International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 34 No. 1 Oct. 2021, pp. 51-61

http://www.ijias.issr-journals.org/

Evaluating the Predictive Accuracy of Heston Stochastic Volatility Model for Currency Options using Local and Global Calibration

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ABSTRACT: This paper analyses the implementation and calibration of the Heston Stochastic Volatility Model. We first introduce the model, provides theoretical motivation for its robustness and hence popularity and explain how characteristic functions can be used to estimate option prices. Then we consider the implementation of the Heston model, showing that relatively simple solutions can lead to fast and accurate currency option prices. In this work, we perform several tests, using both local and global calibration to evaluate the Predictive Accuracy of the Heston Stochastic Volatility Model for currency options.
Our analyses show that straightforward setups deliver excellent calibration results. All calculations are carried out in MATLAB and included in the paper. All the MATLAB's codes required to implement the model are provided in the appendix A.

KEYWORDS: Heston Model; Currency option; MATLAB Calibration; Local calibration; Global calibration.

1 INTRODUCTION

The Garman-Kohlhagen model has been adopted as the standard model for pricing foreign currency options as it is a modification of the famous, Black-Scholes model (1973). However, this method is based on several assumptions that are not representative of the real world. In particular, the Garman-Kohlhagen model assumes that volatility is deterministic and remains constant through the option's life, which clearly contradicts the behavior observed in financial markets. During the last decades several alternatives have been proposed to improve volatility modelling in the context of derivatives pricing. One of such approaches is to model volatility as a stochastic quantity. By introducing uncertainty in the behavior of volatility, the evolution of financial assets can be estimated more realistically. In addition, using appropriate parameters, stochastic volatility models can be calibrated to reproduce the market prices of liquid options and other derivatives contracts. One of the most widely used stochastic volatility models today was proposed by Heston in 1993. In the Heston model, volatility is assumed to be stochastic and is defined by a stochastic differential equation. The success of the Heston model is based on the calibration of its parameters.

In this paper we analyze the valuation of foreign currency options using the Heston model. Our aim is to illustrate the use of the model, with an emphasis on the implementation and calibration, and to make it a better suited foreign currency option pricing model for the FX market using both local and global optimization. First, we introduce the Heston model and discusses the implementation of its closed-form solution. Secondly, we introduce bouth local and global calibrations methods and finally, we analyze the calibration problem, considering both local and global optimization methods. For all relevant sections, generic and ready-to-use MATLAB's codes have been developed to illustrate the use of the MATLAB routines.

2 THE HESTON MODEL

In this paper the main model used in determining the price for a currency option is the Heston model (1993), which assumes that the process $\boldsymbol{S_t}$ follows a lognormal distribution, and the process $\boldsymbol{V_t}$ follows a Cox-Ingersoll-Ross process (CIR process) (1985). The model is given as:

$$dS_t = (r_d - r_f)S_t dt + \sqrt{V_t} S_t dB_t \; (2.1)$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dZ_t \; (2.2)$$

$dB_t dZ_t = \rho dt$ and the variables:

r_d and r_f are the domestic and forgein interest rates respectively.

θ is the long-term mean of variance.

κ is the rate of mean reversion.

σ is the volatility of volatility.

S_t and V_t are the price and volatility of the process

B_t and Z_t are correlated Wiener process, and the correlation coefficients is ρ.

The variance of the CIR process is always positive and $2\kappa\theta > \sigma^2$ (feller condition) **[4],** then it cannot reach zero. It is assumed that the interest rate is a constant, hence r_d and r_f are fixed values.

2.1 Closed-Form Solution Of The Heston Model

When markets are complete and arbitrage-free, currency option values can be calculated as the present value of their expected payoff under the risk-neutral measure:

$$E^*[S_t/S] = Se^{(r_d - r_f)\,t} \text{ Where: } S = S_0$$

Under the equivalent martingale measure, the dynamics of the Heston model equation defined by equations (2.1) and (II.2) are given by the following set of stochastic differential equations **[6]:**

