International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 8 No. 2 Sep. 2014, pp. 200-209

http://www.ijisr.issr-journals.org/

Arithmetic and Logical Models of Stranded Transmission Line Conductors for Voltage and Voltage-Drop Analysis

Abdul Rasak Zubair and Adeboye Olatunbosun

Department of Electrical/Electronic Engineering,
University of Ibadan,
Ibadan, Oyo State, Nigeria

ABSTRACT: Transmission lines are used for transportation of energy from the point of generation to the point of usage. A transmission line may be single-core conductor or stranded conductor. Stranding reduces line reactance, skin effect and the tendency of occurrence of corona effect. Transmission line parameters such as inductance and capacitance depend on the geometric mean radius (GMR) of the line. GMR is essential for Voltage and Voltage-Drop analysis which is necessary to ensure save operation of power system. Arithmetic and logical models of triangular, hexagonal, circular and sector shaped stranded conductors are presented. The models which accurately predict the Cartesian coordinates of centers of strands are coded into computer programs which facilitate both the accurate graphical drawing and accurate computation of GMR of stranded conductors.

KEYWORDS: Power Transmission, Stranded Conductors, Geometric Mean Radius, Line Parameters, Electric Field, Magnetic Field, Geometry.

1 INTRODUCTION

Energy is required by man to carry out useful work which is essential in his day to day activities. The electric power system is one of the tools for converting and transporting energy which plays an important role in meeting the daily energy demand of man.

An Electric power system consists of three main sub-systems: the generating sub-system, the transmission sub-system and the distribution sub-system [1,2,3]. It cost a lot of money to install any of these sub-systems. Electricity itself is a useful but dangerous thing to leave with, so a lot is at stake in an electric power system. Every care must be taken to ensure safe operation so as to prevent damages of equipment and loss of life.

To design and operate an Electric power system, the various types of calculations that have to be carried out to ensure safe operation may be grouped under the following three headings [1,2,3].

(i) Voltage and voltage-drop calculations: to ensure that the voltage at specified points remain within appropriate limits.
(ii) Load-flow calculations: to ensure that the current in the various branches of a network does not exceed a safe working limit.
(iii) Fault calculations: to determine current and voltage under abnormal conditions, such as short-circuit, in order to ensure safety, also to select suitable fuses or circuit breakers or to set protective gear.

Energy is usually preferred to exist in electrical form due to the ease of transportation of electrical energy and the ease of converting electrical energy to other forms of energy. The electrical energy cannot always be produced where needed. There is, therefore, the need for transportation from the place of production to the place of utilisation. This is where electrical power transmission and transmission lines come in [1,2,3].

Transmission lines are basically electricity conductors made of aluminum in most cases. Copper conductors are sometimes but very rarely used. This is because, despite the fact that copper has almost twice the conductivity of aluminum, aluminium has a weight and price advantage over copper. For equivalent resistance and weight, aluminum has larger diameter and hence lower electric field intensity and consequently a less chance of corona occurrence compared with copper.

There are overhead transmission lines as well as under-ground cables. A conductor may be single-core type or of multiple-strand type. The latter is called stranded conductor.

Around every current-carrying conductor, there are both electric field ($\overline{E}$) and magnetic field ($\overline{B}$) as illustrated in Fig. 1 [1,2,3,4]. Conductors made of same material connected to the same voltage source will have the same number of lines of electric flux. Consider two conductors having equal cross-sectional areas, made of same material; one of which is single-core and the other is multiple-strand. The surface area of the multiple-strand type is larger than that of the single-core type. This implies a lower voltage gradient, electric field intensity and hence less tendency to ionize the surrounding air at the surface of the multiple-strand type than at the surface of the single-core type. Thus stranding reduces the tendency of occurrence of corona effect [1,2,3,5,6]. Corona effect is undesirable in that it results in energy loss and communication interference. Stranding also reduces line reactance and skin effect [1,2,3,7,8]. To further reduce tendency of occurrence of corona effect, bundled conductors are used at very high voltage levels. Two, three or four conductors are said to be bundled together if they serve as a single-phase of a power system.

