International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 46 No. 3 Sep. 2025, pp. 563-582

http://www.ijias.issr-journals.org/

Implication of low intensity rain spectra (R≤1mm/h) in the variability of Z-R relationship linked to the microphysical processes of precipitation: Size-controlled, number-controlled and mixed control of drops

Bakary Bamba, Augustin Kadjo Koffi, Brahima Kone, Modeste Kacou, Ibrahima Bamba, Eric-Pascal Zahiri, and Abé Delfin Ochou

Laboratoire des Sciences de la Matière, de l'Environnement et de l'Energie Solaire, Université Félix Houphouët-Boigny, UFR SSMT, 22 BP 582 Abidjan 22, Côte d'Ivoire

ABSTRACT: Understanding the necessary relationship between the radar reflectivity factor Z and the rain rate R is still important today and constitutes an interesting challenge for the scientific community. Despite this interest and the advances already obtained in this field, understanding the variability of this Z-R law is not yet trivial. This work therefore consists of studying the implication of rain spectra at low intensities R≤1mm/h in this variability. Thus, from a large historical database of raindrop size distribution (DSD) acquired in tropical Africa, we have shown that rain spectra R≤1mm/h lead to high values of the coefficients A and b of the Z-R power relationship. However, the variability of pre-factor A is pronounced in agreement with those of the N_T and D_0/N_T parameters of the DSD; while the exponent b remains quasi-constant in such rainy samples, independently of their percentage. Establishing the link between the observation parameters D_0, N_T and D_0/N_T of the DSD and the pre-factor A of the Z-R relationship made it possible to arrive at close power relationships D_0 – A, N_T – A and D_0/N_T – A corresponding respectively to the microphysical modes «D_0 constant, N_T variable», «D_0 variable, N_T constant» and «D_0 variable, N_T variable». The last microphysical condition seems to be more suited to the power relationship A – α where α, is the pre-factor of the power relationship D_0/N_T – R found in the literature.

KEYWORDS: Rain spectra, Low intensity R≤1mm/h, Raindrop Size Distribution (DSD), Tropical Africa, Z – R relationship Variability, observation parameters, microphysical modes.

1 INTRODUCTION

The objective of this work is to show the importance of raindrop size distributions (DSD) at low intensities (R≤1mm/h) in the variability of the relationship between the radar reflectivity factor Z and the rain rate R, frequently formulated by $Z = AR^b$ **[1]; [2].** This Z-R relationship is essential for the scientific community because it makes it possible to convert measurements obtained by meteorological radar into rain rate, useful in many physical applications (hydrometeorology, telecommunications, etc...). But, it continues to be a preoccupation due to its great spatio-temporal variability. On this subject, the very recent work of **[2]** and **[3]** showed that this variability is not arbitrary but rather consistent. To demonstrate this, they used the median volume diameter D_0 and the total number of drops per unit volume of air N_T as the characteristic parameters of the observed DSD. These two parameters materializing the size of the drops and their number, constitute the result on the ground of the various microphysical processes (coalescence, fractionation, accretion, etc...) whose common seat is the atmosphere. Like **[4],** they showed the existence of three modes of variability of the DSD in precipitation and which seem to control the variability of the Z-R relationship. These are modes where the rain rate R depends solely on either the number of drops N_T, their diameter D_0 or their ratio D_0/N_T. Thus, on the basis of a large sample of entire rain events and/or subdivided into convective/stratiform part, the illustrative approach developed by **[2]** then **[3]** consisted of determining the functional relationship $D_0/N_T=\alpha R^{\beta}$ which, confronted with the relationship $Z=AR^b$, allowed them to develop the functional relationships A-α and b-β to which they attributed a quasi-constant character and this, regardless of precipitating systems, climatic zones and the type of DSD measuring instrument.

Furthermore, observations always indicate the presence of high and low values of the coefficient A of the Z-R relationship in precipitation in relation to these microphysical modes **[5]; [6]; [4].** To this end, it has been revealed in the literature that Z-R relationships are extremely sensitive to intensity classes **[5]; [7]; [8]; [9].** For example, **[9]** showed that for a rainy sample, the high intensity values

(R>10mm/h), representative of convective situations, contribute to significantly reducing the pre-factor A of the Z-R relationship and raising the exponent b compared to the overall sample. On the other hand, low intensities (R≤1mm/h), representative of stratiform situations, contribute to significantly increasing the values of coefficients A and b. In order to explain the origin of the large values of pre-factor A obtained in the so-called stratiform and transition portions of their studied squall line, **[9]** showed that a rainy sample whose intensities hardly exceed 10 mm/h (R≤10mm/h) and having a percentage of spectra R≤3mm/h greater than 80% generally leads to high values of the pre-factor A. Such behavior of the pre-factor A with respect to the rain intensity ranges R would find an explanation in the very nature of the relationship $Z=AR^b$ sought. Indeed, the linear relationship linking Z to R on a log-log scale is such that:

$$ZdB = AdB + bRdB \tag{1}$$

where if X designates Z, R or A, we have: XdB = 10logX. In this relationship, the values of R≤1mm/h are such that RdB<0 and contribute to increasing AdB, that is to say A, for a given value of b, when those of R>1mm/h such that RdB>0 produce the opposite effect. Such results suggest that there is a link between this increase or decrease in the value of pre-factor A and the percentage of these two intensity classes (R≤1mm/h and R>1mm/h) in the overall sample studied. The present study, devoted mainly to DSDs at low intensities (R≤1mm/h), contributes to providing a solid basis for their real impact in the microphysical interpretation of the variability of the Z-R power law coefficients, in particular that of the pre-factor A, given that the exponent b remains quasi-constant in a climatic zone. Indeed, it emerges from the work of **[10]** that R≤1mm/h spectra have a no less important contribution in a global sample.

In this work, the description of the database as well as the brief presentation of the observation sites will be the subject of section 2. This section also presents the method used for sampling individual spectra from a rainfall event. Section 3 displays the main results of this paper, among other things, the sensitivity of the DSD parameters to rain spectra at low intensities R≤1mm/h and, the microphysical interpretation of the variability of the coefficients of the Z-R relationship in relation to the percentage of such rain spectra. A discussion is also conducted in the said section. The conclusion of this study is summarized in section 4.

2 METHODOLOGY

2.1 DATABASE

The DSD data selected for our analysis were collected using the first modern instrument, the RD-69 disdrometer developed by **[11].** It was installed during individual DSD measurement campaigns on four study sites; namely Abidjan (Guinean coastal site), Boyélé (equatorial continental site), Dakar (Sahelian coastal site) and Niamey (Sahelian continental site). Its operating principle is based on the conversion of the momentum of the drops into an electrical impulse. In fact, depending on the amplitude of their impacts on the horizontal surface (50 cm^2) of the sensor, the raindrops received each minute are distributed according to their size in 25 channels covering the diameter range between 0.3 and 5.1 mm, in constant steps of 0.2 mm. Under these conditions, this instrument integrates very poorly small drops, i.e. those with diameters less than 0.3 mm. Also, it has been recognized in the work of **[12]** that during intense precipitation, small drops with a diameter of less than 1.5 mm are underestimated, due to environmental noise. Fortunately, **[13]** and **[7]** were able to find arguments to correct this anomaly. They showed that under optimal conditions of use of the RD-69 disdrometer, the number of small drops always remains low in tropical precipitation. In other words, small drops have a negligible contribution compared to the total number of drops detected. In this respect, their inclusion or not in the processing of the acquired data has less impact on the quality of the results obtained. Details on the limitations of the RD-69 disdrometer are beyond the scope of this study. They have been well developed and discussed in many previous works **[14]; [15]; [13]; [7]; [16].** A total of 412 rainfall events of distinct natures (Thunderstorms, Squall lines and Stratiform rains) were sampled using this instrument, corresponding to 50412 minute spectra (or DSDs) or 840h12min and a cumulative rainfall of 4505 mm.

The technique for identifying precipitating systems is based on the hyetogram of the rainfall event observed on the ground by the disdrometer as done by **[9]** and **[2].** Their structures and morphologies have been well documented by many authors **[9]; [16]; [17];** among many others.

Furthermore, disdrometers are very effective for measuring rainfall spectra at low intensities R≤1mmh^{-1} **[18]; [10].** As indicated in the introduction, such spectra, which are the subject of this work, are important for an accurate estimation of mean rainfall by radar remote sensing through the Z-R relationship.

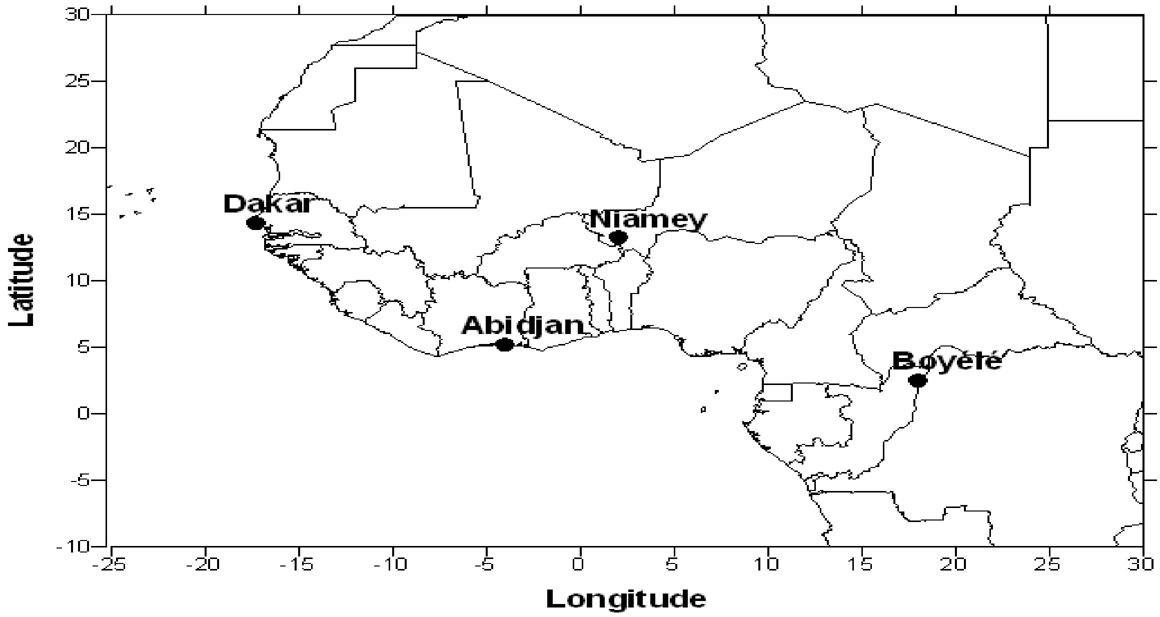

Fig. 1. ***Geographical location of the different cities (in black dots) where the measurement of the raindrop size distribution (DSD) was observed using a JW-RD 69 mechanical impact disdrometer***

Figure 1 shows the locations of the instruments; and Table 1 presents the essential characteristics of this database. Such a database, although not acquired simultaneously at the different sites, constitutes the guarantee of a good statistical study on the implication of the spectra of low-intensity rains R≤1mmh^{-1} in the variability of the Z-R relationship in relation to the modes of variability of the DSD.

