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Moroccan Financial Market: Stochastic Modeling and Prediction Interval for Future Values of MASI index

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ABSTRACT: Since the sixties, debates have been born on the models, which determine the evolution of the stock prices. In this work we will focus on one of the best performances in the region of the Middle East and North Africa (MENA), is Africa's third largest Bourse: Casablanca Stock Exchange (CSE), which had the “Index de la Bourse des Valeurs de Casablanca” (IGB) as an index. IGB was replaced in January 2002 by two indexes: MASI (Moroccan All Shares Index) comprises all listed shares, allows investors to follow all listed values and to have a long-term visibility. MADEX (Moroccan Most Active Shares Index) comprises most active shares listed continuously with variations closely linked to all the market serves as a reference for the listing of all funds invested in shares.

Firstly, it aims at the investigation of stochastic model to show the variation of MASI index values, and, secondly, we will achieve a prediction interval of 95% of chance for Moroccan index future values. Here, the geometric Brownian motion (stochastic process without mean reversion propriety) is used to model the stochastic variation of MASI index values. In order to calculate models’ parameters daily close values of the Moroccan index from 02/01/2003 to 05/11/2019 can be taken from Casablanca Stock Exchange and, hence, stochastic models for MASI index variation is to be derived.

KEYWORDS: MASI index, Modeling, Brownian Motion, Casablanca Stock Exchange, Stochastic Process.

1 INTRODUCTION

Over the last decade, the transparency of the financial markets has become one of the major concerns that have characterized the world scene. Theorists and practitioners have been interested in this subject by studying information efficiency.

This article will be devoted to the application of the stochastic modeling to the MASI index (Moroccan All Share Index), the overall indicator of the Casablanca Stock Exchange. It should be noted, indeed, that our study period is between 02/01/2003 and 05/11/2019 with daily data, a period that is crucial in the economic and financial world having most significant events.

Our contribution is concretized by elaborating a stochastic model of MASI index. To do this, we assume that the MASI variation is a stochastic process (random variable time-dependent).

More concretely, the estimation of the model is carried out by Geometric Brownian Motion (GBM), we must first, give the theoretical principles of the GBM, then we elaborate a stochastic model for the MASI index using historical data, and finally, building a prediction interval for future values of MASI index.

2 THEORETICAL PRINCIPLES OF RESEARCH

2.1 THE GEOMETRIC BROWNIAN MOTION (GBM)

The Geometric Brownian Motion (GBM) is a fundamental example of a stochastic process without mean reversion properties. The GBM is the underlying process from which is derived to form the Black and Scholes formula for pricing European options [1]. Let the exchange rate be assigned as x_t where ln (x_t) obeys the following defined equation.

$$\boldsymbol{dln(x_t) = \mu dt + \sigma dw_t}$$

Here μ and σ are constants and Wt is a standard Brownian motion.

2.2 METHODOLOGY

2.2.1 GEOMETRIC BROWNIAN MOTION.

Let the continuous-time exchange rate be assigned as x_t where ln (x_t) obeys the following equation:

$$dln(x_t) = \mu dt + \sigma dw_t \text{ (1)}$$

Here, μ and σ are constants and dwt is a standard Brownian motion. In ordinary calculating, one can derive that:

$$dln(x_t) = \frac{dx_t}{x_t} \qquad \text{So} \qquad \frac{dx_t}{x_t} = \mu dt + \sigma dw_t$$

If we adopt Ito's Lemma as mentioned in J.C. Hull [1], the equation will be as follows:

$$dln(x_t) = (\mu - \tfrac{1}{2}\sigma^2)\, dt + \sigma dw_t \qquad \text{with} \qquad \gamma = \mu - \tfrac{1}{2}\sigma^2$$

This means that $ln\ (x_t)$ is an Arithmetic Brownian Motion. By integrating equation between u and t, and according to Damiano Brigo et al [4], gives:

$$\ln(x_u) - \ln\ (x_t) = (\mu - \frac{1}{2}\sigma^2)\,(u-t) + \sigma(w_u - w_t) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)(u-t);\ \sigma^2(u-t)\right)$$

By considering $u = T$, $t = 0$ and taking the exponent on equation above leads to:

$$x_T = x_0 exp\left(\left(\mu - \tfrac{1}{2}\sigma^2\right)T + \sigma w_T\right) \qquad \text{(w0=0)}$$

