left[\left(\int_0^t e^{-\alpha(t-s)}\sigma dw_s\ \right)^2\right] = \int_0^t\left(e^{-\alpha(t-s)}\sigma\right)^2 ds = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha t})$. *(*Ito isometric property)

The conditional mean and variance of x_t given x_0 is:

$$E_0[x_t] = \mu + (x_0 - \mu_t)e^{-\alpha t}$$

$$Var_0[x_t] = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha t}) \quad \alpha > 0$$

The conditional mean and variance of $\mathbf{x_t}$ given $\mathbf{x_s}$ are:

$$E_s[x_t] = \mu + (x_s - \mu_t)e^{-\alpha(t-s)}$$

$$Var_s[x_t] = \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha(t-s)}\right) \quad \alpha > 0$$

Calculations for the maximum likelihood estimates are made according to M.A. van den Berg (2007) "Calibrating the Ornstein-Uhlenbeck model" **[6]**. As follows, the conditional density functions for x_{t_i} given $x_{t_{i-1}}$ is:

$$f\left(x_{t_i} \middle| x_{t_{i-1}};\ \mu, \alpha, \sigma\right) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[\frac{-(x_{t_i} - x_{t_{i-1}} e^{-\alpha\Delta t} - \mu(1 - e^{-\alpha\Delta t}))^2}{2\sigma^2}\right]$$

where $\sigma^2 = \hat{\sigma}^2 \frac{(1 - e^{-2\alpha\Delta t})}{2\alpha}$ *(*)*

The log-likelihood function is given by:

$$\ln L(\mu, \alpha, \sigma) = \sum_{i=1}^{n} \ln f(x_{t_i} | x_{t_{i-1}}; \mu, \alpha, \sigma) = -\frac{n}{2}\ln(2\pi) - n ln(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left((x_{t_i} - x_{t_{i-1}} e^{-\alpha\Delta t} - \mu(1 - e^{-\alpha\Delta t})\right)^2$$

The log-likelihood function have to be maximized by taking partial derivatives of equation with respect to μ, α and σ and which yield three equations all equal to zero:

$$\begin{cases} \dfrac{\partial lnL(\mu, \alpha, \sigma)}{\partial \mu}|_{\hat{\mu}} = 0 \\ \dfrac{\partial lnL(\mu, \alpha, \sigma)}{\partial \alpha}|_{\hat{\alpha}} = 0 \\ \dfrac{\partial lnL(\mu, \alpha, \sigma)}{\partial \sigma}|_{\hat{\sigma}} = 0 \end{cases}$$

Then, the estimators will be:

$$\hat{\mu} = \frac{\sum_{i=1}^{n}\left(x_{t_i} - x_{t_{i-1}} e^{-\hat{\alpha}\Delta t}\right)}{n\left(1 - e^{-\hat{\alpha}\Delta t}\right)} \quad \hat{\alpha} = -\frac{1}{\Delta t} ln\left[\frac{\sum_{i=1}^{n}\left(x_{t_i} - \hat{\mu}\right)\left(x_{t_{i-1}} - \hat{\mu}\right)}{\sum_{i=1}^{n}\left(x_{t_{i-1}} - \hat{\mu}\right)^2}\right] \quad \bar{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_{t_i} - \hat{\mu} - e^{-\hat{\alpha}\Delta t}\left(x_{t_{i-1}} - \hat{\mu}\right)\right)^2$$

The following formulas are used to simplify further calculations:

$$S_x = \sum_{i=1}^{n} x_{t_{i-1}} \quad S_{xx} = \sum_{i=1}^{n} {x_{t_{i-1}}}^2 \quad S_{yy} = \sum_{i=1}^{n} {x_{t_i}}^2 \quad S_{xy} = \sum_{i=1}^{n} x_{t_{i-1}} x_{t_i} \quad S_y = \sum_{i=1}^{n} x_{t_i}$$

By using equations above, the MLE's parameters are:

$$\hat{\mu} = \frac{S_y - e^{-\hat{\alpha}\Delta t} S_x}{n\left(1 - e^{-\hat{\alpha}\Delta t}\right)} \quad (a) \qquad \hat{\alpha} = -\frac{1}{\Delta t} ln\left[\frac{S_{xy} - \hat{\mu}S_x - \hat{\mu}S_y + n\hat{\mu}^2}{S_{xx} - 2\hat{\mu}S_x + n\hat{\mu}^2}\right] \quad (b)$$

$$\bar{\sigma}^2 = \frac{1}{n}\left[S_{yy} - 2e^{-\hat{\alpha}\Delta t} S_{xy} + e^{-2\hat{\alpha}\Delta t} S_{xx} - 2\hat{\mu}\left(1 - e^{-\hat{\alpha}\Delta t}\right)\left(S_y - e^{-\hat{\alpha}\Delta t} S_x\right) + n\hat{\mu}^2\left(1 - e^{-\hat{\alpha}\Delta t}\right)^2\right] \quad (c)$$