Table 1. Description of the database. SL (Squall Lines), ALL (whole rainy events)

Location	coordinates	Observing period	Events number/1 min. Spectra number			
			SL	Thunderstorm	Stratiform	ALL
Abidjan (Côte d'Ivoire)	5°25N-4°W	1986 (Jun., Sept.-Dec.)	05/681	15/1098	02/179	22/1958
		1987 (Feb.-Dec.)	20/2633	82/5198	17/2006	119/9837
		1988 (Feb.-Jun.)	10/2000	26/1748	05/548	41/4296
		1986-1987-1988	35/5314	123/8044	24/2733	182/16091
Boyele (Congo)	2°50N-18°04E	1988 (May-Jul., Sept.-Dec)	25/5525	25/1434	11/1717	61/8676
		1989 (Mar.-Jun.)	10/2243	19/2530	6/644	35/5417
		1988-1989	35/7768	44/3964	17/2361	96/14093
Niamey (Niger)	13°30N-2°10E	1989 (Jul.-Sept.)	09/1379	13/1328	01/55	23/2762
		1991 (Aug.)	03/357	02/94	02/146	07/597
		1989-1991	12/1736	15/1422	03/201	30/3359
Dakar (Senegal)	14°34N-17°29W	1997 (Jul.-Oct.)	04/699	12/1795	03/466	19/2960
		1998 (Jul.-Sept.)	11/2581	16/1921	02/136	29/4638
		1999 (Jul.-Sept.)	08/1839	13/1972	03/591	24/4402
		2000 (Jul.-Oct.)	14/2247	11/1470	07/1152	32/4869
		1997-2000	37/7366	52/7158	15/2345	104/16869
					TOTAL	412/50412

2.2 Method Of Sampling Event Spectra

2.2.1 Definition

Several methods have been proposed in the literature to sample event spectra. Some eloquent examples are the recent methods proposed by **[19]** and **[20].** For the former, their method consisted in sorting the rain spectra according to whether they were convective or stratiform and for the latter, it is based on the stratification of storm spectra by increasing time steps with the minimum size of granulometric spectra considered equal to 5 minutes. In this study, we also propose to introduce a new technique for sampling event spectra, independently of their nature (convective/stratiform) and their origin (continental/maritime) which is defined as follows:

If n (n≥10) denotes the total number of chronological minute spectra of a given rain event, and k = 10 minutes, the reason for the numerical series thus defined, the total number of terms in this series is therefore n-k+1. Under these conditions, if i is the first minute spectrum of a term then i+k-1 is the last.

Thus, for each term of the series, we calculate the main integrated precipitation parameters, including the radar reflectivity factor Z and the rain intensity R, then by power regression, we determine the values of the coefficients A and b. Also, for each term, we calculate the proportion (or percentage) of low intensity rain spectra (R≤1mm/h). This method is applied to all 412 events in our database. The details of the calculation of Z and R knowing the DSD, as well as the other integrated precipitation parameters such as the median volume diameter D_0, and the total number of drops per unit volume of air N_T used in this work are generally calculated from the following theoretical formulas:

$$Z = \int_0^{+\infty} D^6 N(D) dD \quad (2)$$

$$R = 6\pi 10^{-4} \int_0^{+\infty} D^3 N(D) v(D) dD \quad (3)$$

$$\int_0^{D_0} D^3 N(D) dD = \int_{D_0}^{+\infty} D^3 N(D) dD \quad (4)$$

$$N_T = \int_0^{+\infty} N(D) dD \quad (5)$$

where $v(D)$ is the limiting falling velocity of raindrops of diameter D and, N (D) representing the DSD, denotes the number of drops per unit volume and per diameter interval.

Furthermore, this technique of sampling event spectra is called in this work, the "DSD sliding window" method. Figure 2 shows us an application of this method on a simulated rain event (represented here by a matrix with one row and n columns designating the chronological minute spectra of the event), with presentation of the first two terms of the series called "sliding series". We note that one of the particularities of this method lies in the fact that at each sliding of the window, the ith spectrum is automatically substituted by the (i+k) th spectrum; thus allowing to keep constant the width of the sliding window. This fact makes it possible to appreciate the importance of a minute spectrum in a given sample and consequently the impact that its substitution can generate on the state of the previous system assumed to be its initial state.

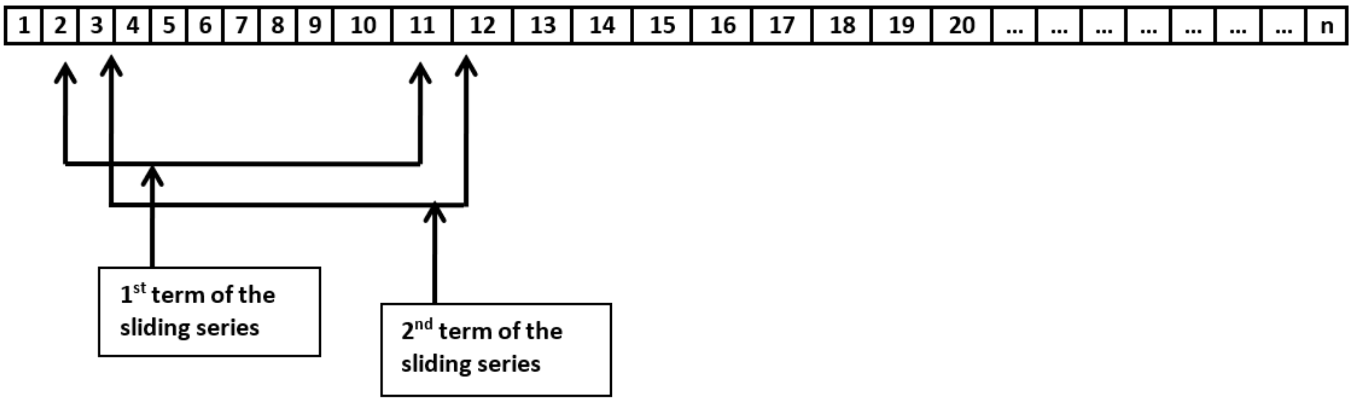

Fig. 2. Simplified diagram of the sliding window DSD method, with presentation of the first two terms of the series. The DSD window has a width of 10 minutes and slides in 1-minute increments until the last spectrum n of the rain event considered is recovered

It should also be remembered that this method makes it possible to increase the range of values of the integrated parameters of the DSD, as well as the coefficients A and b of the Z-R power law for a given rain event when the width of the sliding window is less than the size of said event as is the case in this paper.

2.2.2 APPLICATION

To assess the relevance of our method for sampling event spectra, we focused on analyzing the values of the coefficients A and b as a function of the sliding series number. For this, we used as an example a stratiform rain observed on May 29, 1987 in Abidjan on which we applied our method with a window width of 10 minutes. This width can vary from 10 minutes to the entire event. However, a smaller width would reduce the number of spectra in the window and therefore obscure some information. In this regard, **[21]** proved that the calculation of the DSD parameters is very sensitive to the sample size, especially when it is small. Indeed, for each window displacement, the coefficients A and b of the Z-R relationship are calculated by the least squares method as well as the correlation coefficients r. Figures 3a, b,c show respectively the hyetogram of the rain event considered, and the evolutions as a function of the series number, of the different parameters cited above (A, b and r). The analysis of figure 3c clearly justifies that the correlation coefficient r is not a function of the position of the sliding window considered for the determination of the Z-R relationship. Indeed, we observe acceptable correlation values at the beginning, middle and end of the sliding series of minute spectra of the event. Similarly, the analysis of Figure 3b shows remarkably that the values of coefficients A and b do not seem to be related to the considered position of this sliding window. However, strong and weak values of coefficients A and b are observed independently of the sliding window considered. Such results suggest that the current values of A (or b) depend on the percentage (or ratio) of rain spectra $R \leq 1$mm/h contained in the window if we refer to equation (1). A similar study was done by **[9]** and **[20].** It follows from their different works that the values of coefficients A and b are not an increasing function of the number of minute spectra considered in the rain samples. All these observations show that the obtained Z-R relationships depend more on the microphysical processes translated by the size, the number of drops and the combination of the two as proposed by **[4]; [2]** and **[3]** than on the sampling method of the spectra. Thus, this dependence of the Z-R power law with the DSD will be related to the weight of the spectra at low intensities ($R \leq 1$mm/h) in this paper. Then, the analysis of equation (1) reveals that the pre-factor A varies globally in an inverse direction with the rain intensity R, thus suggesting that a high proportion (or percentage) of low-intensity rain spectra in a global sample would contribute to increasing the value of A. This suggestion will be confirmed or refuted in the section dealing with the results and discussion. Furthermore, since this study is carried out independently of the intrinsic form of the DSD (exponential, gamma or lognormal), it can therefore be used to understand the variability of the Z-R relationship at other latitudes.

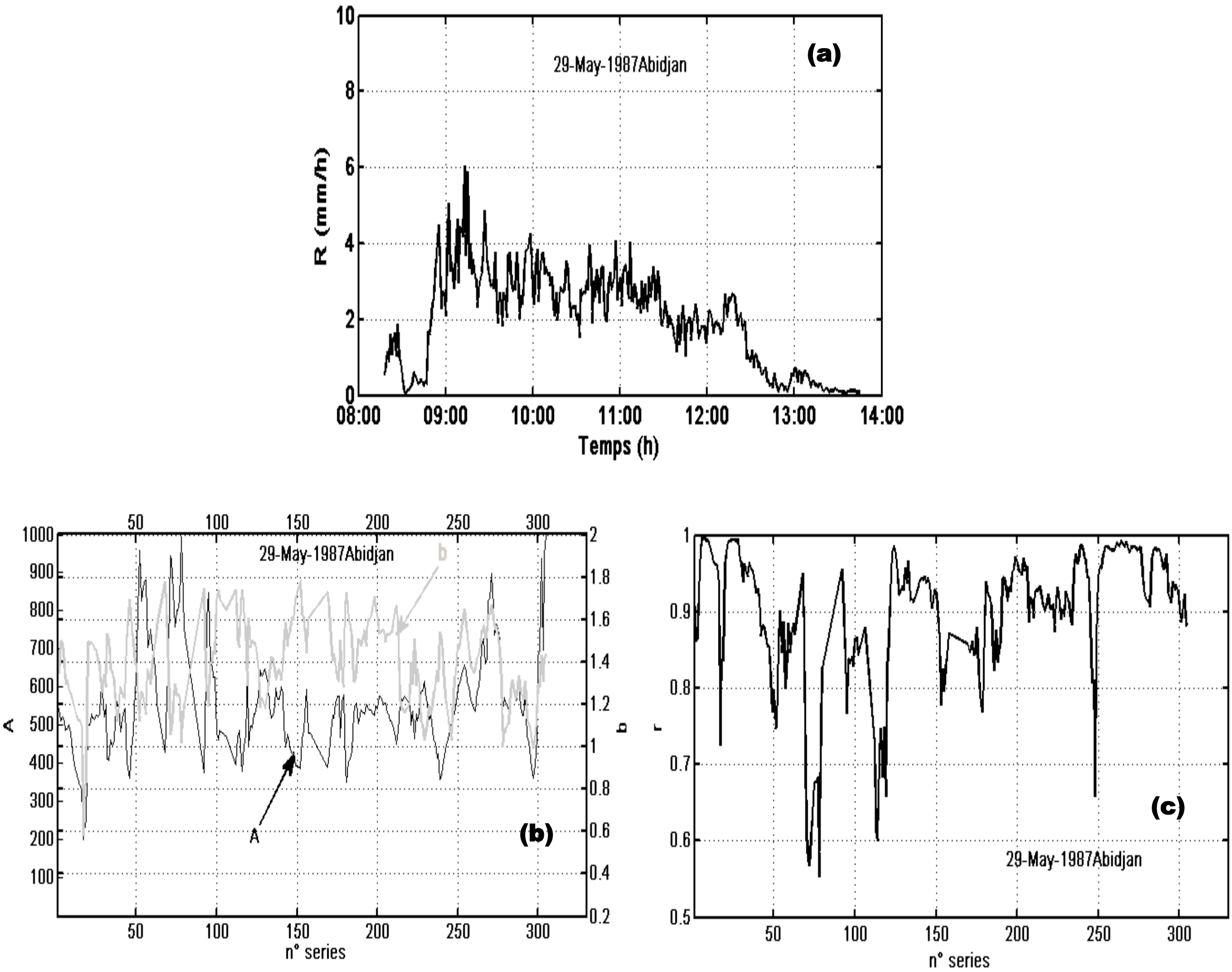

Fig. 3. Example of application of the DSD method with a sliding window of width 10 minutes on the stratiform rain event of May 29, 1987 observed in Abidjan. (a) hyetogram of the rain event, (b) variability of the coefficients A and b of the Z-R power relationship as a function of the position of the sliding window (or series) (the number of DSDs in a series is fixed at 10 minutes) and, (c) current values of the correlation coefficient r of the Z-R relationship

3 RESULTS AND DISCUSSION

3.1 SENSITIVITY OF DSD PARAMETERS TO LOW INTENSITY RAIN SPECTRA R⩽1MM/H: IMPACT ON THE Z-R RELATIONSHIP

To conduct this study on the sensitivity of DSDs to low intensity rain spectra (R≤1mm/h), two methods were adopted. The first is based on the stratification of spectra according to 10 classes of low intensity R≤1mm/h and the second, on their percentage. For the latter, 10 classes of percentage are also proposed. Table 2 presents these different classes of low intensity R≤1mm/h and percentage thus considered. The importance of these classes lies in the fact that they can represent different types of rain R≤1mm/h ranked on the one hand, from very low to low intensity R≤1mm/h and on the other hand, from low to high proportions of spectra R≤1mm/h. After that, we calculated for each type of process and for each of the defined classes, the characteristic parameters of the DSD (size, number of drops and combination of the two) as well as the coefficients A and b of the Z-R power law. Since the study is carried out independently of the analytical form of the DSD, the drop size and number parameters used are respectively the median volume diameter D_0 and the total number of drops per unit volume of air N_T. Given that it has been revealed in many works that the variability of the Z-R relationship is more linked to the simultaneous variations of the number and diameter of drops in precipitation than to their variations taken individually **[4]; [2]; [3],** we therefore considered the D_0/N_T parameter as a third parameter like **[2]** and **[3].** The advantage of subdividing the rainfall rates R≤1mm/h into different intensity classes and percentage is that it could help us capture with less uncertainty the potential information on the characteristics of DSDs, as well as their influences on the Z-R relationship.