The mean and the variance of x_T according to Damiano Brigo et al (2007) [4] are:

$$E(x_T) = x_0 e^{\mu T} \qquad \text{And} \qquad Var(x_T) = e^{2\mu T} {x_0}^2 \left(e^{\sigma^2 T} - 1\right)$$

Therefore, the version of a simulation equation for the GBM, using the fact that is $dW = Z\sqrt{\Delta t}$ [**1**]:

$$\ln\left(x_{t_{i+1}}\right) - \ln\left(x_{t_i}\right) = \gamma\Delta t + \sigma Z_i \;\; Z_i \sim N(0{,}1) \qquad \text{and} \qquad \gamma = \mu - \tfrac{1}{2}\sigma^2$$

By taking the exponent of both sides, it results:

$$\boldsymbol{x_{t_{i+1}} = x_{t_i} exp\left(\gamma\Delta t + \sigma Z_i \sqrt{\Delta t}\right)} \qquad Z_i \sim N(0{,}1)$$

2.2.2 MAXIMUM LIKELIHOOD ESTIMATION (MLE) – GEOMETRIC BROWNIAN MOTION

According to Damiano Brigo et al (2007) [4], the parameters that must be optimized are $\theta(\mu,\sigma)$ for the GBM. Let the logarithmic return be given as:

$$y_{t_i} = ln\,(x_{t_i}) - \ln\,(x_{t_{i-1}})$$

Which is normally distributed for all $y_{t_1}, y_{t_2} \dots \dots .. y_{t_n}$. And these later values assumed independent. The likelihood function will be denoted as:

$$L(\theta) = f_\theta(y_{t_1}, y_{t_2} \dots \dots .. y_{t_n}) = \prod_{i=1}^{n} f_\theta(y_{t_i}) = \prod_{i=1}^{n} f(y_{t_i} | \, \theta)$$

Here, f_θ is the probability density function. Let $\theta = (\mu, \sigma)$, then the probability density function f_θ is:

$$f_\theta(y_{t_i}) = \frac{1}{x_{t_i}\sigma\sqrt{2\pi t}} exp\left[-\frac{\left(\left(\frac{y_{t_i}}{y_{t_0}}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)t\right)^2}{2\sigma^2 t}\right]$$

The likelihood function needs to be maximized to obtain the optimal estimators $\hat{\theta}\,(\hat{\mu}, \hat{\sigma})$.

First, we have to determine $\hat{w}$ and $\hat{\gamma}$:

$$\hat{w} = \left(\hat{\mu} - \frac{1}{2}\hat{\sigma}^2\right)\Delta t \quad \text{with} \quad \hat{w} = \sum_{i=1}^{n} \frac{y_{t_i}}{n} = \frac{\ln(x_{t_n}) - \ln\,(x_{t_0})}{n}$$

$$\hat{\gamma} = \hat{\sigma}^2 \Delta t \quad \text{with} \quad \hat{\gamma} = \sum_{i=1}^{n} \frac{(y_{t_i} - \hat{w})^2}{n}$$

Then the MLE's parameters are:

$$\hat{\sigma}^2 = \frac{\hat{\gamma}}{\Delta t} \quad \text{And} \quad \hat{\mu} = \frac{1}{2}\hat{\sigma}^2 + \frac{\hat{w}}{\Delta t}$$

2.2.3 MASI INDEX VARIATION: GEOMETRIC BROWNIAN MOTION.

In order to calculate $\hat{\mu}$ *and* $\hat{\sigma}$ daily close values of MASI index from 02/01/2003 to 05/11/2019 can be taken directly from Casablanca Stock Exchange market. And considering $\Delta t = \frac{1}{252}$ (daily data)

2.2.3.1 SIMULATION RESULTS.

Using the daily close MASI index values from 02/01/2003 to 05/11/2019 and Microsoft Excel's solver, we obtain:

$$\boldsymbol{\hat{\mu} = 0,08841822} \quad \text{and} \quad \boldsymbol{\hat{\sigma} = 0,117664371}$$

The simulation equation for MASI index according to GBM is:

$$(**)\ \boldsymbol{x_{t_{i+1}} = x_{t_i} e^{(0{,}0000549.\Delta t + 0{,}117664371.Z_i.\sqrt{\Delta t})}} \quad with \quad Z_i \sim N\,(0{,}1) \quad \text{And} \quad \Delta t = \frac{1}{252}$$

Real MASI index, simulated MASI index and simulation of equation for MASI index according to GBM (with R) model is shown in this Figure. With xo=2970,26 [at 02/01/2003].

