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Comparison of Wavelets for Medical Image Compression Using MATLAB

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ABSTRACT: This study addresses some mathematical and statistical techniques of medical image compression and their computational implementation. Fundamental theories have been presented, applied and illustrated with examples. To make the report as self-contained as possible, key terminologies have been defined and some classical results and theorems are stated, in the most part, without proof. Some algorithms and techniques of image processing have been described and substantiated with experimentation using MATLAB. Medical image compression is necessary for huge database storage in Medical Centers and medical data transfer for the purpose of diagnosis. Wavelet transforms present one such approach for the purpose of compression. The same has been explored in study with respect to wide variety of medical images. In this approach, the redundancy of the medical image and DWT coefficients are reduced through thresholding and further through Huffman encoding. In this study our main goal is to compare different types of wavelets for medical image compression. Finally, implementation of the above-mentioned concepts is illustrated.

KEYWORDS: Wavelet Transform, DWT, Thresholding, Huffman Encoding, Lossy Compression, MSE, PSNR, BPP, CR.

1 INTRODUCTION

Image compression algorithms have been the subject of research both in academia and industry for many years. Today, while significantly improved algorithms have been achieved and compression performance is better than a decade ago, there is still room for new technologies. The first widely adopted international image compression standard was JPEG which was introduced in the late eighties. JPEG is based on DCT followed by entropy coding based on either Huffman coding or binary arithmetic coding. It has been widely used from the printing industry to Internet applications. For example all high-end printers compress the image to be printed before they actually send it to the print engine, and most images transmitted through the internet are JPEG compressed. JPEG is intended for continuous tone images of more than one bit depth. Algorithms for binary images work in a different way, JBIG-1 and JBIG-2 are the standards covering this area. JPEG and JBIG are part of other standards, such as facsimile transmission standards, the FlashPix file format, the TIFF _le format, and page description languages like PDF [11], [14].

In recent years researchers have been using the discrete wavelet transform in compression systems. In 1983 Burt and Anderson were the first to introduce multiresolutional analysis in image compression. While their approach seemed counter intuitive at the first glance, given that it increased the number of samples to be coded, their results were promising. Mallat was the first to point out the connection between multiresolutional analysis and the wavelet transform. Daubechies has studied the discrete wavelet transform and has made it a popular tool in the scientific community. Some of the first papers on wavelet image compression presented excellent compression performance results and gave a lot of intuition behind the use of the wavelet transform in image compression. A number of researchers have described the same principles of wavelet image compression by looking at it from a system perspective, using filter banks, and sub band decomposition, and refer to wavelet coding as sub band coding. Sub band coding and wavelet coding essentially refer to the same system, the description of the system is from a slightly different point of view. In sub band coding the emphasis is in the frequency domain unlike wavelet coding where the emphasis is in the space domain [4], [7], [13].

Numerous organizations have been using wavelet compression algorithms as their own, internal compression standards. An example is the FBI where there was a need for storing large data-bases of -finger-prints and JPEG did not satisfy their requirements. Only more recently was there a decision by the ISO to standardize a wavelet coder in JPEG2000. Until recently all proposed wavelet coders would require buffering the whole image, computing the wavelet transforms in a frame buffer, application of a quantization on the wavelet coefficients and entropy coding of the generated indexes. Wavelet coders could indeed perform very well, but their complexity was well above the complexity of the current JPEG standard. Complexity issues on wavelet coders have only been emphasized by researchers in the last few years, as the JPEG2000 standard process has exposed the complexity of wavelet coders. Our work was among the first to address low memory wavelet coding [6].