Table 2. Different classes of rain rate R and percentage P of the R≤1mm/h rain spectra

Class name	Low intensity classes of the R≤1mm/h rain spectra	Class name	Percentage classes of the R≤1mm/h rain spectra (%)
R_1	R<0.1	P_1	0≤P<10
R_2	0.1≤R<0.2	P_2	10≤P<20
R_3	0.2≤R<0.3	P_3	20≤P<30
R_4	0.3≤R<0.4	P_4	30≤P<40
R_5	0.4≤R<0.5	P_5	40≤P<50
R_6	0.5≤R<0.6	P_6	50≤P<60
R_7	0.6≤R<0.7	P_7	60≤P<70
R_8	0.7≤R<0.8	P_8	70≤P<80
R_9	0.8≤R<0.9	P_9	80≤P<90
R_{10}	0.9≤R≤1	P_{10}	90≤P≤100

Thus, to highlight the importance of low-intensity rain spectra R≤1mm/h in the variability of the Z-R relationship, we were initially interested in their proportion in all the detected event spectra as we indicated above. Thus, if P denotes their percentage in the rain sample, the operation consists of establishing a functional relationship between the DSD observation parameters (D_0, N_T and D_0/N_T) and this variable P. Under these conditions, the results obtained are presented in Figures 4a, b,c respectively for the D_0 – P, N_T – P and D_0/N_T – P relationships. The functional relationships and the corresponding correlation coefficients are indicated in the figures.

The analysis of the variation profiles of the three DSD parameters studied as a function of the percentage of rain spectra R≤1mm/h shows diverse situations. It can be noted that the median volume diameter D_0 and the total number of drops per unit of air volume N_T evolve in accordance with a decreasing power law with the percentage P unlike the ratio D_0/N_T which varies in the opposite direction. The correlation coefficients greater than 0.70 show that the three DSD parameters are well correlated with the percentage P of rain spectra R≤1mm/h. In other words, by first looking at D_0 (see Figure 4a), we notice that it decreases very slowly when the percentage increases and this is justified by a very low slope. This state of affairs is materialized by the functional relationship $D_0 = 2.2P^{-0.11}$. It is also noted that the average drop diameter rarely exceeds 2.0 mm in low intensity rain spectra. This result therefore shows that the rain spectra R≤1mm/h observed in tropical regions of Africa are rich in small drops and are, like the other spectra (R>1mm/h), representative of the theoretical gamma or lognormal forms.

Concerning the N_T parameter (see Figure 4b), its decrease with the percentage P is notable and rapid compared to D_0. The characteristic equation that justifies this relationship is given by $N_T = 634P^{-0.52}$. This result shows that the high values of the average number of drops N_T observed in tropical precipitation are linked to a low ratio of rain spectra R≤1mm/h contained in the global sample.

As for the D_0/N_T parameter, its growth occurs rapidly with the percentage P like the number of drops N_T and this is justified by identical slopes (see Figure 4c). This variation profile is translated by the function $D_0/N_T=0.004P^{0.52}$. This result clearly justifies that the low average values of the D_0/N_T parameter observed by **[2]** in the convective spectra of squall line type events are due to low values of percentage P of the spectra R≤1mm/h contained in such rain samples. For example, to verify this, we were interested in the squall line of August 7, 1998 observed in Dakar where, for a low value of percentage of spectra R≤1mm/h (P = 6%) in the convective region, the average value of D_0/N_T obtained is also low (D_0/N_T = 0.04 mm.m^3). On the other hand, that obtained for the stratiform spectra of the same event is strong (D_0/N_T = 0.11 mm.m^3) corresponding to a higher percentage (P = 50%).

To verify whether this variability of the DSD with the percentage P of the spectra R≤1mm/h influences the Z-R power relationship, we have presented through figures 4d, 4e respectively, the evolution profile of the coefficients A and b of the Z-R law as a function of the percentage P of the spectra R≤1mm/h. The values of the said coefficients relative to the different classes of percentage P thus defined are listed in table 3. As we can see, the analysis of these values shows that they are strong (or high) in the domain of low intensities and this, independently of the defined percentage classes. We note from the analysis of figure 4e that the exponent b is quasi-constant and close to the value 1.39 because the slope of the line is almost zero. On the other hand, through Figure 4d, we notice that the multiplicative factor A increases very slowly with the percentage P of the spectra R≤1mm/h following the power law $A = 261P^{0.11}$. However, this growth becomes notable when the sample contains more than 80% of spectra R≤1mm/h. Therefore, it results by comparison that the variations of the physical parameter D_0/N_T as a function of the percentage P of the spectra R≤1mm/h contained in the samples of tropical rains are similar to those of the coefficient A. These observations are similar to those found in the recent works of **[2]** and **[3].**

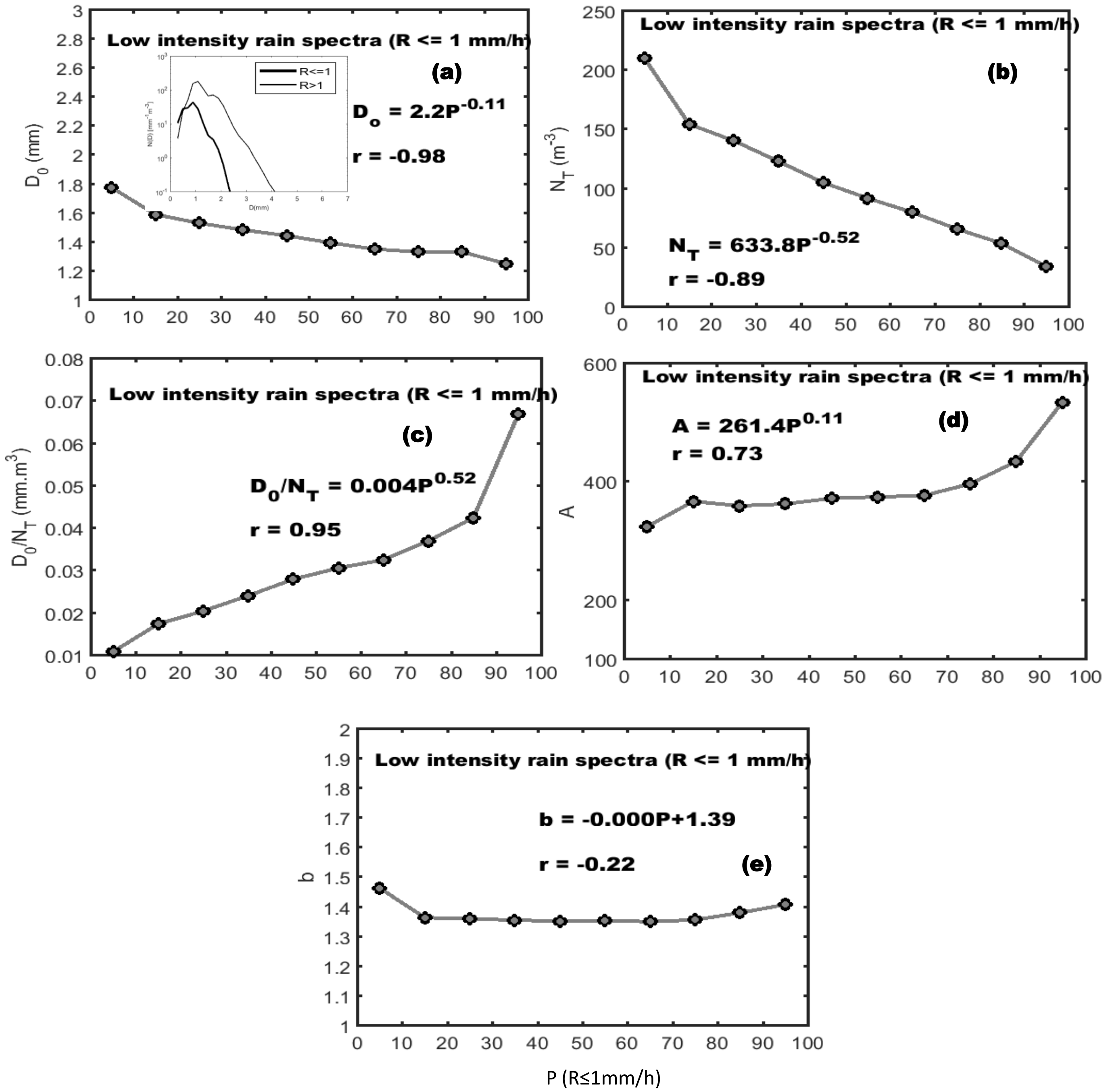

Fig. 4. Functional relationships between DSD parameters (D_0, N_T and D_0/N_T) and the percentage P of low intensity rain spectra (R≤1mm/h): Impact on $Z=AR^b$ relationships

Table 3. Z-R relationships for different percentage classes of low intensity rain spectra (R≤1mm/h)

Z-R relationships		Percentage classes of low intensity rain spectra (R≤1mm/h)									
		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
$Z=AR^b$	A	325	367	360	363	372	375	377	397	435	535
	b	1.46	1.36	1.36	1.35	1.35	1.35	1.35	1.36	1.38	1.41

To highlight in another way the importance of low-intensity DSDs in the variability of the Z-R relationship, we were interested in determining the relationships between the integrated parameters of the DSD (D_0, N_T and D_0/N_T) and the rain intensity R≤1mm/h. Figures 5a, b,c present the variation profiles of the three observation parameters of the DSD (D_0, N_T and D_0/N_T) as a function of the rain intensities

R≤1mm/h. The functional relationships and the corresponding correlation coefficients are marked on the same figures. Their analysis shows that the median volume diameter D_0 and the total number of drops per unit volume of air N_T are increasing power functions of the intensities R≤1mm/h unlike the D_0/N_T ratio which evolves according to a decreasing power law with strong correlations for the three cases. In other words, by first looking at D_0 (see Figure 5a), we notice that it increases when the low intensities increase. However, this growth is slow and less accentuated with regard to a very low slope. This behavior is in good agreement with those found by other authors **[19]; [22]; [10].** Also, it can be observed in this figure that D_0 tends towards an equilibrium distribution in the range of intensities close to R=1mm/h; and this can be illustrated by the quasi-constant character of the D_0 – R curve in the vicinity of these so-called low intensities. Such a shape of the curve is typical of what is called the equilibrium distribution where the collision-coalescence processes are exactly balanced with the fractionation of the drops **[23]; [24]; [25]; [26]; [27].** In other words, this equilibrium distribution shows that the microphysical mode where the variability of the DSD is controlled only by the number of drops **[4]** exists in low intensity rain samples. The analysis of Figure 5a also shows that the average diameter of the drops hardly exceeds 2.0 mm in the spectra of rain at low intensities as we observed above in the calculation of the functional relationship D_0 – P. As already indicated above, this result also confirms that the rain spectra R≤1mm/h are rich in small drops as has been found elsewhere by a number of researchers **[7]; [9]; [28]; [29].** For example, **[9]** showed that in the transition region (with R≤1mm/h) of the continental squall line of February 22, 1998 observed in Darwin (Australia), the DSD was characterized by small drops (D_0≈1mm). On the other hand, **[10]** managed to find in temperate precipitation, drops whose diameters can reach 3 mm for the same types of intensity regimes but, they specified that such situations are almost non-existent in tropical precipitation.