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$$dS_t = (r_d - r_f)S_t dt + \sqrt{V_t} S_t d\bar{B}_t \; (2.3)$$

$$dV_t = \kappa^*(\theta^* - V_t)dt + \sigma\sqrt{V_t} d\bar{Z}_t \; (2.4)$$

Where $d\bar{B}_t d\bar{Z}_t = \rho dt$

Each stochastic volatility model will have a unique characteristic function that describes the probability density function of that model. Heston and Nandi **[9]** utilize the characteristic function of the Heston model when proposing the following formula for the fair value of a currency call option at time **t**, given a strike price **K**, that expires at time **T**:

$$C(S, v, t) = Se^{-r_f \tau} P_1 - Ke^{-r_d \tau} P_2$$

where $\tau = T - t$, P_1 and P_2 can be defined via the Fourier Inversion Transformation **methodol**:

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re \left[\frac{e^{-i\psi lnK} f_j\,(x, v, \tau, \psi)}{i\psi}\right] d\psi$$

Where j=1,2. And the caracteristic function for the logarithm of exchange rate, $x = ln\,(S_t)$ is given by **[11]:**

$f_j(x, v, \tau, \psi) = e^{C(\tau,\phi) + D(\tau,\phi)v_t + i\phi x}$ Where:

$$C(\tau, \phi) = (r_d - r_f)\phi i\tau + \frac{a}{\sigma^2}\left[(b_j - \rho\sigma\phi i + d)\tau - 2\ln\left(\frac{1 - ge^{d\tau}}{1 - g}\right)\right]$$

$$D(\tau,\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2} \left(\frac{1 - e^{d\tau}}{1 - g e^{d\tau}}\right)$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{\left(\rho\sigma\phi i - b_j\right)^2 - \sigma^2 \left(2u_j\phi i - \phi^2\right)}$$

$$u_1 = \frac{1}{2};\ u_2 = -\frac{1}{2};\ a = \kappa\theta;\ b_1 = \kappa\lambda - \rho\sigma;\ b_2 = \kappa + \lambda$$

To find the price for a put option we use the call/put parity theorem and the price of the put option is:

$$\boldsymbol{P(S,v,t) = C(S,v,t) - Se^{-r_f\tau}P_1 + Ke^{-r_d\tau}P_2}$$

Remark: the Heston characteristic function can be easily evaluated using numerical software. The function charasteristic (charasteristic.m), provided in the appendix A, shows how to compute the Heston characteristic function in MATLAB **[3].**

3 METHODOLOGY

3.1 CALIBRATION TO MARKET PRICES

3.1.1 CALIBRATION PROCEDURE IN THE HESTON MODEL

There are five parameters $\boldsymbol{v}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \boldsymbol{\theta}$ and $\boldsymbol{\rho}$ that need to be estimated in the Heston model. The change for each parameter will bring a big impact on the correctness of the model, so the estimation of parameters becomes very important. A variety of methods can be chosen. For instance, one can observe the real market data, and use statistic tool to fit data in the Heston model (Ait-Sahila & Kimmel, 2005) **[1].** Monte Carlo simulation is another famous method to do the calibration. In this study we use a commonly used method called the Inverse Problem, which means that the data is collected from the real market first, and then used to estimate parameters. The most popular approach to solving this inverse problem is to minimize the error or discrepancy between Heston model prices and real market prices. This usually turns out to be a non-linear least-squares optimization problem. More specifically, the squared differences between European currency option market prices and that of the model are minimized over the parameter space. Assume is a set of realization for the parameters in the Heston model. For a call option that is calculated from the Heston model, the optimization problem can be described as:

$$\boldsymbol{Min\ S(\Omega) = \min\ (\Omega) \sum_{i=1}^{N} \left(C_i^H\ (K_i, T_i)\ - C_i^M\ (K_i, T_i)\right)^2}$$

Where Ω is a vector of parameter values $C_i^H(K_i, T_i)$, and $C_i^M\ (K_i, T_i)$ are the i currency option prices from the model and market, respectively, with strike K_i and maturity T_i. N is the number of options used for calibration. Minimizing the objective function is clearly a nonlinear programming (NLP) problem with the nonlinear constrain $2\kappa\theta - \sigma^2 > 0$. Unfortunately, this function is far from being convex and it turned out, that usually there exist many local extrema depending on the initial guess. Therefore, a good initial guess might be critical and, even then, in some cases the convergence to the global optimum is not guaranteed. As a consequence, we decide to try both local and global optimizers.