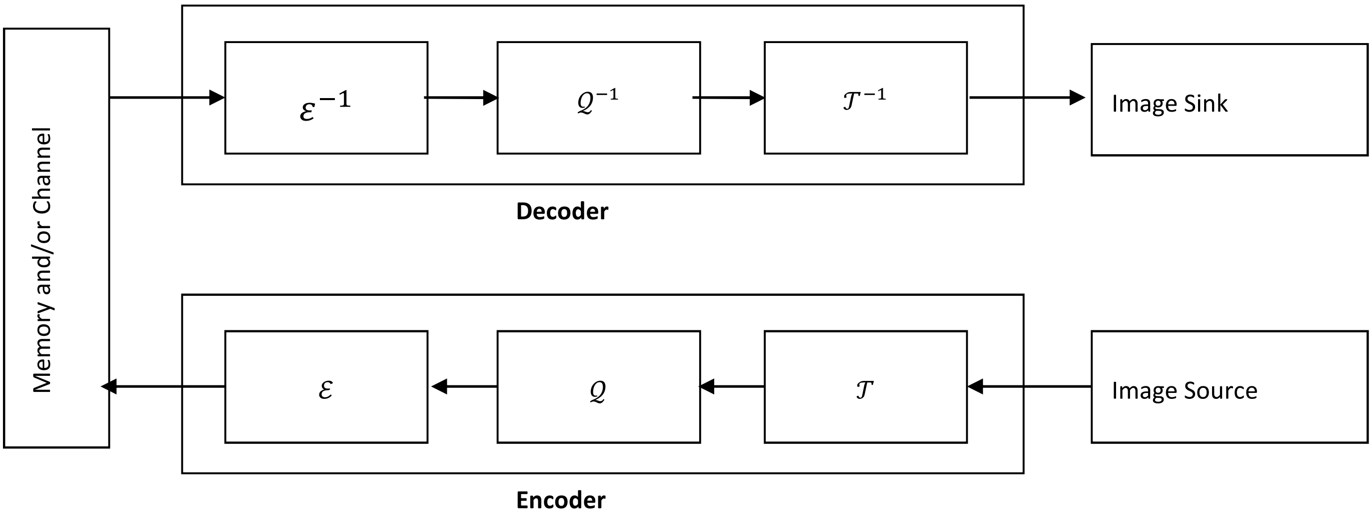


Fig. 1.Generic Image Encoder and Decoder.** $\mathcal{E}$ → **Entropy Encoder,** $\mathcal{Q}$ →**Quantizer**,$\mathcal{T}$ →**Transform.** ε^{-1} →**Entropy Decoder,** $\mathcal{Q}^{-1}$ →**Inverse Quantize,**$\mathcal{T}^{-1}$ →**Inverse Transform

There are two major classes of image compression algorithms, namely lossy and lossless algorithms. Lossless algorithms preserve the image data, i.e. original and reconstructed images are exactly the same. In lossy image compression original and reconstructed images may or may not be identical in a strict mathematical sense, but to a human observer they may look the same, so the goal is to achieve compression that is visually lossless. Both lossy and lossless compression algorithms are used today in a broad range of applications, from transmitting satellite images, to web browsing to image printing and scanning. With lossy compression algorithms we can achieve significantly larger compression ratios compared to lossless algorithms [2], [15].

2 MEDICAL IMAGE COMPRESSION

Traditional image compression techniques have been designed to exploit the statistical redundancy present within real world images. The discrete cosine transforms (DCT), DPCM, and the entropy coding of subband images are all examples of this statistical approach. Removing redundancy can only give a limited amount of compression; to achieve high ratios, some of the non-redundant information must also be removed. Wavelet transform provides one such approach for image compression. Medical image compression is a challenge as the high frequency components may contain details relevant for medical diagnosis. In medical image compression applications, diagnosis is effective only when compression techniques preserve all the relevant and important image information needed. Thus, most of applications such as telemedicine and fast searching and browsing of medical volumetric data suffer from this limitation. For these kinds of applications, lossy compression seems to be an appropriate alternative. DICOM permits lossy image compression by a JPEG baseline system for ultrasonic echo images but suffers due to their inherent poor resolution. Important properties of wavelet transforms such as multiresolution representation, energy compaction, blocking artifacts and decorrelation, has made the discrete wavelet transform (DWT) one of the most important techniques for image and video compression in the last decade and it has been adopted by JPEG 2000 standard. In fact there is no single wavelet, which will always provide the best performance. Since there are many wavelet filters available, each with the different set of basic functions, the choice of wavelet filters is very vital factor to gain a good coding performance. Therefore, one of the intent of this thesis is also to investigate the effect of applying different types of wavelet for lossy compression of medical images [1], [5], [8].