Concerning N_T (see Figure 5b), its increase with low intensities is accentuated and rapid compared to D_0 because its slope is higher. This result is in line with those obtained by **[22].** Also, we note that N_T is almost constant in the range of low intensities close to R=1mm/h. This fact clearly justifies that the microphysical mode where the rain rate depends only on the diameter of the drops (see **[4])** can be observed in such rain samples. Furthermore, as observed in other latitudes by **[9]** and **[10],** the average number of drops is relatively high in our sample of rain spectra R≤1mm/h, with a large range of N_T values (20 – 70 m-3) compared to D_0.

For the D_0/N_T parameter, its decrease is moderate with low intensities R≤1mm/h (see Figure 5c). Without classifying DSDs according to the rainfall intensity regimes R as we did here, **[2]** found the same result for different types of rain samples (global, convective and stratiform). For all classes R≤1mm/h, the D_0/N_T values are relatively strong or moderate. It can be observed in the same figure that in the vicinity of R = 1mm/h, the D_0/N_T ratio is almost constant, thus suggesting that the microphysical mode where the variability of the rain rate is linked to the simultaneous variations of the diameter and number of drops also exists in tropical rainfall at low intensities. To this end, **[2]** showed that the majority of precipitation observed in tropical Africa is the result of this process and that the first two modes mentioned above are very rare. **[4]** found similar results from a theoretical study. These different studies agreed that the high variability of the Z-R relationship from one precipitation to another or within the same precipitation is due to this process.

To verify this in the specific case of low intensity precipitation studied here, we present through Figures 5d, e respectively, the behavior of the coefficients A and b of the Z-R power law as a function of the intensities R≤1mm/h. This exercise will allow us to appreciate the real impact of the sensitivity of the DSD to low intensities on the variability of the Z-R relationships. The values of the said coefficients (A and b) corresponding to the different classes of low intensities thus defined are listed in Table 4. It follows from these results (Figures 5d, e and Table 4) that the values of the coefficients A and b are high in the domain of low intensities and this, independently of the defined classes. However, the highest values are observed in the classes with very low intensities, notably R1 and R2. Also, if the exponent b remains almost constant (b ≈ 1.39), the multiplicative factor A decreases very slowly with intensities R≤1mm/h following the power law $A = 422R^{-0.15}$. As can be seen, the changes in D_0/N_T and A with low intensities are in agreement. This fact shows that the variations in the multiplicative factor A of the Z-R law in low-intensity precipitation are more due to the D_0/N_T ratio than to the D_0 and N_T parameters taken individually. This is therefore reminiscent of the above-mentioned observations of **[4]** and **[2].**

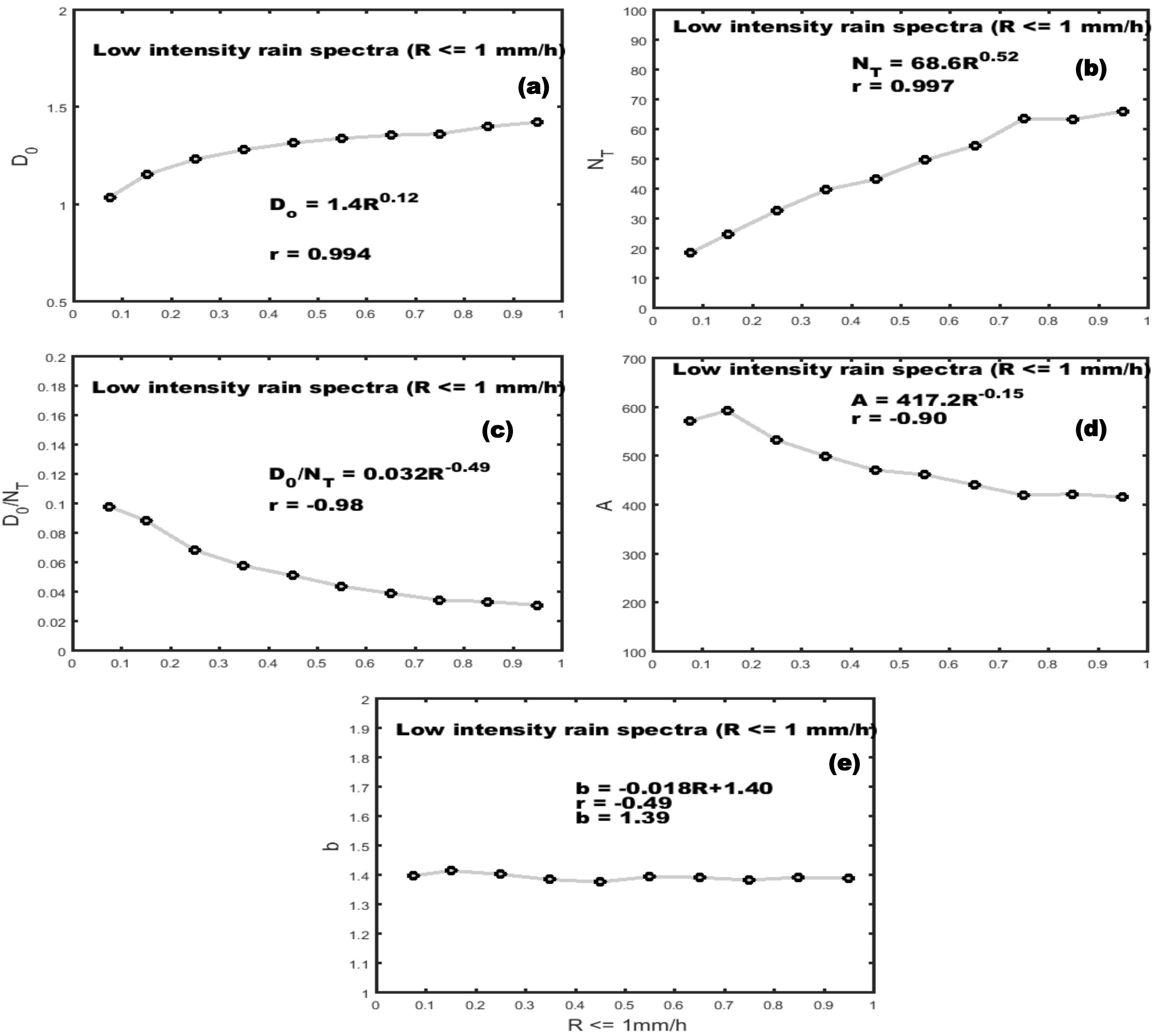

Fig. 5. Functional relationships between DSD parameters (D_0, N_T and D_0/N_T) and intensity R of low-intensity rain spectra (R≤1mm/h): Impact on $Z=AR^b$ relationships

Table 4. Z-R relationships for different classes of low intensity rain spectra (R≤1mm/h)

Z-R relationships		Low intensity classes of the R≤1mm/h rain spectra									
		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
$Z=AR^b$	A	585	591	539	501	469	461	442	429	433	425
	b	1.40	1.42	1.40	1.39	1.38	1.39	1.39	1.38	1.39	1.39

This study clearly shows that if the low intensity rain spectra R≤1mm/h are predominant in the rain sample, this favors a rapid increase in the values of the coefficients of the Z-R relationship (with a high and quasi-constant exponent b). Otherwise, the pre-factor A will undergo a rapid decrease under the weight of the R>1mm/h spectra. This is why we note that certain stratiform parts of the squall lines, although characterized by a high proportion of R>1mm/h spectra, cannot favor an increase in As and cause it to be greater than Ac (As<Ac: **[7]; [8]; [30].** On the other hand, other stratiform parts can be characterized by a significant proportion of R≤1mm/h spectra contributing to considerably increase As such that it is greater than Ac (As>Ac: **[31]; [9]; [10].** For the rest of this work, the link between the variability of the Z-R relationship and the percentage of R≤1mm/h spectra in relation to the modes of variability of the DSD (or microphysical processes), will only concern the pre-factor A in view of the quasi-constant character of the exponent b in such samples.

3.2 Microphysical Interpretation Of The Variability Of The Z-R Relationship In Relation To The Proportion Of Spectra R⩽1mm/H

In the work of **[32],** it was highlighted that the very fine-scale variability of the DSD has a very significant impact on radar rain estimation algorithms through the Z-R relationship. Thus, in this section, we set ourselves the objective of establishing for each of the ten ratio samples defined previously, analytical relationships between the DSD observation parameters (size, number of drops and combination of the two) and the coefficient A of the Z-R law as done theoretically by **[4].** This operation will make it possible to assess the impact of the ratio of the spectra R≤1mm/h on the said empirical relationships, and will therefore constitute a means for a microphysical interpretation of the variability of the pre-factor A of this Z-R power law. The median volume diameter D_0 and the total number of drops per unit volume of air N_T, capable of capturing all the microphysical information of the rain, are the observation parameters of size and number of drops always retained to conduct this study.

3.2.1 1^ST Special Microphysical Condition: D0 – A Relations

Some authors have succeeded in analytically establishing the relationship between the drop size and the coefficient A of the Z-R power law, based on numerous theoretical considerations **[4].** For example, for a gamma distribution, these authors have shown that if the variations of the multiplicative factor A are solely due to the number of drops (N_T in the case of the work of **[4]**), then the functional relationship between the average mass diameter D_m and the pre-factor A is of increasing power form, with an exponent b equal to unity. From then on, the Z-R relationship becomes linear.

In this work, we repeat the same exercise. However, we do it independently of the analytical form of the DSD. This amounts to considering the median volume diameter D_0 as the observation parameter of the drop size instead of the theoretical gamma parameter D_m. Thus, the functional relationship D_0-A that we found for each of our ten samples of percentage of spectra R≤1mm/h, also obeys an increasing power law as follows:

$$D_0 = d_1 A^{d_2} \text{ with } d_2 > 0 \qquad (6)$$

where d_1 and d_2 are the coefficients of the D_0 – A law. They are determined for each sample by simple power regression of D_0 on A. Figure 6a shows the D_0 – A regression curves of all the ratio samples considered. The different values of the coefficients d_1 and d_2 of equation 6, as well as the corresponding correlation coefficients r are listed in Table 5.

Table 5. Multiplicative factors and exponents of the functional relationships between the median volume diameter D_0 of the DSD and the pre-factor A of the Z-R power law as well as the associated correlation coefficients. The results are presented for different percentage classes of low intensity rain spectra (R≤1mm/h)

Samples types	$D_0 = d_1A^{d_2}$		
	d_1	d_2	r
P(0 – 10)	1.063	0.089	0.31
P(10 – 20)	0.206	0.349	0.81
P(20 – 30)	0.166	0.381	0.85
P(30 – 40)	0.142	0.402	0.89
P(40 – 50)	0.133	0.406	0.92
P(50 – 60)	0.127	0.409	0.95
P(60 – 70)	0.124	0.407	0.95
P(70 – 80)	0.131	0.392	0.93
P(80 – 90)	0.150	0.364	0.90
P(90 – 100)	0.275	0.244	0.67

It should be noted from this table that the power relationships D_0 – A relating to all the ratio classes considered are very well established with regard to the high correlation coefficients. On the other hand, as can be seen, the power relationship D_0 – A relating to the low intensity rain spectra P (0 – 10) (i.e. those with a percentage lower than 10%) is poorly established. This could probably be justified by a sampling or measurement problem **[6].** However, we can generally note that the classification of spectra R≤1mm/h according to their percentage contributes enormously to making the power relationship D_0 – A consistent. To this end, the analysis of Figure 6a shows remarkably well that the median volume diameter D_0 increases monotonically with the pre-factor A and this, for all the percentage samples considered. We also note that the slope of the fitting curve increases when the percentage decreases. In other words, D_0 increases rapidly as a function of A when the percentage of spectra R≤1mm/h decreases sharply in the rainy sample. Under these conditions, a given value of the pre-factor A can be due either to large drops in spectra R≤1mm/h at low percentage, or to small drops in spectra R≤1mm/h at high percentage. Such a result suggests that low spectra percentage R≤1mm/h are the catalysts for the large drops we observe in tropical precipitation. In contrast, high spectra percentage R≤1mm/h tend to generate small drops. This suggests that a good knowledge of the percentage of low-intensity rain spectra (R≤1mm/h) in any distribution allows us to predict the average size of the measured drops.