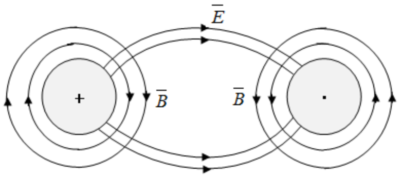


Fig. 1. Magnetic and Electric Fields Associated with Conductors

The basic line constants are inductance, capacitance and resistance per unit length. These constants are essential in voltage and voltage-drop calculations and they can be computed from the dimensions of the line [1,2,3,9,10,11,12].

A current carrying conductor is surrounded by a magnetic field ($\overline{B}$) as shown in Fig. 1. This field is due to the current flowing in the conductor. The field links with the conductor and results in flux linkage. The flux linkage gives rise to reactive voltage drop along the conductor. The flux linkage, F is proportional to the current I in the conductor. That is $F = LI$ (1)

The proportionality constant L is the self-inductance of the conductor. The reactive voltage drop is given as $V_{drop} = \frac{\partial NF}{\partial t}$.

N is the number of turns linked by the flux. F_e is the external flux linkage at distance D from the one-metre long cylindrical conductor shown in Fig. 2, F_i is the internal flux linkage, r is the radius of the conductor and μ is the permeability of the space around the conductor. F_e and F_i [1,2,3] are given as

$$F_i = \frac{\mu I}{8\pi} = \frac{\mu I}{2\pi} \ln\left(e^{1/4}\right) \qquad (2)$$
$$F_e = \frac{\mu I}{2\pi} \ln\left(\frac{D}{r}\right)$$

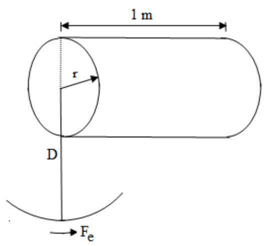


Fig. 2. A Cylindrical Conductor External Flux

At distance d from the conductor, total flux linkage [1,2,3] is therefore given by

$$F_t = F_i + F_e = \frac{\mu I}{2\pi} \ln\left(e^{1/4}\right) + \frac{\mu I}{2\pi} \ln\left(\frac{\mathcal{D}}{r}\right) = \frac{\mu I}{2\pi} \ln\left(\frac{\mathcal{D}}{e^{-1/4} r}\right) = \frac{\mu I}{2\pi} \ln\left(\frac{\mathcal{D}}{GMR}\right) \tag{3}$$

Where $GMR = e^{-1/4} r$ and is called the geometric mean radius (GMR) or self geometric distance (Self GMD) of the conductor. GMR accounts for the presence of internal flux linkages. It is the radius of a fictitious conductor assumed to have no internal flux but with the same total flux as the actual conductor of radius r. Comparing equation (1) and (3) gives

$$L = \frac{\mu}{2\pi} \ln\left(\frac{\mathcal{D}}{GMR}\right) \tag{4}$$

Thus line inductance depends on the geometric mean radius of the conductor. Line capacitance also depends on geometric mean radius (GMR). Evaluation of geometric mean radius is therefore very important for voltage and voltage-drop calculations [1,2,3].

The Geometric Mean Radius (GMR) is a measure of line inductance and line capacitance. For a single core conductor, GMR is the radius of the conductor multiplied with $e^{-¼}$. For a stranded conductor containing N strands, the GMR is N^2- root of the product of each strand's GMR and distances from each strand to other strands [1,2,3]. This is expressed mathematically as