3 BASIC MODEL OF COMPRESSION SYSTEM

Most of the compression systems are based upon reducing the redundant information present in the signal whether it is 1D signal or 2D signal like image. Sometime redundancy reduction process is performed over the transformed signal rather than the original signal itself. The redundancy depends upon the entropy of the signal. Fig. 2 shows the basic model of the compression system based on redundancy in data [12].

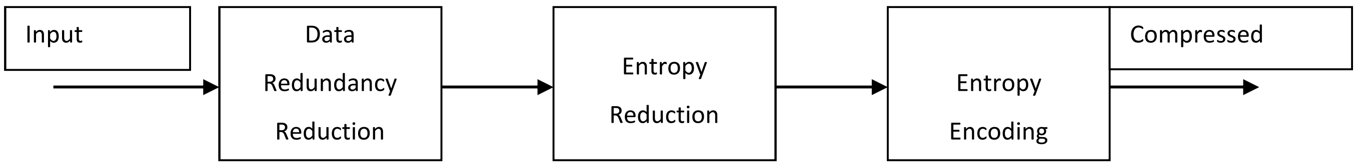


Fig. 2.Basic Model of Compression System

Redundancy reduction removes highly correlated data which is more in case of image due to much of the low frequency content in it. Discrete Wavelet Transform (DWT) has emerged as a popular technique for redundancy reduction. DWT has high decor relation and energy compaction efficiency. Non-significant information is removed from the data by this process but it is a non-reversible process electronically for review. The popular entropy coding techniques are Huffman coding and Arithmetic coding which are used many times in compressing data and also in image compression techniques.

4 WAVELET TRANSFORMS FOR IMAGE

One-dimensional wavelet theory defines a function ψ, the wavelet, and its associated scaling function φ, such that the family of functions $\{\psi^j(x)\}_{j\epsilon z}$, where $\psi^j(x) = \sqrt{2^j}\,\psi(2^j(\,x\,)$, are orthonormal. The continuous wavelet transform is defined by eq. 4.1.

$$C\big(a, b; f(t), \psi(t)\big) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{a}}\psi^*\left(\frac{t-b}{a}\right)\,dt \tag{4.1}$$

Where, *a* is the Scaling Function, *b* is Shifting Function and$\psi(t)$ is the Wavelet function.

For the formulation of Discrete Wavelet Transform, the scale and shift parameters are discretized as,

$$a = a_0^m,\ \ and\ \ \ b = nb_0$$

Thus the Analyzing Wavelet is also discretized as follows:

$$\psi_{m,n}(t) = {a_0}^{-m/2}\psi\left(\frac{t-nb_0}{a_0^m}\right) \tag{4.2}$$

Where, m and n are integer values.

Thus discrete wavelet transform and its inverse transform are defined as follows:

$$S_{m,n} = \int_{-\infty}^{\infty}\psi'_{m,n}\,(t)\ \ s(t)\ \ dt$$

$$s(t) = k_\psi \textstyle\sum_m \sum_n S_{m,n}\psi_{m,n}(t) \tag{4.3}$$

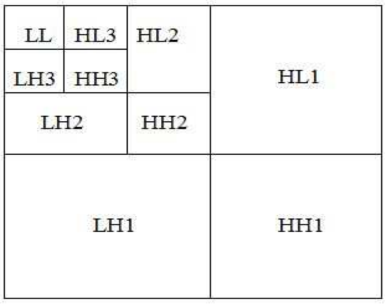
Where, k_ψ is a constant value for normalization.

The function $\psi_{m,n}(t)$ provides sampling points on the scale-time plane; these are linear sampling points in the time (b-axis) direction but logarithmic in the scale (a-axis) direction.