4 LOCAL OPTIMIZATION

When a function exhibits several minima, local optimizers face the problem that once a solution has been found, we cannot be sure whether such solution is the best available. In other words, we cannot distinguish if the solution is a local minimum or a global one, or consequently, if we have reached a local solution, there is no easy way to measure how far we are from the global one. An alternative to tackle this problem is to define a criterion for acceptable solutions. If we select a priori which solutions can be deemed acceptable, we can at least ensure that any accepted solution will be consistent with our tolerance bounds. Conversely, if we found a non-acceptable solution, we can run the algorithm with a different starting point and keep searching for solutions that comply with our criteria [7].

In our tests, we will require that the difference between model and market prices falls on average within the observed bid-ask spreads. Therefore, we will consider the following set of acceptable solutions:

$$\frac{1}{N}\sum_{i=1}^{N}|C_i^H(K_i,T_i) - C_i^M(K_i,T_i)| \leq \frac{1}{2N}\sum_{i=1}^{N}|bid_i - ask_i|$$

bid and ask are the market observed bid and ask prices.

As a local optimizer we will use the MATLAB lsqnonlin function (least-squares non-linear) **[13],** which implements a trust-region reflective minimization algorithm (See Yuan (1999) **[10]** for an overview on the use of trust-region algorithms for solving non-linear problems). In addition, we will also define lower and upper bounds for the optimal parameters. These thresholds are included in the calibration in order to avoid possible solutions that, while mathematically feasible, are not acceptable in an economic sense. In particular, we will use the following bounds **[8]:**

- **Long-term variance and initial variance:** Acceptable solutions for variance levels should take a possible value. However, given its mean-reversion, the volatility of most financial asset rarely reaches levels beyond 100%. Consequently, we will use bounds of **0** and **1** for both for $\boldsymbol{\theta}$ and $\boldsymbol{V_0}$.
- **Correlation:** Statistical correlation takes values from **-1** to **1**. As previously mentioned, the correlation between volatility and stock prices tends to be negative. However, positive correlations might also be possible in particular cases. Therefore, the full range of acceptable solutions will be used in the calibration.
- **Volatility of variance:** Being a volatility, this parameter should exhibit positives values. However, the volatility of financial assets may change dramatically in short time periods (i.e. the volatility itself is very volatile). Consequently, high upper bounds are required for this parameter. In order to avoid potential restrictions, a broad set of solutions, from **0** to **5**, will be used in the calibration.
- **Mean-reversion speed:** To ensure mean-reversion the parameter $\boldsymbol{\kappa}$ should take positive values (negative values will cause mean aversion). However, we have not found clear evidence regarding which upper value could be an appropriate bound. Consequently, instead of fixing an upper level, maximum values for κ will be dynamically set in the calibration as a by-product of the non-negativity constraint.
- **Non-negativity constraint:** In addition to the parameter bounds, another condition is required to ensure that the variance process in the Heston model does not reach zero or negative values. In this regard, Feller (1951) shows that a constraint: $2\kappa\theta - \sigma^2 > 0$, guarantees that the variance in a CIR process is always strictly positive **[4].**

5 GLOBAL OPTIMIZATION

The main advantage of global optimization is that it does not exhaust its search on the first minimum attained. Generally, global optimizers include stochastic movements in their search pattern, which make it possible to overcome local minimums and continue searching even if a potential solution has already been found. However, the use of stochastic methods also entails certain drawbacks. The mathematical properties of these algorithms are less tractable than those of local (deterministic) ones. In addition, despite its name, their convergence to the global minimum is not guaranteed. In fact, since the exit sequence is determined stochastically, the algorithm might decide to terminate early and, in some cases, the solution attained might underperform a local search. All in all, even if global optimization is theoretically more powerful, when working with functions of unknown shape, it is not easy to establish which calibration method will perform better **[7].**

In order to test the results of global optimization we employ the Simulated Annealing framework (SA). This algorithm conducts a guided search, where new iterations are generated by taking into account the previous information but also introducing randomization. Initially, the algorithm starts with high tolerance for random shocks, and different regions are surveyed during the first phase. As a consequence, even if a minimum is found, the algorithm keeps searching for better solutions. As time evolves, the algorithm decreases its tolerance until it eventually settles in the best optimum attained.