If the equation (b) is substituted into (a), it yields:

$$\hat{\mu} = \frac{S_y S_{xx} - S_x S_{xy}}{n\left(S_{xx} - S_{xy}\right) - \left({S_x}^2 - S_x S_y\right)} \quad (c)$$

$$\hat{\alpha} = -\frac{1}{\Delta t} ln\left[\frac{S_{xy} - \hat{\mu}(S_x + S_y) + n\hat{\mu}^2}{S_{xx} - 2\hat{\mu}S_x + n\hat{\mu}^2}\right] \quad (d)$$

And using (*), the third estimate parameter $\hat{\sigma}^2$ is:

$$\hat{\sigma}^2 = \frac{2\hat{\alpha}}{n\left(1 - e^{-2\hat{\alpha}\Delta t}\right)}\left[S_{yy} - 2e^{-\hat{\alpha}\Delta t} S_{xy} + e^{-2\hat{\alpha}\Delta t} S_{xx} - 2\hat{\mu}\left(1 - e^{-\hat{\alpha}\Delta t}\right)\left(S_y - e^{-\hat{\alpha}\Delta t} S_x\right) + n\hat{\mu}^2\left(1 - e^{-\hat{\alpha}\Delta t}\right)^2\right]$$

3.2.1 VASICEK SIMULATION EQUATION

According to M.A. van den Berg **[6]**, the linear relationship between two consecutive observations $x_{t_{i+1}}$ and x_{t_i} is derived from (4) and is given as:

$$x_{t_{i+1}} = x_{t_i} e^{-\hat{\alpha}\Delta t} + \hat{\mu}\left(1 - e^{-\hat{\alpha}\Delta t}\right) + \hat{\sigma}\sqrt{\frac{1 - e^{-2\hat{\alpha}\Delta t}}{2\hat{\alpha}}} Z_i \qquad Z_i \sim N(0,1)$$

Note: Z_i are the same random values used for GBM simulated equation.

To calculate $\hat{\mu}$, $\hat{\alpha}$ and $\hat{\sigma}$ the same daily close price in Moroccan dirham of Euro from 12/01/2008 to 03/01/2016 are used with Microsoft Excel's solver, and hence stochastic model for exchange rate is to be derived.

3.2.2 SIMULATION RESULTS AND DISCUSSION

MLE values of the data set can be computed using Microsoft Excel's solver as:

$\hat{\alpha} = 26,8928501 \qquad \hat{\mu} = 11,1157017 \qquad \hat{\sigma} = 1,172973$

$dx_t = 26,8928501(11,1157017 - x_t)dt + 1,172973dw$

Consequently, due to this stochastic model, the exchange rate is fluctuating around the value 11,1157. With a mean reversion speed equals to 26,8928501 and an intensity of stochastic part is 1,17297. Moreover, this model can be used to forecast the exchange rate in Morocco. Variation of simulated exchange rate (with R) is shown in this Figure. With x_o=11,056 [at 01/12/2008]

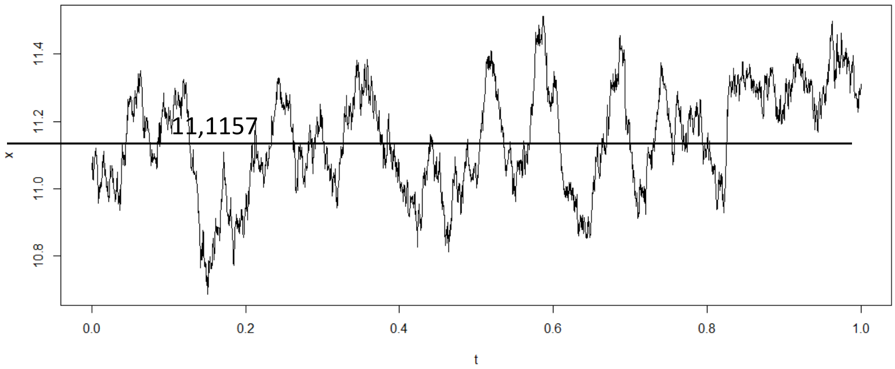

Fig. 2. Exchange rate EURO/MAD simulation using R

Deterministic part of the final stochastic differential equation is going to be:

$dx_t = 26,8928501(11,1157017 - x_t)dt$

And hence deterministic process is going to be (with x_o=11,056 [at 01/12/2008])

$x_t(t) = -0,0597\, e^{-26,8928501.t} + 11,1157017$

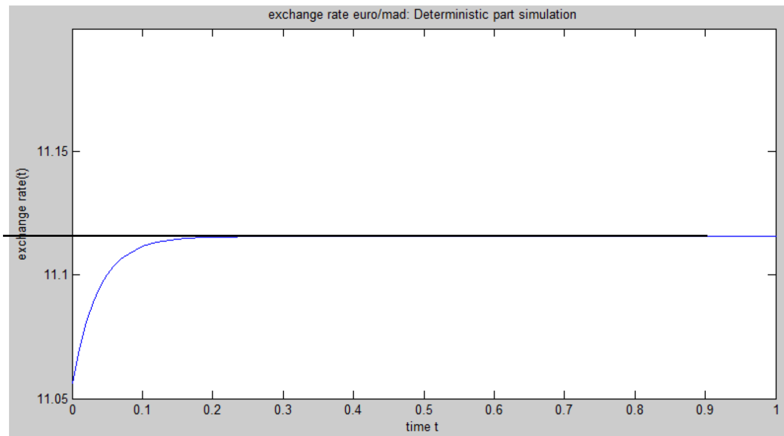

Fig. 3. Exchange rate EuroMad: deterministic part simulation

Therefore, it can be strongly concluded that exchange rate attains to the value 11,1157 for long time periods.