The most common situation is when a_0 is chosen as $a_0 = 2^{1/v}$. Where v is an integer value; and those v pieces of $\psi_{m,n}(t)$ are processed together as one group. For images, we use the hierarchical wavelet decomposition suggested by

Mallat. The high pass filters, H and the low pass filters, L are applied to the image in both the horizontal and vertical directions i.e. over rows and columns of the image matrix. The filtered outputs are subsampled signals, each by a factor of two, generating selective high-pass sub bands details oriented in three directions viz. horizontal, diagonal and vertical; LH, HH and HL respectively. And also a low-pass approximation sub band, LL obtained by applying low pass filter in horizontal and vertical direction. Fig. 3(a) shows wavelet sub band decomposition of an image. The process is then repeated over the LL band, approximation, to generate the next level of the decomposition sub band. Fig. 3(b) illustrates this wavelet image decomposition for any arbitrary jth level sub band. Thus three octaves of decomposition will lead to ten decomposed sub bands. Fig. 3(c) and 3(d) shows a typical MRI image decomposed in this way, up to first and third octave of decomposition respectively [9].

Huffman Encoding:



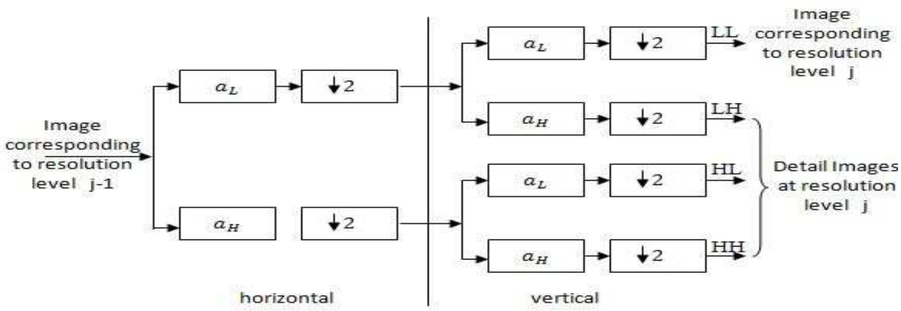
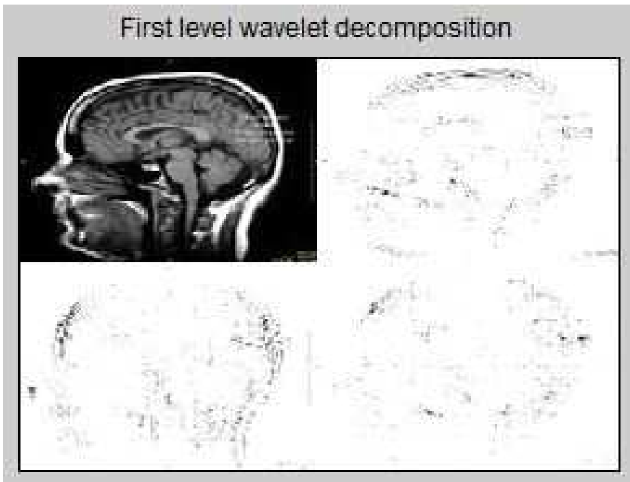


Fig. 3(a) *Fig. 3(b)*



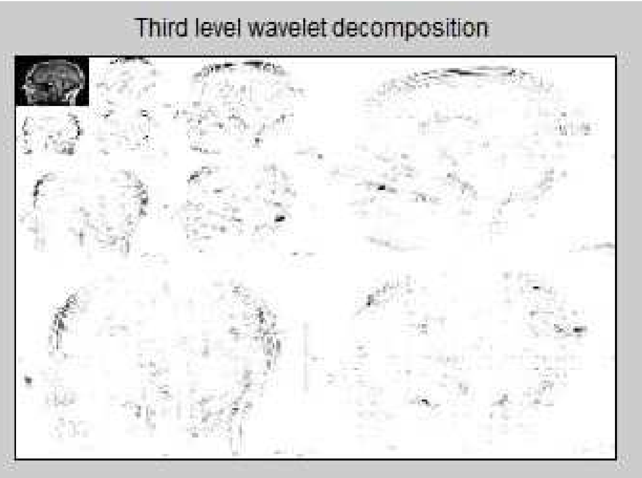


Fig. 3(c) *Fig. 3(d)*

Fig. 3 (a) Wavelet Subband Decomposition of image, (b) Decomposition of image to jth level
(c) Wavelet Decomposition of MRI brain image upto first level, (d) Wavelet Decomposition upto third level

As Huffman codes belong to a family of codes which are variable in code word length, which means that individual symbols which makes a message are represented (encoded) with bit sequences that have distinct length. This helps to decrease the amount of redundancy in message data. Decreasing the redundancy in data by Huffman codes is based on the fact that distinct symbols have distinct probabilities of incidence. This helps to create code words, which really contribute to redundancy. Symbols with higher probabilities of incidence are coded with shorter code words, while symbols with higher probabilities are coded with longer code words.