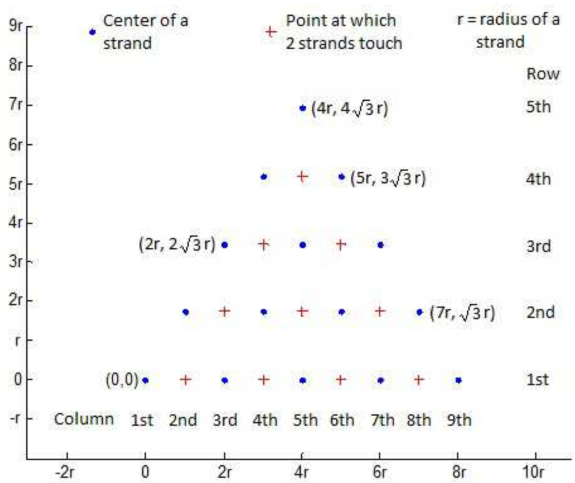
$$GMR = \sqrt[N^2]{(e^{-1/4} r)^N \text{ x } D_1 \text{ x } D_2 \text{ x } D_3 \text{ x} \ldots \text{x } D_N} \tag{5}$$

r is the radius of each strand. D_k is the product of distances between the k^{th} strand and other strands. Distance between two strands is the distance between their centers. To compute the GMR of a conductor, its geometric configuration must be known. In this paper, arithmetic and logical models are developed for the prediction of the Cartesian coordinates of the centers of strands in a stranded conductor and automatic computation of GMR of the conductor. Furthermore, the model facilitates graphical drawing of stranded conductors. Hexagonal, circular, triangular and sector shaped stranded conductors are considered.

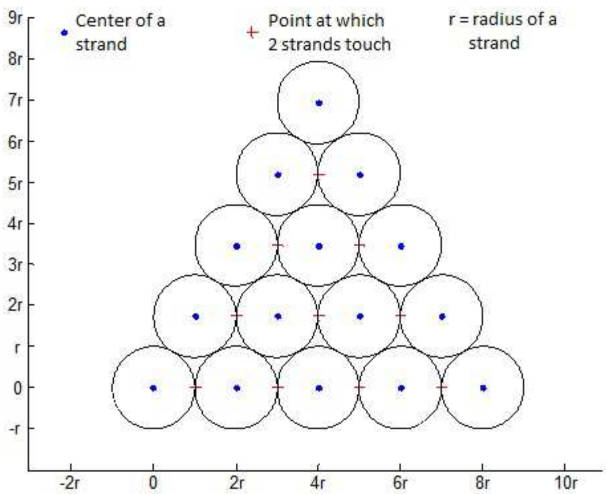
2 Arithmetic and Logical Models of Stranded Conductors

2.1 Triangular Shaped Stranded Conductor

A careful study of triangular shaped stranded conductors reveals some pattern with regard to locations of centers of strands as illustrated in Fig. 3 and Table 1. The centers of strands are located at points along w rows and c columns. N is the number of strands in the stranded conductor. s is serial number and r is the radius of each strand.



(a) Centers of Strands



(b) Strands

Fig. 3. Triangular Shaped Stranded Conductor with s = 5, N = 15, w = 5 and c = 9

Based on the sequence in Table 1, N and s are found to be related as in Eqns. (6) and (7). Similarly, w and c are found to be related to s as in Eqns. (8) and (9) respectively. A center of a strand is at the intersection of a row and a column but not all intersections of rows and columns are centers of strands. Fig. 3 is drawn for s=5, N=15, w=5 and c=9. In Fig. 3, blue dots indicate intersections which are the centers of strands while red plus symbols indicate intersections which are not centers of strands but points at which strands on the same rows meet tangentially. Two adjacent intersections along a row or a column cannot be centers of strands. The distance between two adjacent intersections along a row is r while the distance between two adjacent intersections along a column is $\sqrt{3}r$ The number of intersections along a row reduces by two as you move from one row to the next. The first intersection on the first row and first column is a center of a strand and has the Cartesian coordinate (0, 0). All these observations are used to develop the flowchart of Fig. 4 to compute the Cartesian coordinates of the centers of the strands. The output, Center is an N by 2 matrix containing x and y coordinates of the centers. The distance between two centers (x_1, y_1) and (x_2, y_2) is given as in Eqn. (10). The N by 2 matrix Center facilitates both the graphical drawing of the stranded conductor and the computation of the GMR based on Eqns. (5) and (10).