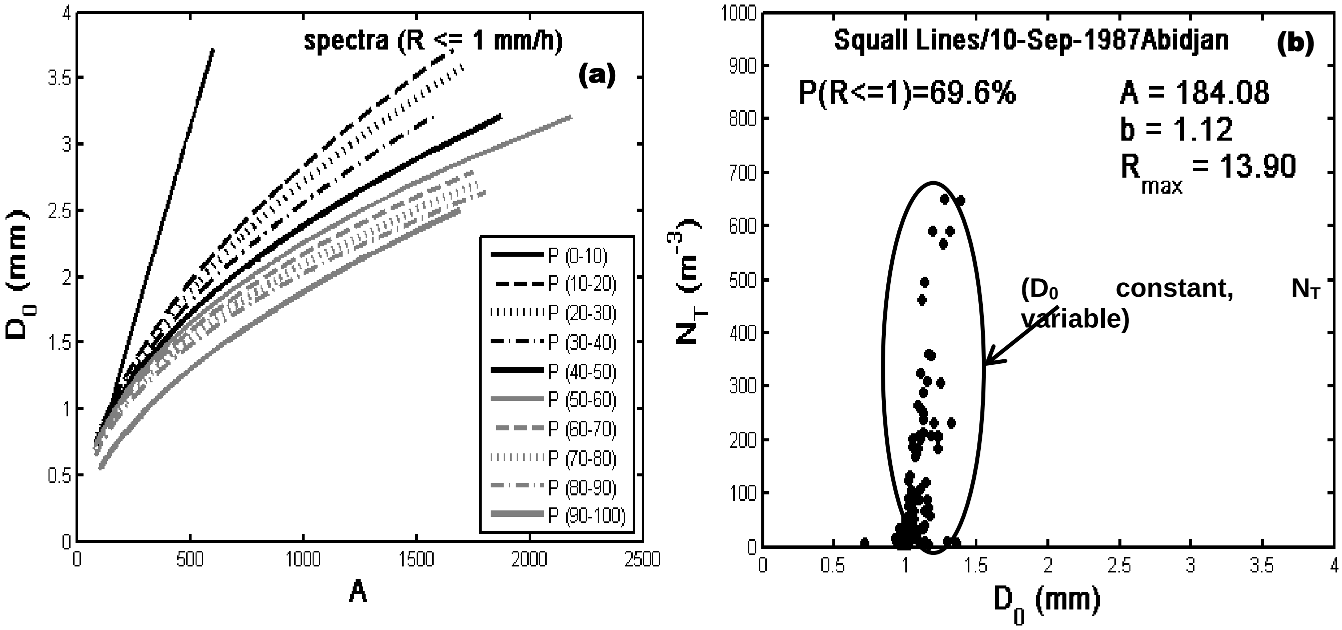

Fig. 6. (a) Functional relationships between the median volume diameter D_0 of the DSD and the multiplicative factor A of the Z-R power law for different percentage samples of low intensity rain spectra (R≤1mm/h). (b) Example of variability of raindrop size distribution (size and number) according to the D_0-A microphysical mode which can exist in precipitation: case of the squall line of September 10, 1987 observed in Abidjan

For example, to demonstrate this in this work, we rely on the uncertainty values of the coefficient A reported in the works of **[33]; [34]; [35]** and **[4].** It results from their different works that the multiplicative factor A of the Z-R relation can typically be estimated with an approximate uncertainty of 30% and the form factor μ of the DSD gamma with an uncertainty of ±2. The latter is related to an uncertainty of ±0.1 for the exponent b of the Z-R law. Under these conditions, for a value of A = 400, the value of D_0 obtained for the percentage P (10 – 20) is D_0 = 1.67mm. Assuming an uncertainty of 30% for A (i.e., 280 < A < 520), the estimated median volume diameter D_0 can be 1.47 < D_0 < 1.83mm, corresponding to an approximate uncertainty of 18%. For the same value of A = 400, another percentage such as P (80 – 90), allows to arrive at D_0 = 1.33mm with an approximate uncertainty of 14.5% (i.e., 1.17 < D_0 < 1.46mm). This difference observed in the estimation of D_0 for the same value of the pre-factor A illustrates the importance of taking into account the percentage of spectra R≤1mm/h in rainfall estimation algorithms.

This study is therefore of interest insofar as it allows us to show that equation 6 has a real physical meaning only in homogeneous precipitation in terms of drop size. In other words, the rain rate R seems to be controlled by the number of drops N_T **[4].** Furthermore, the strong dependence of the power law D_0 – A on the percentage of spectra R≤1mm/h also contributes to the non-uniqueness of the linear relationship Z-R in precipitation.

CONSISTENCY OF THE STUDY

To demonstrate the consistency of this study, we present in Figure 6b the scatter plot of the (N_T, D_0) pair of an event from our JW RD-69 database apparently characterized by the microphysical process for which the rain rate R varies as a function of the number of drops (i.e., the size of the drops is constant). This is the rain event of September 10, 1987 observed in Abidjan. According to our classification method, this event is considered as convective rain. Its essential characteristics are listed in Table 6. It contains 115 spectra, its maximum intensity is R_{max} = 13.90 mm/h, its Z-R relationship (determined by linear regression) is $Z = 184R^{1.12}$ with an exponent b close to unity (quasi-balanced DSD), its percentage of spectra R≤1mm/h is P = 69.6%, its average diameter is $\langle D_0 \rangle$ = 1.07mm and its average number of drops is $\langle N_T \rangle$ = 113.56m^{-3}. Knowing that the exponent b is independent of the percentage of spectra R≤1mm/h, by replacing D_0 by its average value of 1.07mm in the relationship $D_0 = 0.124A^{0.407}$ (see Table 5) corresponding to the percentage class of the event, we obtain: $Z = 199R^{1.12}$. We note that this relation is in agreement with the Z-R relation of the event. However, pre-factor A (A = 199) is estimated with a low approximate uncertainty of 8% compared to pre-factor A (A = 184) of the studied event.

Table 6. Example of a rain event illustrated by the specific microphysical mode D_0-A in accordance with the work of [4]. The statistical parameters of the DSD characteristics (size and number of drops), the Z-R relationship obtained by linear regression, the percentage of spectra R≤1mm/h, as well as the analytical coefficient A of the Z-R law of the event are also indicated

Evts/ (spectra number)	Z-R (observed event)		Drop diameter D_0 (mm)			Number of drops N_T (m^{-3})			Microphysical mode	P(R≤1) (%)	Coefficient A of the Z-R law (estimated)
	A	b	mean $<D_0>$	σ	cv	mean $<N_T>$	σ	cv			A
SL/ 10-Sep-1987 Abidjan (115 spectra)	184	1.12	1.07	0.09	0.09	113.56	148.90	1.31	(D_0 constant, N_T variable)	69.6	199

σ: Standard deviation; cv: coefficient of variation

3.2.2 2ND SPECIAL MICROPHYSICAL CONDITION: NT – A RELATIONS

In the theoretical work of **[4],** it was also shown the existence of an analytical relationship between the number of drops in precipitation and the pre-factor A of the Z-R power law. Indeed, they showed, assuming that the distribution is of gamma form, that if the variations of the pre-factor A are due only to the diameter of the drops (D_m in the case of the work of **[4])** then, the functional relationship between the number of drops per unit volume of air N_T and the pre-factor A is of a decreasing power form, with an exponent b equal to 1.63. Therefore, the Z-R relationship obeys a power law.

In the theoretical work of **[4],** it was also shown the existence of an analytical relationship between the number of drops in precipitation and the pre-factor A of the Z-R power law. Indeed, they showed, assuming that the distribution is of gamma form, that if the variations of the pre-factor A are due only to the diameter of the drops (D_m in the case of the work of **[4])** then, the functional relationship between the number of drops per unit volume of air N_T and the pre-factor A is of a decreasing power form, with an exponent b equal to 1.63. Therefore, the Z-R relationship obeys a power law.

$$N_T = n_1 A^{n_2} \text{ with } n_2 < 0 \quad (7)$$

where n_1 and n_2 respectively denote the multiplicative factor and the exponent of the N_T – A law. They are determined by simple power regression between the parameters N_T and A. In Figure 7a, we present the fitting curves corresponding to the N_T – A relationship of the different percentage samples considered. All the values of the coefficients of this relationship relative to the different rain samples R≤1mm/h are confined in Table 7, as well as the corresponding correlation coefficients r. All of these values show that the number of drops N_T is well linked with the pre-factor A in view of the acceptable correlation coefficients. To this end, the analysis of Figure 7a clearly shows that the total number of drops per unit volume of air N_T decreases monotonically with the multiplicative factor A and this, independently of the percentage. We also note that the slope of the fitting curve increases when the percentage increases. In other words, the N_T parameter decreases rapidly as a function of the coefficient A of the Z-R law when the percentage of the spectra R≤1mm/h increases sharply in the rainy sample. Therefore, for a given value of the pre-factor A, a high number of drops in the low percentage R≤1mm/h spectra is necessary, while a lower number in the high percentage R≤1mm/h spectra would be sufficient. Such conditions suggest that low percentage are responsible for the high number of drops in precipitation, while high percentage are rather related to a lower number. This fact confirms that a good knowledge of the percentage of the rain spectra R≤1mm/h in any distribution can help to predict the average number of drops in precipitation and therefore, the shape of the measured DSD as suggested in the previous section. Indeed, like the special microphysical mode D_0 – A discussed above, this last result also suggests that by not taking into account the percentage of spectra R≤1mm/h in a rainy sample, additional errors would be generated in the estimation of the average number of drops, and therefore in the estimation of the coefficient A of the Z-R power relationship.

Table 7. Multiplicative factors and exponents of the functional relationships between the total number of drops per unit volume of air N_T of the DSD and the pre-factor A of the Z-R power law as well as the associated correlation coefficients r. The results are presented for different percentage classes of the low intensity rain spectra (R≤1mm/h)

Samples types	$N_T = n_1 A^{n2}$		
	log(n_1)	n_2	r
P(0 – 10)	8.9984	-0.7023	-0.63
P(10 – 20)	11.2209	-1.1146	-0.74
P(20 – 30)	11.6982	-1.2152	-0.80
P(30 – 40)	11.8789	-1.2654	-0.84
P(40 – 50)	11.7692	-1.2676	-0.86
P(50 – 60)	11.6619	-1.2711	-0.87
P(60 – 70)	11.0168	-1.1825	-0.86
P(70 – 80)	10.1593	-1.0577	-0.83
P(80 – 90)	9.2746	-0.9289	-0.76
P(90 – 100)	7.8243	-0.7527	-0.59

For example, to justify this, we use the same uncertainty value related to the estimation of the coefficient A, i.e. 30%, already indicated previously. Thus, for a value of A = 400, the value of N_T associated with the percentage P (10 – 20) is N_T = 94 m^{-3}. Assuming an uncertainty of 30% for the pre-factor A (i.e., 280 < A < 520), the total number of drops per unit volume of air N_T estimated can be 70 m^{-3} < N_T < 140 $m^{-3,}$ corresponding to an approximate uncertainty of 37% or ±35m^{-3}. However, for the same value of A = 400, another percentage such as P (80 – 90) gives N_T = 41m^{-3} with an approximate uncertainty of ±12.5m^{-3} (i.e., 32 m^{-3} < N_T < 57m^{-3}). This difference in the estimated values of the N_T parameter for the same value of the pre-factor A in precipitation further denotes the importance of taking into account the percentage of spectra R≤1mm/h in radar or satellite rainfall estimation algorithms.