5 HARD THRESHOLDING AND SOFT THRESHOLDING

Thresholding is the simplest method of image denoising .In this from a gray scale image, thresholding can be used to create binary image. Thresholding is used to segment an image by setting all pixels whose intensity values are above a threshold to a foreground value and Thresholding is mainly divided into two categories all the remaining pixels to a background value [10].

Hard threshold is a "keep or kill" procedure and is more intuitively appealing. The transfer function of the Hard thresholding is shown in the figure. Hard thresholding may seem to be natural. Sometimes pure noise coefficients may pass the hard threshold and this thresholding method is mainly used in medical image processing. Hard thresholding can be defined as follow:

$$D(U,\lambda) = \begin{matrix} U & for\ all\ |D| > \lambda \\ o & othrwise \end{matrix} \tag{5.1}$$

Soft threshold shrinks coefficients above the threshold in absolute value. The false structures in hard thresholding can be overcomed by soft thresholding. Now a days, wavelet based denoising methods have received a greater attention. Important features are characterized by large wavelet coefficient across scales in most of the timer scales. Soft thresholding can be defined as follow:

$$D\ (U,\ \lambda) = sgn\ (U) max(0, |U| - \lambda) \tag{5.2}$$

6 ALGORITHM FOR COMPRESSION

The compression algorithm for medical image compression based on the wavelet transforms is given in following steps.

- For Compressing the Medical Image:
  - The DWT of the medical image is generated by obtaining wavelet decomposition coefficients for the desired levels. The numbers of levels are decided by the entropy of the image.
  - A threshold for the decomposed image coefficients is selected, below which all the coefficients are made zero. This reduces the band space of the image signal, as large number of coefficients are made zero.
  - Huffman encoding on the thresholded coefficients is applied to reduce the redundancy in the coefficient data.
  - The thresholded and Huffman encoded coefficients are saved instead of the image.
- For Uncompressing the Medical Image:
  - When the image is to be uncompressed, the Huffman decoding is done on the coefficients and the threshold coefficients are obtained.
  - Image is regenerated from these threshold coefficients by taking inverse discrete wavelet transform (IDWT).

7 PERFORMANCE ESTIMATION

Results have been obtained by calculating few parameters obtained by the comparing original image and uncompressed image. They are defined as follows [3], [16]:

7.1 MEAN SQUARE ERROR (MSE)

It is the cumulative squared error between original and recovered image. It is defined by eq.

$$MSE = \frac{1}{mn}\ \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|I(i,j) - K(i,j)\|^2 \tag{7.1}$$

Where, I is the original image and K is the uncompressed image. The dimension of the images is $m \times n$. Thus MSE should be as low as possible for effective compression.

7.2 PEAK SIGNAL TO NOISE RATIO (PSNR)

It is most commonly used as a measure of quality of reconstruction of lossy compression. It is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation; defined by eq.

$$PSNR = 20\ log_{10}\ \left(\frac{MAX_i}{\sqrt{MSE}}\right) \tag{7.2}$$

Where, *MAXi* is the maximum possible pixel value of the image. PSNR should be as high as possible.

7.3 BIT PER PIXEL (BPP)

It is number of bits used to encode each pixel value. Thus for the purpose of compression BPP should be less to reduce storage on the memory.

7.4 COMPRESSION RATIO (CR)

It is defined as the rate of the size of the original image data over the size of the compressed image data.

$$C_R = \frac{n_1}{n_2} \tag{7.3}$$

Where, C_R is the compression ratio, n_1 and n_2 is the number of information carrying units in the original and encoded images respectively. CR is express in percentage.