Fig. 1. MASI index between 02/01/2003 and 05/11/2019

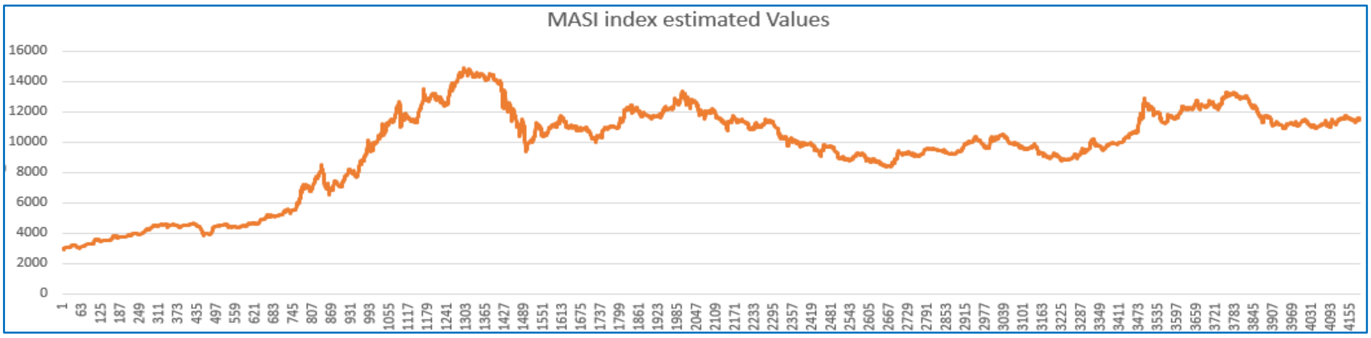


Fig. 2. MASI index estimated values between 02/01/2003 and 05/11/2019

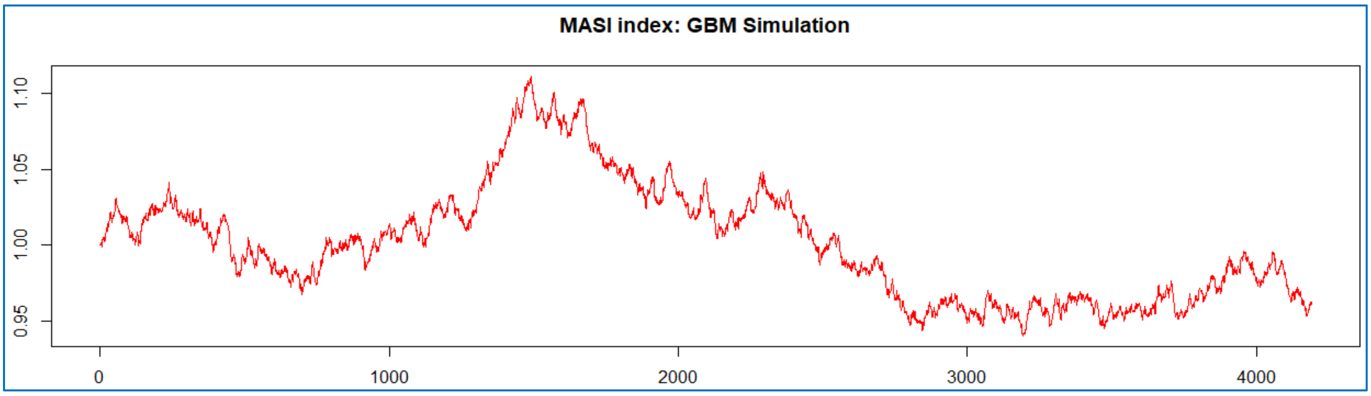


Fig. 3. MASI index Geometric Brownian Motion simulation with R [equation (**)]

R Script

```
> t<-1
> n<-4190
> dt<-t/n
> dw<-sqrt(dt)*rnorm(n)
> x<-numeric(n)
> param<-c(0.0884182195397345,0.117664370978569)
> x[1]<-1
> for(i in 1:n) { x[i+1]<-x[i]+param[1]*x[i]*dt+param[2]*x[i]*dw[i]}
> plot.ts(x, xlab="", ylab="", main="Simulation d'un MBG", col=2)
```

2.2.3.2 GBM Model, Error Performances.

To measure forecast accuracy, we use here the mean absolute percentage error (MAPE) as follow:

Let be the estimated value MASI index at time ti:

The mean absolute percentage error (MAPE) [3].