Finally, the simulated equation is:

$$x_{t_{i+1}} = x_{t_i} e^{-\hat{\alpha}\Delta t} + \mu\left(1 - e^{-\hat{\alpha}\Delta t}\right) + \hat{\sigma}\sqrt{\frac{1-e^{-2\hat{\alpha}\Delta t}}{2\hat{\alpha}}} Z_i \qquad Z_i \sim N(0,1)$$

$$x_{t_{i+1}} = 0,9289698 * x_{t_i} + 0,78954996 + 0,05920237 * Z_i$$

3.2.3 VASICEK MODEL, ERROR PERFORMANCES

Let $\hat{x}_{t_i}$ be the estimated value of exchange rate euro-mad at time t_i:

The sum of squared errors (SSE).

$$\boldsymbol{SSE = \sum_{i=1}^{2243}(x_{t_i} - \hat{x}_{t_i})^2 = 16,044}$$

The mean absolute percentage error (MAPE).

$$\boldsymbol{MAPE = \frac{1}{2243}\sum_{i=1}^{2243}\frac{|\,x_{t_i} - \hat{x}_{t_i}\,|}{x_{t_i}} * 100 = 0,5458}\,\%$$

4 COMPARISON AND DISCUSSION

4.1 COMPARISON

EURO/MAD Exchange rate : GBM Model	EURO/MAD Exchange rate :Vasicek Model
Simulated equation: $x_{t_{i+1}} = x_{t_i} e^{(0,000029288959.\Delta t + 0,1033947295.Z_i.\sqrt{\Delta t})}$ $Z_i \sim N(0,1)$ And $\Delta t = \frac{1}{365}$	Simulated equation: $x_{t_{i+1}} = 0,9289698 * x_{t_i} + 0,78954996 + 0,05920237 * Z_i$ $Z_i \sim N(0,1)$
The sum of squared errors (SSE). $SSE = \sum_{i=1}^{2243}(x_{t_i} - \hat{x}_{t_i})^2 = 16,8722$	The sum of squared errors (SSE). $SSE = \sum_{i=1}^{2243}(x_{t_i} - \hat{x}_{t_i})^2 = 16,044$
The mean absolute percentage error (MAPE). $MAPE = \frac{1}{2243}\sum_{i=1}^{2243}\frac{\|\,x_{t_i} - \hat{x}_{t_i}\,\|}{x_{t_i}} * 100 = 0,5548\%$	The mean absolute percentage error (MAPE). $MAPE = \frac{1}{2243}\sum_{i=1}^{2243}\frac{\|\,x_{t_i} - \hat{x}_{t_i}\,\|}{x_{t_i}} * 100 = 0,5458\,\%$

4.2 DISCUSSION AND RECOMMENDATION

To measure forecast accuracy, we use here two selection criteria: The sum of squared errors (SSE), which is the sum of the squares deviations of predicted from empirical values of data, it is a measure of the discrepancy between the data and an estimation model. A small SSE indicates a tight fit of the model to the data it is used as an optimality criterion in model selection, and the mean absolute percentage error (MAPE) which is a relative measure which expresses errors as a percentage of the actual data, this is its biggest advantage as it provides an easy and intuitive way of judging the extent, or importance of errors **[7]**.

In this study it seems that for both criteria SSE and MAPE, Vasicek model gives the better measure of forecast accuracy compared to GBM model (16,044<16,8722 and 0,5458%<0,5548%), and as a conclusion, we recommend this later for exchange rate EURO/MAD, as follow:

$$\boldsymbol{dx_t = 26,8928501(11,1157017 - x_t)dt + 1,172973dw}$$ *dw is a standard Brownian motion*

$$\boldsymbol{x_{t_{i+1}} = 0,9289698 * x_{t_i} + 0,78954996 + 0,05920237 * Z_i} \qquad Z_i \sim N(0,1)$$

5 CONCLUSION

This work has focused on two stochastic models. The first is a model without a mean reversion property (GBM model), and the second is a model based on the Vasicek process with speed of mean reversion, these theatrical models are used and calibrated with daily close prices for exchange rate EURO / MAD, and as a result elaborating of two models for EURO/MAD exchange rate forecast.

Equally important, the work introduces a comparison in terms of estimation error (deviations of predicted from empirical values of data) to choose one of these two models, it turned out that the model of Vasicek presents the lowest square sum of errors and also the smallest mean absolute average percentage error.

As a consequence, and according to our measure of forecast accuracy, we can recommend the Vasicek model compared to GBM model for modeling exchange rate EURO / MAD.

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