8 RESULTS AND DISCUSSIONS

The results over the images have been obtained using Haar, Doubechies 4, Symlet 8 and Biorthogonal 5.5 wavelets. Results are analyzed in tabular form, and images. The Image results are shown in Appendix. The First test image is that of a MRI of a human skull. The best result was obtained with Doubechies4 wavelet. The MSE is as low as 2.818 and PSNR is 43.63dB at the BPP of 1.7581. The compression ratio at this point is 21.98%. The image is illustrated in Fig. 4(a), in the Appendix. The results of various parameters of the Skull MRI image are recorded in Table 1 over different wavelets.

Table 1. Values of MRI skull Image Parameters

Img	Wavelet	MSE	PSR (dB)	BPP	CR%
MRI Skull	Harr	19.16	35.31	1.9376	24.22
		39.99	32.11	0.86011	10.75
	db 4	2.818	43.63	1.7581	21.98
		21.36	34.83	0.7119	8.90
	Sym 8	2.876	43.54	1.6965	21.21
		20.69	34.97	0.68372	8.55
	Bior 5.5	3.468	42.73	1.6516	20.65
		26.36	33.92	0.63782	7.97

In the second test Image that of Cardiac MR of Vertical Left outflow tract of heart, the best results are obtained with Doubechies4 wavelet. The MSE is as low as 3.925 and PSNR is 42.19dB at the BPP of 2.2996. The compression ratio at this point is 28.74%. The image is illustrated in Fig. 4(d), in the Appendix. The results of various parameters of the Cardiac MR image are recorded in Table 2 over different wavelets.

Table 2. Values of Cardiac MR Image Parameters

Img	Wavelet	MSE	PSR (dB)	BPP	CR%
Cardiac MR	Harr	5.543	40.69	2.7375	34.22
		35.45	32.64	1.001	12.51
	db 4	3.925	42.19	2.2996	28.74
		28.42	33.59	0.80005	10.00
	Sym 8	4.605	41.5	2.1936	27.42
		27.79	33.69	0.76843	9.61
	Bior 5.5	5.193	40.98	2.1326	26.66
		37.15	32.43	0.66882	8.36

In the third Image, that of Ultrasound of Liver Cyst the best results are obtained with Symlet 8 wavelet. Biorthogonal5.5, Doubechies4 and Harr showed a poorer result. The MSE is as low as 10.32 and PSNR is 37.99dB at the BPP of 1.5902. The compression ratio at this point is 19.88%. The image is illustrated in Fig. 4(g), in the Appendix. The results of various parameters of the Ultrasound Image are recorded in Table 3 over different wavelets.

Table 3. Values of Ultrasound Image Parameters

Img	Wavelet	MSE	PSNR (dB)	BPP	CR%
Ultra Sound	Harr	12.84	37.04	1.795	22.44
		79.74	29.11	0.6483	8.10
	db 4	14.99	36.37	1.6649	20.81
		80.4	29.08	0.5960	7.45
	Sym 8	10.32	37.99	1.5902	19.88
		73.6	29.46	0.5749	7.19
	Bior 5.5	16.32	36.00	1.5612	19.51
		99.45	28.15	0.5272	6.59

In the 4th Image, that of X-ray the best results are obtained with Symlet 8 wavelet. Biorthogonal5.5, Doubechies4 and Harr showed a poorer result. The MSE is as low as 5.322 and PSNR is 40.87dB at the BPP of 0.61243. The compression ratio at this point is 7.66%. The image is illustrated in Fig. 4(j), in the Appendix. The results of various parameters of the X-ray Image are recorded in Table 4 over different wavelets.

Table 4. Values of X-ray Image Parameters

Img	Wavelet	MSE	PSNR (dB)	BPP	CR%
X-ray	Harr	7.646	39.3	0.8181	10.23
		34.11	32.8	0.2441	3.05
	db 4	5.6	40.65	0.6178	7.72
		25.63	34.04	0.2263	2.83
	Sym 8	5.322	40.87	0.6124	7.66
		24.91	34.17	0.2196	2.75
	Bior 5.5	7.64	39.3	0.5194	6.49
		34.18	32.79	0.1940	2.43

9 CONCLUSIONS

The algorithm works well over the images as shown by the results. Huffman encoding is a lossless data compression technique. At the most optimal compression the original and uncompressed from wavelet coefficient is almost the very same. Future scope depends on developing a trained system which can automatically detect type of medical image and determine which suitable wavelet will produce the best compression on it. One such system could be developed using Neural Networks trained on specific images by different wavelets.