In particular, we will use the Matlab function asamin, which was developed by Prof. Shinichi Sakata **[5].** This function implements an Adaptive Simulated Annealing (ASA), dynamically adjusting the tolerance for random shocks. The ASA framework has been shown by Goel and Stander (2009) to provide good results among a range of different global optimizers. For comparability, we will use the same parameter bounds that we defined above.

6 DATA ANALYSIS, SIMULATIONS AND RESULTS

6.1 DATA AND DATA SOURCES

In order to produce fairly accurate results, the Heston model needs to be calibrated so as to estimate numerical values for its parameters. In calibrating the Heston model, EUR/USD was used as the underlying asset to estimate $\boldsymbol{v}, \boldsymbol{\kappa}, \boldsymbol{\sigma}, \boldsymbol{\theta}$ and $\boldsymbol{\rho}$ and

In determining the currency option price, Eur Libor (US Libor) data are used to determine the domestic (foreign) interest rate and the following market data had to be extracted from the foreign currency market. This data includes **[12]:**

- EUR/USD Exchange rate,
- Strike price.
- Bid and Ask

Currency call option quotes were obtained from the Bloomberg website. This data will be used to calibrate the model. The period runs from May 28, 2020 to May 07, 2021 and we use 5 maturities and for each maturity we use 5 strike prices (in total 25 currency options). The currency options dataset (Dataset D) that we use in the calibration are shown in Appendix B.

The domestic and foreign risk-free interest rates are provided by Bloomberg and the available maturities match those of the options. We use In-The-Money (ITM) currency call options sorted by S_t/K with the following expiry dates: Options that expire in:

- 1 month, [at 06/26/2020]
- 3 months currency options, [at 09/04/2020]
- 6 months currency options, [at 12/04/2020]
- 9 months currency options, [at 03/05/2021]
- One-year currency options. [at 05/07/2021]

6.2 HESTON MODEL CALIBRATION RESULTS

6.2.1 LOCAL CALIBRATION

Using the bounds described above, the implementation of the local calibration algorithm is shown in Local calibration script (Local_calibration.m). In addition, function (costfloc.m) provides the objective function required for this script. For dataset D, the results obtained with local optimization are the following:

Parameters	$\boldsymbol{v}$	$\boldsymbol{\theta}$	$\boldsymbol{\sigma}$	$\boldsymbol{\rho}$	$\boldsymbol{\kappa}$
Values	**0**	**0.0041**	**0.3163**	**0.9925**	**12.4467**

Using these results, the model predicted values and its comparison with the market prices are shown below:

Table 1. The model predicted values and its comparison with the market prices using local calibration

Option	$Mid\ price$	$Heston\ model\ price\ (HMP)$	$difference\ (abs)$	$HMP\ \epsilon\ [bid\ ask]$?
1	0,0213	0,023321215	0,002071215	Yes
2	0,0192	0,020602937	0,001402937	Yes
3	0,0173	0,018003745	0,000703745	Yes
4	0,0155	0,015556239	0,0000562387	Yes
5	0,0138	0,013289389	0,000460611	Yes
6	0,0491	0,050678284	0,001628284	Yes
7	0,0446	0,045674045	0,001124045	Yes
8	0,0402	0,040862243	0,000662243	Yes
9	0,036	0,036338303	0,000338303	Yes
10	0,032	0,032180043	0,000230043	Yes
11	0,0509	0,050130595	0,000769405	Yes
12	0,0468	0,046318915	0,000481085	Yes
13	0,0429	0,04272339	0,00017661	Yes
14	0,0392	0,039340564	0,000190564	Yes
15	0,0355	0,036164384	0,000664384	Yes
16	0,0566	0,055651559	0,000948441	Yes
17	0,0527	0,052113067	0,000586933	Yes
18	0,0489	0,048738756	0,000161244	Yes
19	0,0453	0,045530252	0,000280252	Yes
20	0,0417	0,042487951	0,000787951	Yes
21	0,0571	0,056029498	0,001070502	Yes
22	0,0534	0,052748018	0,000601982	Yes
23	0,0497	0,049613322	0,00003,6678	Yes
24	0,0461	0,046624579	0,000524579	Yes
25	0,0427	0,043780006	0,001080006	Yes

INTERPRETATION OF THE TABLE

As the table shows, the calibrated Heston model provides an excellent match for all traded options. All currency options have a predicted value that falls within the observed bid-ask spread. In addition, when evaluated in terms of our acceptance criterion, the model's average distance from the mid-market price is 0,00068153, which is lower than the average deviation in the bid-ask spreads (0,003978). The Elapsed time for the local calibration is 2.377027 seconds.