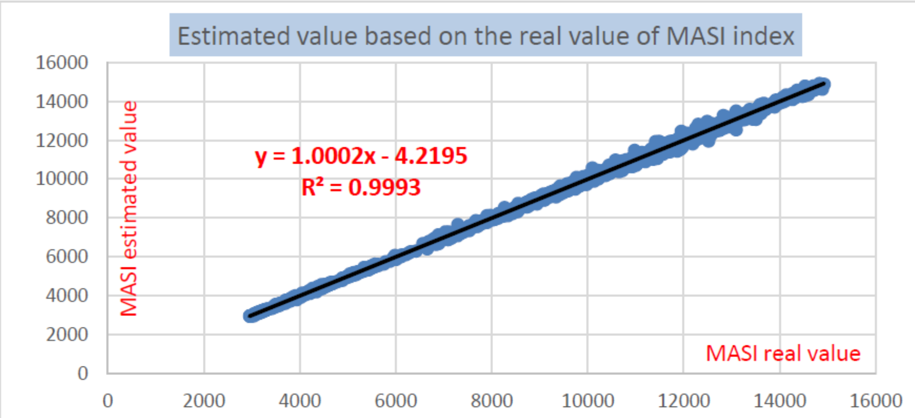
$$MAPE = \frac{1}{4189}\sum_{i=1}^{4189}\frac{|\,x_{t_i} - \hat{x}_{t_i}\,|}{x_{t_i}} * 100 = 0{,}8084\,\%$$

3 Prediction Interval Of Exchange Rate Values

First we look at the link between the values predicted by our model (GBM Model) and the real values of the MASI index. We note $Y = \{y_1, y_2 \dots \dots . y_n\}$ *where* y_i for $i \in \{1, \dots \dots n\}$ (with $n = 4189$) the real values of the exchange rate, and $X = \{x_1, x_2 \dots \dots . x_n\}$ *where* x_i for i ∈ {1, … … n} the predicted values by GBM model.

To do this, we plot the point cloud and calculate the determination coefficient R^2.

The figure below shows a strong linear relationship between Y and X.



According to the figure above, the cloud of point form a straight line, for which we can derive the following equation:

$$\boldsymbol{y = 1{,}0002.x_i - 4{,}2195}\ (*)$$

The relationship between the two variables is positive. An increase in the value of x is likely to be related to an increase in the value of y which is confirmed by the determination coefficient very close to 1 ($R^2 = 0.9993$).

After having shown that there is a strong linear relationship between X and Y, we will use this result to calculate exchange rate values (using linear regression equation) and consequently the prediction intervals of the real values of MASI index.

We note $\hat{y}_i$ for $i \in \{1, \dots \dots .. n\}$ the values given by equation (*), so we have:

$$\boldsymbol{\hat{y}_i = 1{,}0002.x_i - 4{,}2195\ (*)}$$

Using this relationship, we can calculate all the values $\hat{y}_i$ corresponding to x_i, for $i \in \{1, \dots \dots .. n\}$

To find a prediction interval for a future value y_{n+1} of MASI index we use the following result [4]:

$$y_{n+1} \in \left[\hat{y}_{n+1} \pm t_{n-2,1-\alpha/2}\, s\sqrt{1 + \frac{1}{n} + \frac{(x_{n+1} - \bar{x}_n)^2}{\sum_{i=1}^{n}(x_i - \bar{x}_n)^2}}\right]$$

With:

- $\hat{y}_{n+1}$: Value given by least square equation.
- $t_{n-2,1-\alpha/2}$: is the $(1 - \frac{\alpha}{2}) -$ quantile of the student with n-2 degree of freedom.

- $s = \sqrt{\frac{SSR}{n-2}}$: is the root of the sums of squared residuals (SSR) divided by $n - 2$.
- $n = 4189$.

To deliver results, Microsoft Excel Solver is used as follows:

- To calculate the $\hat{y}_i$ we call function: TREND ().
- The Excel function LINEST () provide regression parameters.
- The function T.INV.2T () provide quintiles of Student.
- The function DEVSQ () calculate the sums of squared residuals.