APPENDIX-IMAGE RESULTS

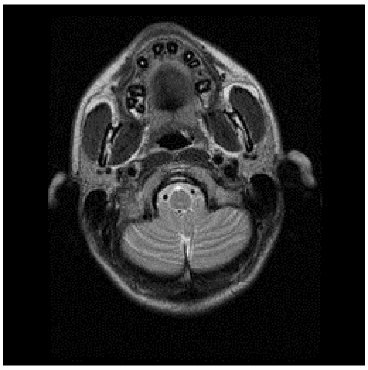


Fig. 4(a)

Fig. 4(b)

Fig. 4(c)

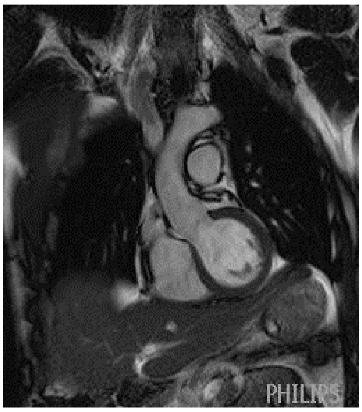


Fig. 4(d)

Fig. 4(e)

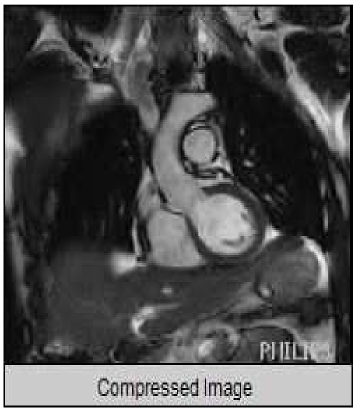


Fig. 4(f)

Fig. 4(g)

Fig. 4(h)

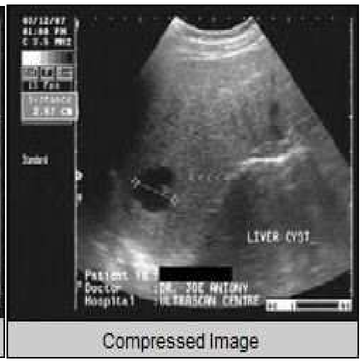


Fig. 4(i)

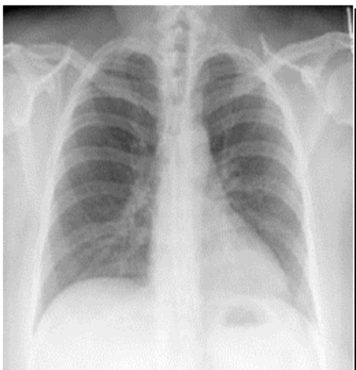


Fig. 4(j)

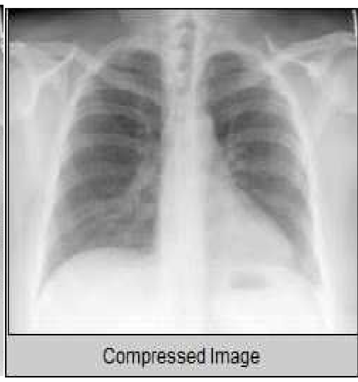


Fig. 4(k)

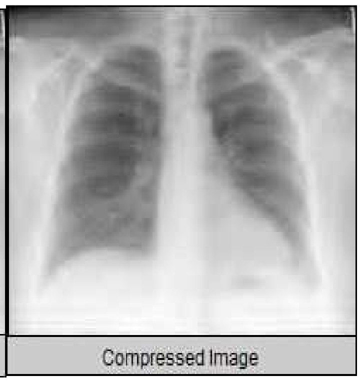


Fig. 4(l)