6.2.2 GLOBAL CALIBRATION

Using the same bounds, the implementation of the global calibration algorithm is shown in global calibration script (global_calibration.m). In addition, function (costfglob.m) provides the objective function required for this script. For dataset D, the results obtained with global optimization are the following:

Parameters	$\boldsymbol{\upsilon}$	$\boldsymbol{\theta}$	$\boldsymbol{\sigma}$	$\boldsymbol{\rho}$	$\boldsymbol{\kappa}$
Values	0	0.0051	0.2898	1	12.5445

Using these results, the model predicted values and its comparison with the market prices are shown below:

Table 2. The model predicted values and its comparison with the market prices using global calibration

Option	$Mid\ price$	$Heston\ model\ price\ (HMP)$	$difference\ (abs)$	$HMP\ \epsilon\ [bid\ ask]$?
1	0,0213	0,023417386	0,002167386	Yes
2	0,0192	0,020845746	0,001645746	Yes
3	0,0173	0,018398617	0,001098617	Yes
4	0,0155	0,016102317	0,000602317	Yes
5	0,0138	0,013979546	0,000229546	Yes
6	0,0491	0,050953453	0,001903453	Yes
7	0,0446	0,046323391	0,001773391	Yes
8	0,0402	0,041935856	0,001735856	Yes
9	0,036	0,037835831	0,001835831	Yes
10	0,032	0,03405105	0,00210105	Yes
11	0,0509	0,052496997	0,001596997	Yes
12	0,0468	0,048859629	0,002059629	Yes
13	0,0429	0,045398533	0,002498533	Yes
14	0,0392	0,042115549	0,002965549	Yes
15	0,0355	0,039011504	0,003511504	Yes
16	0,0566	0,05877213	0,00217213	Yes
17	0,0527	0,055379552	0,002679552	Yes
18	0,0489	0,052129129	0,003229129	Yes
19	0,0453	0,049021112	0,003771112	No
20	0,0417	0,046054862	0,004354862	No
21	0,0571	0,059710824	0,002610824	Yes
22	0,0534	0,056534021	0,003184021	Yes
23	0,0497	0,053482655	0,003832655	Yes
24	0,0461	0,050556218	0,004456218	No
25	0,0427	0,047753714	0,005053714	No

INTERPRETATION OF THE TABLE

As can be seen, the optimal parameters values under ASA are slightly different to those of local calibration. However, there are some divergences in the overall results. Under global calibration 21 out of 25 model values are within the observed bid-ask spreads, and the average distance to the mid-market price is 0,00252278. Therefore, the ASA solution is also acceptable according to our criterion and its quality is less accurate than the results obtained through MATLAB's lsqnonlin. The main drawback of ASA is its substantially higher computational time (590.013684 seconds in ASA vs 6.5 seconds in MATLAB's lsqnonlin). Based on these exercises, we can conclude that MATLAB's lsqnonlin provides excellent calibration results (average distance from the mid-market price is 0,00068153 vs 0,00252278 for ASA and all call currency options prices are within the observed bid-ask spreads), and it also employs lower computational times.

However, these results could be conditioned by an objective function that may not be complex enough to exploit the ASA strengths. In particular, since typically we do not know whether the objective function may exhibit several local minima, a conservative approach will be to run both calibration approaches. The drawback is, of course, that a global search might not necessarily improve the results provided by a local one. However, the advances in computing power and numerical methods keep reducing the time required for global calibration. In our calibration, the running time of ASA was lower than 10 minutes, which for many practical applications makes it worth testing for potentially better solutions.

7 CONCLUSION

This paper provides promising results regarding the application of the Heston model to currency option price estimation. We have adapted the original work of Heston (1993) to a foreign exchange (FX) setting. Simplicity, semi-analytic solution for options price and its ability to capture the option smile make from Heston model One of the most widely used stochastic volatility models today. In this research paper, the EUR/USD exchange rate was considered and both local and global optimization has been explained and used to calibrate the model to the market data.