This study is therefore of interest insofar as it allowed us, on the one hand, to show that there is a link between the number of drops (high or low) and the percentage of spectra R≤1mm/h in the rain sample (Low percentage: High number of drops; High percentage: Low number of drops) and on the other hand, to highlight the importance of this percentage in the relationship of dependence of the pre-factor A in N_T. To this end, it allows us to confirm the results of **[4]** and also to show that equation 7 has a real physical meaning only in homogeneous precipitation in terms of number of drops. In other words, the rain rate R seems this time to be controlled by the size of the drops. This result is in line with that found by **[36],** who showed that this special microphysical condition N_T – A is only applicable to homogeneous stratiform precipitation of low intensities, notably drizzle (R≤1mm/h, Z < 20dbZ). They also revealed that it is not linked to the process of coalescence or to the process of fractionation of drops but rather to the process of accretion of cloud droplets; which process allows to keep constant the number of drops already existing (whether it is high or low). However, the drops that benefit from the accretion effect simply grow larger at the expense of the others; thus varying the diameter of the drops in the rain sample. In the early works of **[37]** and **[38],** it was revealed that this microphysical mode is due to wind shear or turbulence. Other authors have not sought to link this N_T – A law to any process as the first authors cited did; nevertheless, they have devoted themselves to its consequences in precipitation. To this end, **[39]** suggested that such a microphysical mode can promote the disappearance of all convective cells. This supports the results of **[36]** and ours, insofar as the negative power N_T – A law seems to be generally applicable to non-convective rain samples, especially at low intensities. Furthermore, the strong dependence of the N_T – A power law on the percentage of spectra R≤1mm/h also contributes to the non-uniqueness of the Z-R power relationship in precipitation.

CONSISTENCY OF THE STUDY

To demonstrate the consistency of this study, we present in Figure 7b the scatter plot of the (N_T, D_0) pair of an event from our JW RD-69 database apparently characterized by the microphysical process for which the rain rate R varies as a function of the drop diameter (i.e., the number of drops is constant). This is the rain event of July 4, 1988 observed in Boyélé. According to our classification method, this event is considered a stratiform rain. Its essential characteristics are listed in Table 8. It contains 299 spectra, its maximum intensity is R_{max} = 3.27 mm/h, its Z-R relationship (determined by linear regression) is Z = 517$R^{1.53,}$ its percentage of spectra R≤1mm/h is P = 68.6%, its average diameter is <D_0> = 1.44mm and its average number of drops is <N_T> = 35m^{-3}. Knowing that the exponent b is independent of the percentage of spectra R≤1mm/h, replacing N_T by its average value of 35m^{-3} in the relationship $N_T = e^{11.0168}A^{-1.1825}$ (Table 7) corresponding to the percentage class of the event, we obtain: Z = 550$R^{1.53}$. It is noted that this relationship is in agreement with the Z-R relationship of the event. However, the pre-factor A (A = 550) is estimated with a low approximate uncertainty of 6% compared to the pre-factor A (A = 517) of the event studied.

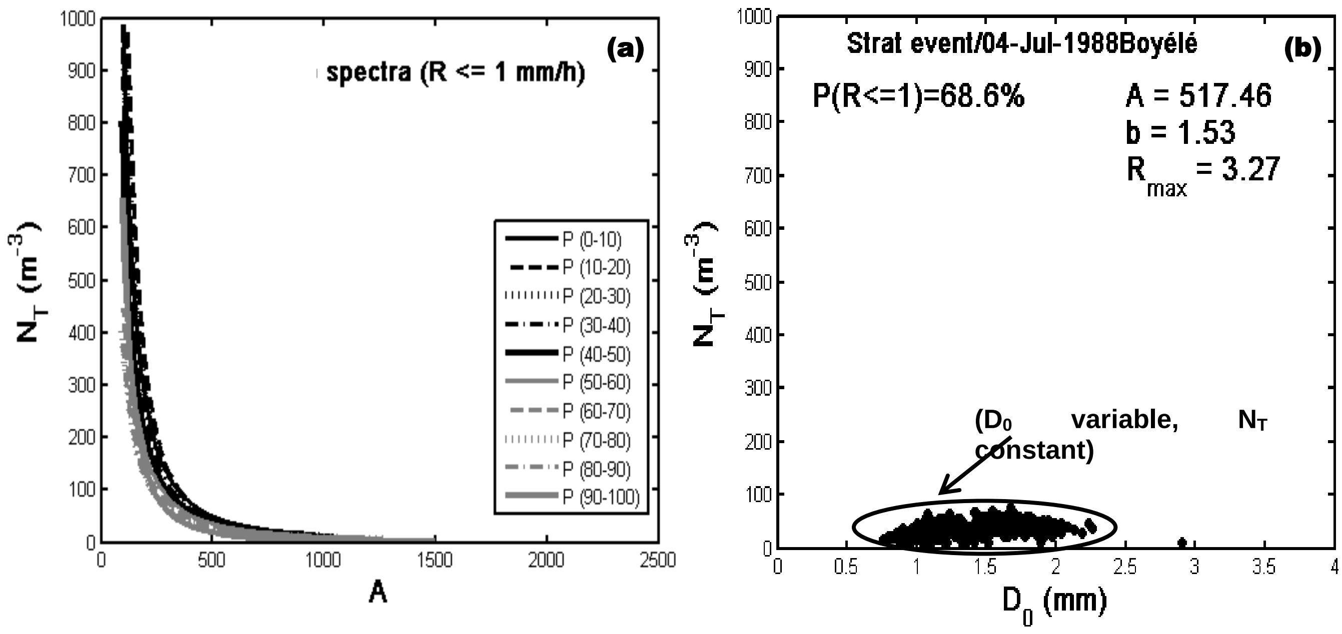

Fig. 7. (a) Functional relationships between the total number of drops per unit volume of air N_T of the DSD and the multiplicative factor A of the Z-R power law for different percentage samples of the low intensity rain spectra (R≤1mm/h). (b) Example of variability of raindrop size distribution (size and number) according to the N_T-A microphysical mode which can exist in precipitation: case of the stratiform rain of July 4, 1988 observed in Boyélé

Table 8. Example of a rain event illustrated by the specific microphysical mode N_T-A in accordance with the work of [4]. The statistical parameters of the DSD characteristics (size and number of drops), the Z-R relationship obtained by linear regression, the percentage of spectra R≤1mm/h, as well as the analytical coefficient A of the Z-R law of the event are also indicated

Evts/ (spectra number)	Z-R (observed event)		Drop diameter D_0 (mm)			Number of drops N_T (m^{-3})			Microphysical mode	P(R≤1) (%)	Coefficient A of the Z-R law (estimated)
	A	b	mean $<D_0>$	σ	cv	mean $<N_T>$	σ	cv			A
Strat event/ 04-Jul-1988 Boyele (299 spectra)	517	1.53	1.44	0.36	0.25	35.06	13.34	0.38	(D_0 variable, N_T constant)	68.6	550

3.2.3 3RD SPECIAL MICROPHYSICAL CONDITION: D0/NT – A AND A – A RELATIONS

It is very instructive to dwell on the theoretical work of **[4]** and to think that there really exists in precipitation a coherent relationship between the simultaneous involvement of the size and number of drops and the pre-factor A of the Z-R power law. Indeed, they showed for a modified gamma distribution, that if the mean mass diameter D_m and the total number of drops per unit volume of air N_T can vary together to influence the pre-factor A then, the functional relationship between the ratio $D_m^{(1+\mu)}/N_T$ and the pre-factor A is of an increasing power form, with an exponent b depending on the form factor μ of the modified gamma DSD [$b = (7+\mu)/(4.67+\mu)$]. Therefore, the Z-R relationship also obeys a power law.

In this study, the D_0/N_T ratio **[2]; [3]** obtained from observations is used instead of the theoretical ratio $D_m^{(1+\mu)}/N_T$. Thus, the functional relationship D_0/N_T – A that we obtained for the same percentage samples of spectra R≤1mm/h is indeed of the form increasing power:

$$D_0/N_T = r_1 A^{r_2} \text{ with } r_2 > 0 \quad (8)$$

where r_1 and r_2 represent the coefficients of the D_0/N_T – A law. They are obtained by power regression between the D_0/N_T and A parameters. In Figure 8a, we show the fitting curves corresponding to the D_0/N_T – A relationship of the different percentage samples considered. Table 9 shows all the values of the coefficients of this relationship, as well as the associated correlation coefficients r. As can be seen, the D_0/N_T ratio is well linked with the pre-factor A. To this end, the analysis of Figure 8a clearly shows that the D_0/N_T ratio increases rapidly and monotonically with the multiplicative factor A and this, independently of the percentage. Like the number of drops N_T, we also note that the slope of the fitting curve increases when the percentage increases. In other words, the D_0/N_T ratio increases rapidly as a function of the coefficient A of the Z-R law when the percentage of the spectra R≤1mm/h increases sharply in the rainy sample. Thus, for a given value of pre-factor A, a low D_0/N_T ratio in low percentage R≤1mm/h spectra is necessary, while a higher D_0/N_T ratio in high percentage R≤1mm/h spectra would be sufficient. This situation suggests that the low values of the D_0/N_T ratio observed in precipitation and more precisely in the work of **[2]** are due to the low percentage of spectra R≤1mm/h, while the high values are due to the high percentage. All this confirms the essential nature of the percentage of rain spectra R≤1mm/h in any distribution and suggests (like the first two special microphysical modes treated) that by not taking into account the percentage of spectra R≤1mm/h in a rain sample, we would undoubtedly generate additional errors in the estimation of the average D_0/N_T ratio, therefore in the estimation of the coefficient A of the Z-R power relationship.

Table 9. Multiplicative factors and exponents of the functional relationships between the D_0/N_T parameter of the DSD and the pre-factor A of the Z-R power law as well as the associated correlation coefficients. The results are presented for different percentage classes of low intensity rain spectra (R≤1mm/h)

Samples types	$D_0/N_T = r_1A^{r_2}$		
	log(r_1)	r_2	r
P(0 – 10)	-9.4115	0.8196	0.67
P(10 – 20)	-12.5142	1.4129	0.89
P(20 – 30)	-12.4512	1.4329	0.89
P(30 – 40)	-12.0852	1.3944	0.86
P(40 – 50)	-11.2141	1.2674	0.81
P(50 – 60)	-10.7063	1.1949	0.78
P(60 – 70)	-10.1587	1.1132	0.75
P(70 – 80)	-9.4666	1.0089	0.71
P(80 – 90)	-8.8506	0.9156	0.66
P(90 – 100)	-7.9261	0.8061	0.61

To illustrate this, we proceed as before, i.e. using the same uncertainty value related to the estimation of coefficient A. For example, for a value of A = 400, the value of D_0/N_T obtained for the percentage P (10 – 20) is D_0/N_T = 0.0174mm.m^3. Assuming an uncertainty of 30% for the pre-factor A (i.e., 280 < A < 520), the estimated D_0/N_T ratio can be 0.0105mm.m^3 < D_0/N_T < 0.0252mm.m^3, corresponding to an approximate uncertainty of 42% or ±0.00735mm.m^3. However, for the same value of A = 400, the percentage P (80 – 90) gives D_0/N_T = 0.0345mm.m^3 with an approximate uncertainty of ±0.0095mm.m^3 (i.e., 0.0249mm.m^3 < D_0/N_T < 0.0439mm.m^3). This difference in the estimated values of the D_0/N_T ratio for the same value of the pre-factor A in precipitation further reveals the interest of taking into account the percentage of spectra R≤1mm/h in radar or satellite rainfall estimation algorithms. Moreover, despite the fact that the microphysical interpretation of the coefficients A and b of the Z-R power law is a prolific subject for the scientific community, we could not find in the literature a microphysical process (coalescence, fractionation among many others) associated with this D_0/N_T – A law (or with this mode of variability of the coefficient A). Indeed, this study, which requires radar observations of the vertical profile of the rain, simultaneous with disdrometer measurements, is not addressed in this work. However, the strong dependence of the D_0/N_T – A law on the percentage of the spectra R≤1mm/h also leads to the non-uniqueness of the Z-R relationship.