(a) Original MRI Skull Image
b) CR%=21.98 BPP=1.7581 PSNR=43.63
c) CR%= 8.90 BPP=0.7119 PSNR=34.83
(d) Original MR Cardiac Image.
e) CR%=28.74 BPP=2.2996 PSNR=42.19
f) CR%=10.00 BPP=0.80005 PSNR=33.59
g) Original Ultrasound Liver Cyst. Image
h) CR%=19.88 BPP=1.5902 PSNR=37.99
i) CR%= 7.19 BPP=0.57495 PSNR=29.46
j) Original X-Ray Image
k) CR%= 7.66 BPP=0.61243 PSNR=40.87
l) CR%= 2.75 BPP=0.2196 PSNR=34.17

REFERENCES

[1] A. Bhardwaj and M. K. Singh, “A Novel approach of medical image enhancement based on Wavelet transform”, *International Journal of Engineering Research and Applications*, 2(3), pp. 2356-2360, 2012.

[2] A. Rehman and Z. Wang, “Reduced-Reference Image Quality Assessment by Structural Similarity Estimation”, *IEEE Transactions on Image Processing*, 21(8), pp. 3378-3389, 2012.

[3] L. Devnath and M. R. Islam, “Fingerprint Image De-noising by Various Filters for Different Noise using Wavelet Transform”, *American International Journal of Research in Science, Technology, Engineering & Mathematics*, 13(1), pp. 39-44, 2015.

[4] V. Sharan, N. Keshari and T. Mondal, “Biomedical Image De-noising and Compression in Wavelet using MATLAB”, *International Journal of Innovative Science and Modern Engineering*, 2(2), pp. 9-13, 2014.

[5] L. Mredhula and M. A. Dorairangaswamy, “An Extensive Review of Significant Researches on Medical Image Denoising Techniques”, *International Journal of Computer Applications*, 64(14), pp. 1-12, 2013.

[6] M. Neelima and M. M. Pasha, “Wavelet Transform Based On Image Denoising Using Thresholding Techniques”, *International Journal of Advanced Research in Computer and Communication Engineering*, 3(9), pp. 7906-7908, 2014.

[7] M. Misiti, Y. Misiti, G. Oppenheim and J. M. Poggi, “Wavelet Toolbox User’s Guide”, The MathWorks, Inc. R2013b, version 4.12, 2013.

[8] Amir Averbuch, *et al.: “*Image Compression Using Wavelet Transform and Multiresolution Decomposition*”, IEEE Trans. Image Processing*, 5(1), pp. 4-15, 1996.

[9] L. Devnath, S. K. D. Nath, A. Kr. Das and M. R. Islam, “Selection of Wavelet and Thresholding Rule for Denoising the ECG Signals”, *Annals of Pure and Applied Mathematics*, 10(1), pp. 65-73, 2015.

[10] S. Arastehfar, A. A. Pouyan and A. Jalalian, “An enhanced median filter for removing noise from MR images”, *Journal of AI and Data Mining (JADM)*, 1(1), pp. 13-17, 2013.

[11] M.D.Adams and R.Ward; “Wavelet Transforms in JPEG 2000 Standard”, *IEEE Pacific Rem Conf. on Comm. Comp. & Signal Pross.*, 2001.

[12] T. Acharya and P. Tsai, “JPEG2000 Standard for Image Compression: Concepts, Algorithms and VLSI Architectures”, John Wiley & Sons, 2005.

[13] S. Mallat, “A Theory for Multiresolution Signal Decomposition by the Wavelet Representation,” *IEEE Trans. Pattern Anal. And Machine Intel.*, 11(7), pp. 674-693, 1989.

[14] S. G. Chang and M. Vattereli, “Adaptive wavelet thresholding for image denoising and compression”, *IEEE Tran. On Image Processing*, 9(2), pp. 1532-1546, 2007.

[15] Y. Yang, Z. Su and L. Sun, “Medical Image Enhancement Algorithm Based on Wavelet Transform”, *IEE Electronics Letters*, 46(2), pp. 120-121, 2010.

[16] Z. Q. Abdullah, ”Quality Assessment on Medical Image Denoising Algorithm: Diffusion and Wavelet Transform Filters”, A thesis submitted in fulfillment of the requirements for the award of the Degree of Master of Computer Science (Software Engineering), *Faculty of Computer Science and Information Technology, Universiti Tun Hussein Onn Malaysia*, Springer-Verlag, 2014.