The calibration parameters founded by both methods provided a very good results within a relatively short timeframe. However, local optimization with MATLAB's lsqnonlin function provides excellent calibration results for all call currency options prices in our data set and it also employs lower computational times.

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APPENDIX A: MATLAB'S CODES USED IN CALIBRATING THE HESTON MODEL

Characteristic function for Heston model (charasteristic.m):

```
function y = charasteristic(s0, v0, vbar, a, vvol, r, rho, t, w)
alpha = -w.*w/2 - 1i*w/2;
beta = a - rho*vvol*1i*w;
gamma = vvol*vvol/2;
h = sqrt(beta.*beta - 4*alpha*gamma);
rplus = (beta + h)/vvol/vvol;
rminus = (beta - h)/vvol/vvol;
g=rminus./rplus;
C = a * (rminus * t - (2 / vvol^2) .* log((1 - g .* exp(-h*t))./(1-g)));
D = rminus .* (1 - exp(-h * t))./(1 - g .* exp(-h*t));
y = exp(C*vbar + D*v0 + 1i*w*log(s0*exp(r*t)));
end
```

Call price in the Heston model

```
function z = call_heston(s0, v0, vbar, a, vvol, r, rho, t, k)
int1 = @(w, s0, v0, vbar, a, vvol, r, rho, t, k) real(exp(-1i.*w*log(k)).* charasteristic(s0, v0, vbar, a, vvol, r, rho, t, w
1i)./(1i*w.* charasteristic(s0, v0, vbar, a, vvol, r, rho, t, -1i)));
int1 = integral(@(w)int1(w,s0, v0, vbar, a, vvol, r, rho, t, k),0,100);
pi1 = int1/pi+0.5;
int2 = @(w, s0, v0, vbar, a, vvol, r, rho, t, k) real(exp(-1i.*w*log(k)).* charasteristic(s0, v0, vbar, a, vvol, r, rho, t,
w)./(1i*w));
int2 = integral(@(w)int2(w,s0, v0, vbar, a, vvol, r, rho, t, k),0,100);
pi2 = int2/pi+0.5;
z = s0*exp(-0.00456058*t)*pi1-exp(-0.00248764*t)*k*pi2;
```

Costfloc function for local calibration (Costfloc.m)

```
function [costex] = costfloc(x)
global hesdata; global finalcostex;
for i=1:length(hesdata)
costex(i)= hesdata(i,5)-
call_heston(hesdata(i,1),x(1),x(2),(x(5)+x(3)^2)/(2*x(2)),x(3),hesdata(i,4),x(4),hesdata(i,2),hesdata(i,3));
end
finalcostex=sum(costex)^2 ;
end
```

Local calibration script (Local_calibration.m)

```
clear

global hesdata;
load('hesdata.mat');
ex0 = [0.5,0.5,1,-0.5,1];
mi = [0, 0, 0, -1, 0];
ma = [1, 1, 5, 1, 20];
tic;
x = lsqnonlin(@costfloc,ex0,mi,ma);
toc;
Heston_sol_ex = [x(1), x(2), x(3), x(4), (x(5)+x(3)^2)/(2*x(2))];
disp('les paramètres de calibration du modèle sont[v theta sigma rho kappa]:') ;
disp(Heston_sol_ex);
```

Costfglob function for global calibration (costfglob.m)

```
function [costex, flag] = costfglob(x)
global hesdata; global finalcostex; global costex; global cost_iex; global Heston_solex2;
for i=1:length(hesdata)
cost_iex(i)= hesdata(i,5)-
call_heston(hesdata(i,1),x(1),x(2),(x(5)+x(3)^2)/(2*x(2)),x(3),hesdata(i,4),x(4),hesdata(i,2),hesdata(i,3));
end
costex = sum(cost_iex.^2);
finalcostex = sum(costex);
flag = 1;
Heston_solex2=[x(1), x(2), x(3), x(4),(x(5)+x(3)^2)/(2*x(2))];
end
```