RESULTS:

The Excel function LINEST () provide the flowing regression parameters.

a	1,000226723	b	-4,219500056
Standard error of the slope	0,000399344	Standard error of b	4,014231835
R^2	0,999333024	SD for residuals values	75,89772638
Fisher	6273404,706	Degree of freedom	4187
is explained variation	36137727428	is unexplained variation	24119066,41

So we can now calculate prediction interval parameters:

- $n = 4189$
- $\bar{x}_n = 9613{,}598434$
- $\sum_{i=1}^{n} (x_i - \bar{x}_n)^2$: the sums of squared residuals $= 36121346525$
- $t_{0,95} = 1{,}960530726$
- $S = 75{,}89772638$

A future value y_{n+1} of MASI index corresponding to x_{n+1} have 95% of the chance to be in the following prediction interval:

$$y_{n+1} \in \left[\hat{y}_{n+1} \pm 1{,}960530726 \times 75{,}89772638. \sqrt{1 + \frac{1}{4189} + \frac{(x_{n+1} - 9613{,}598434)^2}{36121346525}}\right]$$

NOTE:

The value x_{n+1} is predicted by GBM model and the corresponding image $\hat{y}_{n+1}$ is derived by the linear regression relationship.

SIMULATION EXAMPLE:

This figure shows that the estimated values of the MASI index are between a maximum and a minimum value in 95% of cases for three months January, February and March 2019.

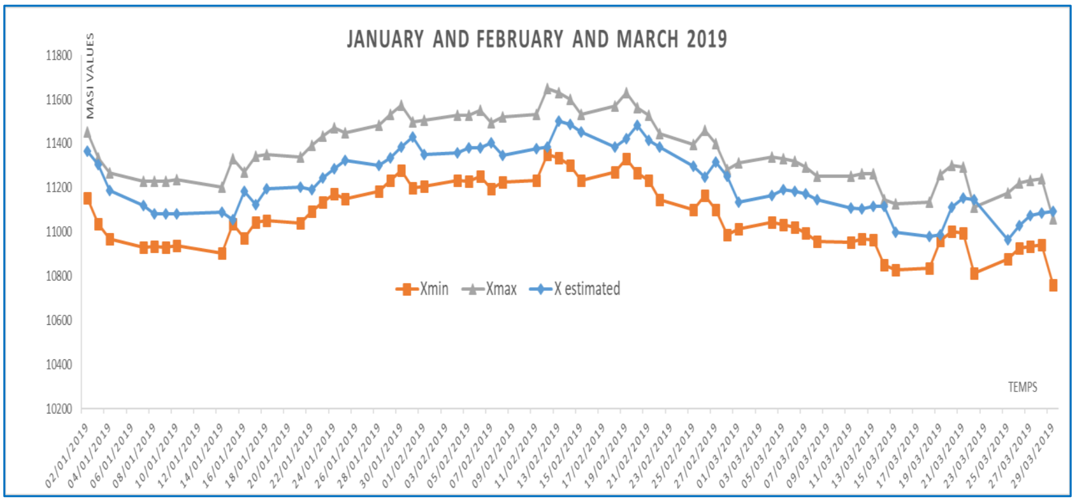


Fig. 4. The estimated values of the MASI index are between a maximum and a minimum value

4 CONCLUSION

This work has focused on stochastic model (GBM model) that is used and calibrated with daily close values of MASI index, and as a result elaborating a model with a measure of forecast accuracy (using mean absolute average percentage error (MAPE)) for MASI index variation.

Finally, thanks to the important relationship between the predicted and real values of MASI index, we have achieved a 95% prediction interval for future values of the MASI index.

We note that, the model we have developed gives a better estimate for the values of the MASI index in the very short term. To conclude, our work, make available to the market analyzer a forecasting tool in order to anticipate the market trend.

REFERENCES

[1] JC Hull Options Futures & Other Derivatives 8th ed .
[2] Available: http://polymer.bu.edu/hes/rp-hull12.pdf.
[3] D Brigo et al “A Stochastic Processes Toolkit for Risk Management” (2007).
[4] Available http://ssm.com/abstract=1109160 (Accessed 9 September 2009) or http://arxiv.org/pdf/0812.4210.pdf.
[5] S. MAKRIDAKIS, M. Hibon "evaluating accuracy (or error) measures.
[6] Available: http://sites.insead.edu/facultyresearch/research/doc.cfm?did=46875.
[7] “Introduction to Linear Regression Analysis” Par Douglas C. Montgomery, Elizabeth A. Peck, G. Geoffrey Vining.