Table 1. Triangular Shaped Stranded Conductors data

Serial Number (s)	Number of Strands (N)	Number of Rows (w)	Number of Columns (c)
1	1	1	1
2	3	2	3
3	6	3	5
4	10	4	7
5	15	5	9

$$N = s(s+1)/2 = 0.5s^2 + 0.5s \quad (6)$$

$$s = -0.5 + \sqrt{0.25 + 2N} \quad (7)$$

$$w = s \quad (8)$$

$$c = 2s - 1 \quad (9)$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \quad (10)$$

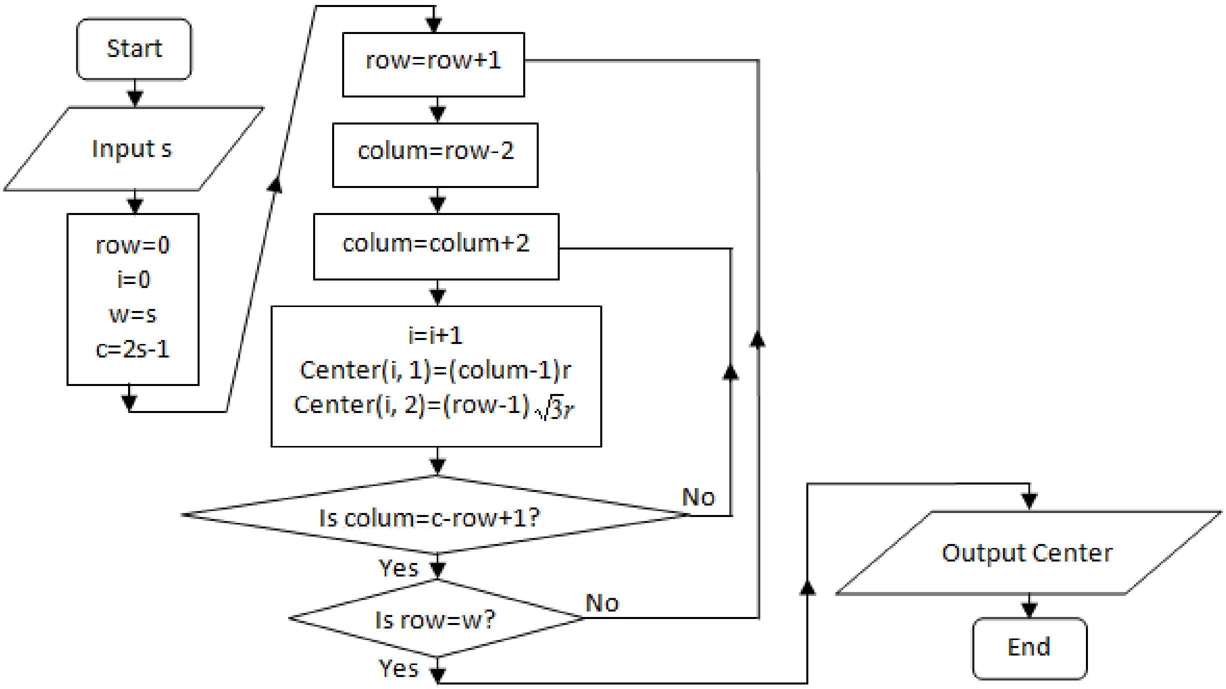
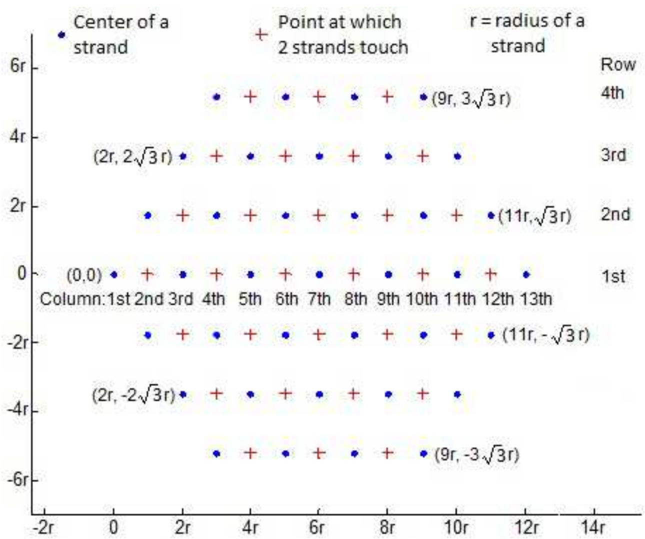