CONSISTENCY OF THE STUDY

To demonstrate the consistency of this study, we show in Figure 8b the scatter plot of the (N_T, D_0) pair of an event in our JW RD-69 database apparently characterized by the microphysical mode for which the rain rate R depends on the simultaneous variations of the diameter of the drops and their number. This is the rain event of August 7, 1998 observed in Dakar. According to our classification method, this event is considered as a convective rain. Its essential characteristics are listed in Table 10. It contains 234 spectra, its maximum intensity is R_{max} = 127 mm/h, its Z-R relationship (determined by linear regression) is $Z = 410R^{1.26}$, its percentage of spectra R≤1mm/h is P = 41.9%, its average diameter is $\langle D_0 \rangle$ = 1.55mm and its average number of drops is $\langle N_T \rangle$ = 110m^{-3}. Knowing that the exponent b is independent of the ratio of the spectra R≤1mm/h, by replacing D_0/N_T by its average value of 0.0948mm.m^3 in the relation $D_0/N_T = e^{-11.2141}A^{1.2674}$ (see Table 9) corresponding to the percentage class of the event, we obtain: $Z = 1084R^{1.26}$. It is noted that this relationship is in disagreement with the Z-R relationship of the event because the pre-factor A is overestimated by the D_0/N_T – A law. This could be explained by the difference from a dimensional point of view between the theoretical ratio $D_m^{(1+\mu)}/N_T$ and the observation ratio D_0/N_T used to conduct this study. To this end, it is noted that the observation ratio D_0/N_T has the dimension [L^4] while that of the theoretical ratio $D_m^{(1+\mu)}/N_T$ deduced from the work of **[4]** is [$L^{(4+\mu)}$]. All this shows that the observation ratio D_0/N_T taken individually is not suitable for conducting this study since it has difficulties in integrating the form of the observed DSD (μ represents the form factor of the modified gamma theoretical DSD).

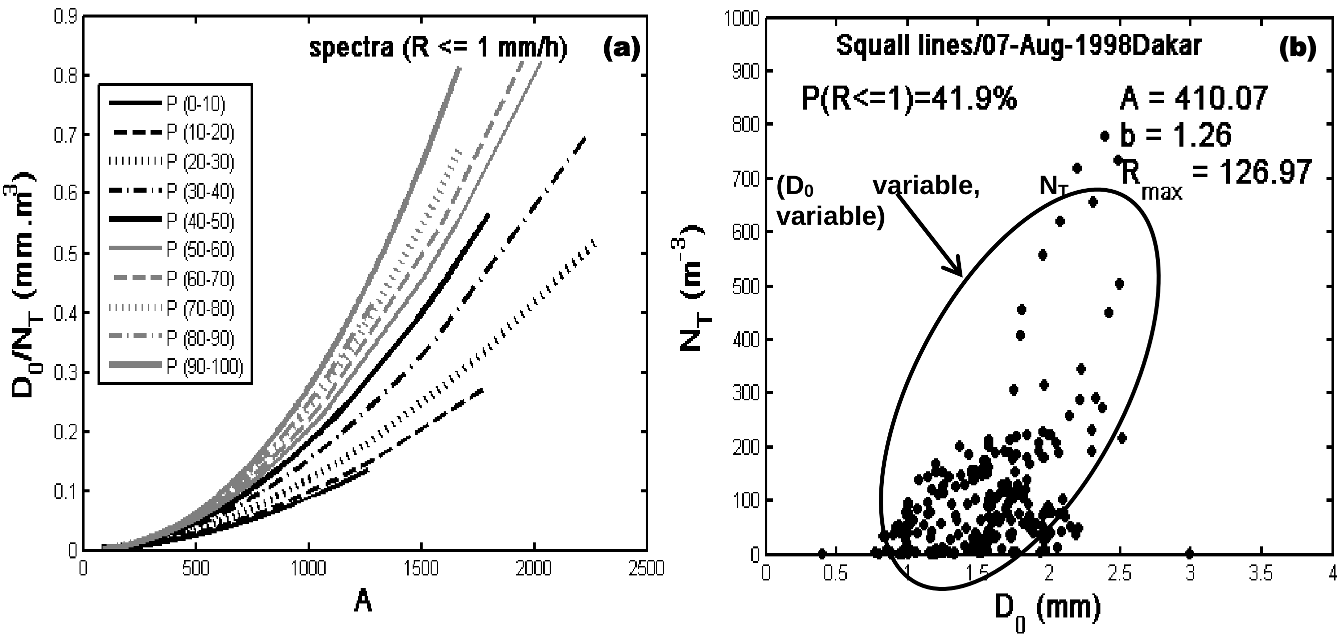

Fig. 8. ***(a) Functional relationships between the D_0/N_T parameter of the DSD and the multiplicative factor A of the Z-R power law for different percentage samples of low intensity rain spectra (R≤1mm/h). (b) Example of variability of raindrop size distribution (size and number) according to the D_0/N_T-A microphysical mode which can exist in precipitation: case of the squall line of August 7, 1998 observed in Dakar***

Table 10. Example of a rain event illustrated by the specific microphysical mode D_0/N_T-A in accordance with the work of [4]. The statistical parameters of the DSD characteristics (size and number of drops), the Z-R relationship obtained by linear regression, the percentage of spectra R≤1mm/h, as well as the analytical coefficient A of the Z-R law of the event are also indicated

Evts/ (spectra number)	Z-R (observed event)		Drop diameter D_0 (mm)			Number of drops N_T (m^{-3})			Microphysical mode	P(R≤1) (%)	Coefficient A of the Z-R law (estimated)
	A	b	mean <D_0>	σ	cv	mean <N_T>	σ	cv			A
SL/ 07-Aug-1998 Dakar (234 spectra)	410	1.26	1.55	0.42	0.27	109.83	127.61	1.16	(D_0 variable, N_T variable)	41.9	1084

Thus, our choice fell on the power law A-α **[2]; [3]** instead of the law D_0/N_T – A to represent this microphysical mode where the rain rate R depends on the mixed variations of the size and number of drops. Indeed, the functional relationship $D_0/N_T=\alpha R^{\beta}$ **[2]; [3]** is such that the shape of the distribution seems to be integrated into the pre-factor α of said relationship because it has been shown in many previous works **[7]; [9]; [40]; [18]** that the shape of the DSD varies with the rain rate R. In Figure 9, we show the regression curves of the A-α relationship of the 10 samples of percentage of rain spectra R≤1mm/h considered. As can be seen from the graph, the shape of the fitting curves corresponding to the A-α relationship is identical to that found by **[2]** and **[3]** and this, for all the samples studied. In other words, the multiplicative factor A of the Z-R relation is well linked with the pre-factor α of the D_0/N_T – R relation. The high correlation coefficients, greater than 0.80, clearly indicate the good correlation between the parameters A and α. All the values of the coefficients of this relationship relative to the different samples of percentage of the rain spectra R≤1mm/h are confined in Table 11. The analysis of the values (orders of magnitude) of the coefficients a_1 and a_2 shows that they are high compared to those obtained in the work of **[2]** where the calculations are made independently of the intensity classes. This recalls the results obtained above and confirms that the rain spectra R≤1mm/h contribute to a rapid increase in the pre-factor A of the power law Z-R in precipitation. However, the values of the coefficients a_1 and a_2 corresponding to the rain sample P (90 – 100) deviate from those obtained for the other rain samples R≤1mm/h. This could be due to a sampling problem **[6].**

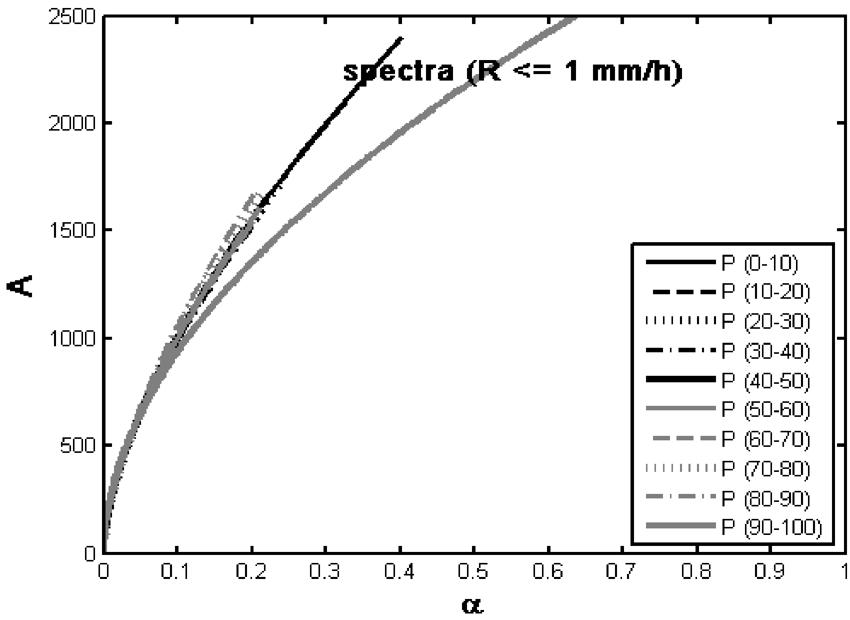

Fig. 9. ***Functional relationships between the multiplicative factors of the power laws Z-R ($Z=AR^b$) and D_0/N_T-R ($D_0/N_T = \alpha R^\beta$) for different percentage samples of the low intensity rain spectra R≤1mm/h considered***

Table 11. Coefficients of the relationship between the multiplicative factors of the power laws Z-R ($Z=AR^b$) and D_0/N_T-R ($D_0/N_T = \alpha R^\beta$) for the different samples of low intensity rainfall R≤1mm/h considered

Sample types	$A=a_1\alpha^{a_2}$		
	a_1	**a_2**	r
P(0 – 10)	4219.0	0.6203	0.89
P(10 – 20)	4208.7	0.6197	0.94
P(20 – 30)	4174.6	0.6216	0.94
P(30 – 40)	4113.8	0.6183	0.94
P(40 – 50)	4248.2	0.6261	0.95
P(50 – 60)	4185.5	0.6227	0.95
P(60 – 70)	4263.8	0.6263	0.95
P(70 – 80)	4516.8	0.6397	0.95
P(80 – 90)	4656.5	0.6436	0.94
P(90 – 100)	3177.4	0.5303	0.85

To illustrate the effectiveness of this law in representing this mode of variability of the Z-R relationship, we use the same rain event of August 7, 1998 observed in Dakar. Its granulometric characteristics were given above. The coefficient α of the power law D_0/N_T – R of the event is 0.0288. Knowing that the exponent b is independent of the percentage of the spectra R≤1mm/h, by replacing the coefficient α by its value in the relationship $A = 4248.2\alpha^{0.6261}$ (see Table 11) corresponding to the percentage class of the event, we obtain: $Z = 461R^{1.26}$. We note that this relationship is in line with the Z-R relationship of the event because the pre-factor A is estimated with a relatively small difference of 12%. This confirms the effectiveness of the A – α relationship **[2]; [3]** in formalizing the approach on the simultaneous involvement of the size and number of drops in the variability of the Z-R power relationship.

4 CONCLUSION

This paper investigated the role of the percentage of low intensity rain spectra (R≤1 mm/h) in the variability of the Z-R relationship in relation to the microphysical processes responsible for droplet formation.