Global calibration script (global_calibration.m)

```
clear
global Heston_solex2;
global finalcost2ex;
load('hesdata.mat');
x0 = [0.5,0.5,1,-0.5,1];
mi = [0, 0, 0, -1, 0];
ma = [1, 1, 5, 1, 20];
asamin('set', 'test_in_cost_func', 0);
xtype = [-1;-1;-1;-1;-1];
tic;
[f, x_opt, grad, hessian, state] = asamin ('minimize','costfgolb' ,x0',mi',ma', xtype);
toc;
disp('les paramètres de calibration du modèle sont [v theta sigma rho kappa] :');
Heston_solex2=[x_opt(1), x_opt(2), x_opt(3), x_opt(4),(x_opt(5)+x_opt(3)^2)/(2*x_opt(2))];
disp(Heston_solex2);
```

APPENDIX B: SAMPLE OF DATA USED IN CALIBRATING THE HESTON MODE

N	Spot	DTM	Strike	r=rd-rf	Mid	rd	rf	bid	ask
1	1.1115	0.115079365079365	1.0875	-0.00706805	0.02125	-0.002781775	0.004286275	0.0171	0.0254
2	1.1115	0.115079365079365	1.09	-0.00706805	0.0192	-0.002781775	0.004286275	0.0152	0.0232
3	1.1115	0.115079365079365	1.0925	-0.00706805	0.0173	-0.002781775	0.004286275	0.0135	0.0211
4	1.1115	0.115079365079365	1.095	-0.00706805	0.0155	-0.002781775	0.004286275	0.0119	0.0191
5	1.1115	0.115079365079365	1.0975	-0.00706805	0.01375	-0.002781775	0.004286275	0.0103	0.0172
6	1.1115	0.392857142857143	1.06	-0.00706805	0.04905	-0.002781775	0.004286275	0.0445	0.0536
7	1.1115	0.392857142857143	1.065	-0.00706805	0.04455	-0.002781775	0.004286275	0.0401	0.049
8	1.1115	0.392857142857143	1.07	-0.00706805	0.0402	-0.002781775	0.004286275	0.0359	0.0445
9	1.1115	0.392857142857143	1.075	-0.00706805	0.036	-0.002781775	0.004286275	0.0318	0.0402
10	1.1115	0.392857142857143	1.08	-0.00706805	0.03195	-0.002781775	0.004286275	0.0279	0.036
11	1.1137	0.753968253968254	1.065	-0.00706805	0.0509	-0.002781775	0.004286275	0.0466	0.0552
12	1.1137	0.753968253968254	1.07	-0.00706805	0.0468	-0.002781775	0.004286275	0.0426	0.051
13	1.1137	0.753968253968254	1.075	-0.00706805	0.0429	-0.002781775	0.004286275	0.0388	0.047
14	1.1137	0.753968253968254	1.08	-0.00706805	0.03915	-0.002781775	0.004286275	0.0352	0.0431
15	1.1137	0.753968253968254	1.085	-0.00706805	0.0355	-0.002781775	0.004286275	0.0317	0.0393
16	1.1164	1.11507936507937	1.065	-0.00706805	0.0566	-0.002781775	0.004286275	0.0525	0.0607
17	1.1164	1.11507936507937	1.07	-0.00706805	0.0527	-0.002781775	0.004286275	0.0487	0.0567
18	1.1164	1.11507936507937	1.075	-0.00706805	0.0489	-0.002781775	0.004286275	0.045	0.0528
19	1.1164	1.11507936507937	1.08	-0.00706805	0.04525	-0.002781775	0.004286275	0.0415	0.049
20	1.1164	1.11507936507937	1.085	-0.00706805	0.0417	-0.002781775	0.004286275	0.038	0.0454
21	1.1186	1.36507936507937	1.07	-0.00706805	0.0571	-0.002781775	0.004286275	0.0531	0.0611
22	1.1186	1.36507936507937	1.075	-0.00706805	0.05335	-0.002781775	0.004286275	0.0494	0.0573
23	1.1186	1.36507936507937	1.08	-0.00706805	0.04965	-0.002781775	0.004286275	0.0458	0.0535
24	1.1186	1.36507936507937	1.085	-0.00706805	0.0461	-0.002781775	0.004286275	0.0424	0.0498
25	1.1186	1.36507936507937	1.09	-0.00706805	0.0427	-0.002781775	0.004286275	0.0391	0.0463