The analysis of the variability of the coefficients A and b of the Z-R relationship, in relation to the percentage of low intensity rain spectra (R≤1 mm/h), showed that rain spectra (R≤1 mm/h) lead to high values of the coefficients of the Z-R relationship. However, the variability of the multiplicative factor A is pronounced, in agreement with that of the N_T and D_0/N_T parameters of the DSD; while the exponent (b) remains almost constant in such rain samples, regardless of the percentage. Such a result suggests that the choice of the exponent (b) should be avoided in this study, and only the pre-factor (A) should be adapted according to the percentage considered.

Following a theoretical study, we formalized the link between the observation parameters D_0, N_T and D_0/N_T of the DSD and the pre-factor A of the Z-R power law. We find exactly close power relationships D_0 – A, N_T – A and D_0/N_T – A (for all the rain samples R≤1mm/h considered) corresponding respectively to the microphysical modes "constant D_0, variable N_T", "variable D_0, constant N_T" and "variable D_0, variable N_T". The analysis of these three special microphysical conditions showed that the low percentage (P (R≤1mm/h) < 50%) of the R≤1mm/h spectra are at the origin of a high number of large drops corresponding to a low D_0/N_T ratio in the precipitation. On the other hand, the few small drops observed in precipitation with a high D_0/N_T ratio are due to high percentage (P (R≤1mm/h) ≥ 50%).

We analyzed the consistency of this study by choosing three rain events from our database, namely the squall line of September 10, 1987 observed in Abidjan ($Z=184R^{1.12}$), the stratiform rain of July 4, 1988 observed in Boyélé ($Z=517R^{1.53}$) and the squall line of August 7, 1998 recorded in Dakar ($Z=410R^{1.26}$), corresponding respectively to the special microphysical modes D_0 – A, N_T – A and D_0/N_T – A. The estimation of the pre-factor A of the Z-R relationship associated with each event from the aforementioned equations (in the same order) in relation to the percentage of the spectra R≤1mm/h, gives as results A=199, A=550 and A=1084. If the first two modes (D_0 – A and N_T – A) give satisfactory results in the estimation of the pre-factor A with respectively low deviations of 8% and 6%, the D_0/N_T – A mode, for its part, enormously overestimates the pre-factor A of the squall line of August 7, 1998 observed in Dakar.

It has been suggested that this disagreement between the observed (A=410) and estimated (A=1084) values of this rainfall event would be related to the difference from a dimensional point of view between the theoretical ratio $D_m^{(1+\mu)}/N_T$ [$L^{(4+\mu)}$] (μ is the shape factor of the gamma DSD) of Steiner et al. (2004) and the observed ratio D_0/N_T [L^4] used in this study. To overcome this inability of the D_0/N_T ratio to integrate the shape of the observed DSD, we chose the A – α relationship to represent this mode of variability of the DSD. Thus, the pre-factor of the Z-R relationship estimated from this A – α law is A=461. This value is close to that of the rain event with a small difference of 12%.

Acknowledgments

The authors are grateful to all of those who contributed to the collecting of JWD data used in this study, under the lead of Prof. Henri Sauvageot from the Atmospheric Research Center (Toulouse, France).

Conflicts Of Interest

There are no conflicts of interest in this work, and the authors agree to the publication of this article in this journal.

References

[1] Marshall, J. S., and W. M. K. Palmer (1948). The distribution of raindrops with size. *J. Meteor.,* 5, 165-166.

[2] Ochou A. D., Zahiri E-P., Bamba B., Koffi M. (2011). Understanding the variability of Z-R relationships caused by natural variations in raindrop size distributions (DSD): Implication of drop size and number. *Atmospheric and Cimate Sciences,* 1, 147-164.

[3] Bamba, B., A.D. Ochou, E.-P. Zahiri, and M. Kacou (2014). Consistency in Z-R Relationship Variability Regardless Precipitating Systems, Climatic Zones Observed from Two Types of Disdrometer. *Atmospheric and Climate Sciences,* 4, 941-955.

[4] Steiner, M., Smith, A. J., Uijlenhoet, R. (2004). A Microphysical Interpretation of Radar Reflectivity-Rain Rate Relationships. *Journal of the Atmospheric Sciences,* Vol. 61, No.10, pp. 1114-1131.

[5] List, R. (1988). A linear radar reflectivity-rainrate relationship for steady tropical rain. *J. Atmos. Sci.,* 45, 3564-3572.

[6] Uijlenhoet, R., M. Steiner, and J. A. Smith (2003b). Variability of raindrop size distributions in a squall line and implications for radar rainfall estimation. *J. Hydrometeor.,* 4, 43-61.

[7] Sauvageot, H., and J.P. Lacaux (1995). The shape of averaged drop size distributions. *J. Atmos. Sci.,* 52, 1070-1083.

[8] Yuter, S. E., and R.A. Houze (1997). Measurements of raindrop size distribution over the Pacific Warm Pool and implementations for Z-R relations. *J. Appl. Meteor.,* 36, 847-867.

[9] Maki, M., Keenam, T. D., Sasaki, Y., and Nakamura, K. (2001). Characteristics of the raindrop size distribution in tropical continental squall lines observed in darwin, australia. *J. Appl. Meteor.* 40, 1393, 1412.

[10] Islam, T., Rico-Ramirez, A., M., Thurai, M., Han, D. (2012). Characteristics of raindrop spectra as normalized gamma distribution from a Joss-Waldvogel disdrometer. *Atmospheric Research,* 108, 57-73.

[11] Joss, J., and A. Waldvogel (1967). Ein Spektrograph für Niederschlagstropfen mit automatischer Auswertung. *Pure Appl. Geophys.,* 68, 240-246.

[12] Williams, R.C., A. Kruger, K. S. Gage, A. Tokay, R. Cifelli, W. F. Krajewski and C. Kummero (2000). Comparison of Simultaneous Rain Drop Size Distributions Estimated from Two Surface Disdrometers and a UHF Profiler. *Geophysical Research Letters,* Vol. 27, N° 12, Pages 1763-1766.

[13] Sheppard, B.E., and P.I. Joe (1994). Comparison of raindrop size distribution measurements by a Joss-Waldvogel Disdrometer, a PMS 2DG Spectrometer, and a POSS Doppler Radar. *J. Atmos. and Oceanic Technol.,* 10, 874-887.

[14] Joss, J., and A. Waldvogel (1969). Raindrop size distribution and sampling size errors. *J. Atmos. Sci.,* 26, 566-569.
[15] Sheppard, B. E. (1990). Effect of irregularities in the diameter classification of raindrops by the Joss-Waldvogel disdrometer. J. Atmos. Oceanic Technol., 7, 180-183.
[16] Lee, G.W., and I. Zawadzki (2005). Variability of drop size distributions: Time-scale dependence of the variability and its effects on rain estimation. *J. Appl. Meteor.,* 44, 241-255.
[17] Moumouni, S., M. Gosset, E. Houngninou (2008). Main features of rain drop size distributions observed in Benin, West Africa, with optical disdrometers. *Geophys. Res. Lett.,* 35, L23807, DOI: 10.1029/2008GL035755.
[18] Tenório, R.S., M.C.S. Moraes, and H. Sauvageot (2012). Raindrop Size Distribution and Radar Parameters in Coastal Tropical Rain Systems of Northeastern Brazil. *J. Appl. Meteor. Climat.,* 51, 1960-1969.
[19] Testud, J., Oury, S., Black, R. A., Amayenc, P., and Dou, X. K. (2001). The concept of normalized distribution to describe raindrop spectra: A tool for cloud physics and cloud remote sensing. *J. Appl. Meteor.,* 40, 1118-1140.
[20] Chapon, B., Delrieu, G., Gosset, M., Boudevillain, B. (2008). Variability of rain drop size distribution and its effect on the Z-R relationship: A case study for intense Mediterranean rainfall. *Atmospheric Research,* 87, 52-65.
[21] Joss, J., and E.G. Gori (1978). Shapes of Raindrop Size Distributions. *J. Appl. Meteorol.,* 17, 1054-1061.
[22] Ochou, A. D., Nzeukou, A., Sauvageot, H. (2007). Parametrization of drop size distribution with rain rate. *Atmospheric Research,* 84, 58-66.
[23] Sekhon, R.S., and R.C. Srivastava (1971). Doppler radar observations of drop-size distributions in a thunderstorm. *J. Atmos. Sci.,* 28, 983-994.
[24] Valdez, M.P., and K.C. Young (1985). Number fluxes in equilibrium raindrop populations. A Markov-chain analysis. *J. Atmos. Sci.,* 42, 1024-1036.
[25] List, R., N.R. Donaldson, and R.E. Stewart (1987). Temporal evolution of drop spectra to collisional equilibrium in steady and pulsating rain. *J. Atmos. Sci.,* 44, 362-372.
[26] Brown, P.S., and S.N. Whittlesey (1992). Multiple equilibrium solutions in Bleck-type models of drop coalescence and breakup. *J. Atmos. Sci.,* 49, 2319-2324.
[27] Hu, Z., and R.C. Srivastava (1995). Evolution of raindrop size distribution by coalescence, breakup, and evaporation: Theory and observations. *J. Atmos. Sci.,* 52, 1761-1783.
[28] Tenório, R.S., M. C. S. Moraes, D. A. Quinta˜o, and B. H. Kwon (2003b). Estimation of the Z–R relation through the disdrometer for the coastal region in the northeast of Brazil. J. Kor. Earth Sci. Soc., 24, 30–35.
[29] Nzeukou, A., Sauvageot, H., Ochou, A. D., Kebe, C. M. F. (2004). Raindrop size distribution and radar parameters at Cape Verde. *J. Appl. Meteorol.,* 43, 90-105.
[30] Sharma, S., M. Konwar, and D.K. Sarma (2009). Characteristics of Rain Integral Parameters during Tropical Convective, Transition, and Stratiform Rain at Gadanki and its Application in Rain Retrieval. *American Meteorological Society,* 48, 1245-1266.
[31] Tokay, A., and Short, D. A. (1996). Convective vs stratiform rain in the West pacific during TOGA COARE: evidence from raindrop spectra. *J. Appl. Meteor.,* 35, 355-371.
[32] Zahiri, E.-P., M. Gosset, J.-P. Lafore, and V. Gouget (2008). Use of a Radar Simulator on the Output Fields from a Numerical Mesoscale Model to Analyse X-Band Rain Estimators. *Journal Atmospheric And Oceanic Technology,* 25, 341-367.
[33] Doelling, I. G., J. Joss, and J. Riedl (1998). Systematic variations of Z-R relationships from drop size distributions measured in northern Germany during seven years. *Atmos. Res.,* 47-48, 635-649.
[34] Steiner, M., and J.A. Smith (2000). Reflectivity, rain rate, and kinetic energy flux relationships based on raindrop spectra. *J. Appl. Meteor.,* 39, 1923-1940.
[35] Hagen, M., and S.E. Yuter (2003). Relations between radar reflectivity, liquid-water content, and rainfall rate during the MAP SOP. *Quart. J. Roy. Meteor. Soc.,* 129, 477-493.
[36] Rogers, R.R., I.I. Zawadzki, and E.E. Gossard (1991). Variation with altitude of the drop-size distribution in steady light rain. *Quart. J. Roy. Meteor. Soc.,* 117, 1341-1369.
[37] Gunn, K.L.S., and J.S. Marshall (1955). The effect of wind shear on falling precipitation. *J. Meteor.,* 12, 339-349.
[38] Atlas, D., and A.C. Chmela (1957). Physical-synoptic variations of raindrop size parameters. *Proc. Sixth Weather Radar Conf.,* Cambridge, MA, *Amer. Meteor. Soc.,* 21-29.
[39] Carbon, R.E., and L.D. Nelson (1978). The evolution of raindrop spectra in warm-based convective storms as observed and numerically modeled. *J. Atmos. Sci.,* 35, 2302-2314.
[40] Kozu T., K.K. Reddy, S. Mori, M. Thurai, J.T.Ong, D.N. Rao, and T. Shimomai (2006). Seasonal and diurnal variations of raindrop size distribution in Asian monsoon region. *Meteorological Society of Japan,* Vol. 84A, pp. 195-209.