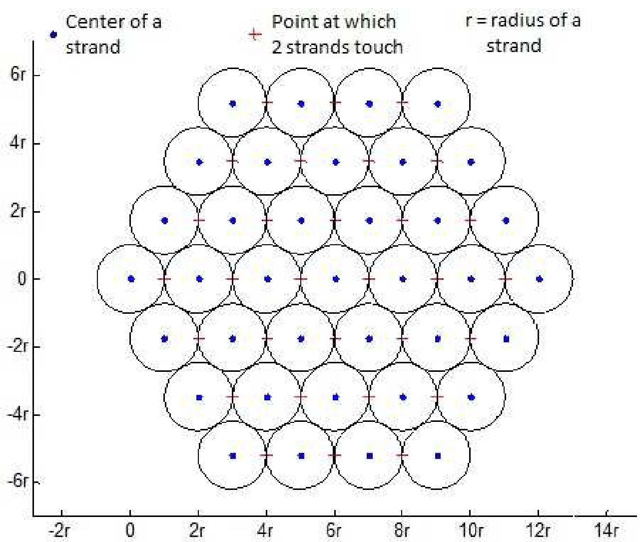
Fig. 4. Flowchart for the Arithmetic and Logical Model of Triangular Shaped Stranded Conductor

2.2 HEXAGONAL SHAPED STRANDED CONDUCTOR

A careful study of hexagonal shaped stranded conductors reveals some pattern with regard to location of centers of strands as illustrated in Fig. 5 and Table 2. The centers of strands are located at points along w rows and c columns in the first quadrant. Each of the centers of strands in [w-1] rows (first row exempted) is duplicated in the fourth quadrant with equal x coordinate but equal and opposite y coordinate.



(a) Centers of Strands



(b) Strands

Fig. 5. Hexagonal Shaped Stranded Conductor with s = 4, N = 37, w = 4 and c =13

Table 2. Hexagonal Shaped Stranded Conductors data

Serial Number (s)	Number of Strands (N)	Number of Rows (w)	Number of Columns (c)
1	1	1	1
2	7	2	5
3	19	3	9
4	37	4	13
5	61	5	17

Based on the sequence in Table 2, N and s are found to be related as in Eqns. (11) and (12). Similarly, w and c are found to be related to s as in Eqns. (13) and (14) respectively. A center of a strand is at the intersection of a row and a column but not all intersections of rows and columns are centers of strands. Fig. 5 is drawn for s=4, N=37, w=4 and c=13. In Fig. 5, blue dots indicate intersections which are the centers of strands while red plus symbols indicate intersections which are not centers of strands but points at which strands on the same rows meet tangentially. Two adjacent intersections along a row or a column cannot be centers of strands. The distance between two adjacent intersections along a row is r while the distance between two adjacent intersections along a column is $\sqrt{3}r$. The number of intersections along a row reduces by two as you move from one row to the next. The first intersection on the first row and first column is a center of a strand and has the Cartesian coordinate (0, 0). All these observations are used to develop the flowchart of Fig. 6 to compute the Cartesian coordinates of the centers of the strands. The output, Center is an N by 2 matrix containing x and y coordinates of the centers. The N by 2 matrix Center facilitates both the graphical drawing of the stranded conductor and the computation of the GMR based on Eqns. (5) and (10).

$$N = 1 + 3s(s-1) = 3s^2 - 3s + 1 \tag{11}$$

$$s = \frac{3 + \sqrt{9 - 12(1-N)}}{6} \tag{12}$$

$$w = s \tag{13}$$

$$c = 1 + 4(s - 1) \tag{14}$$

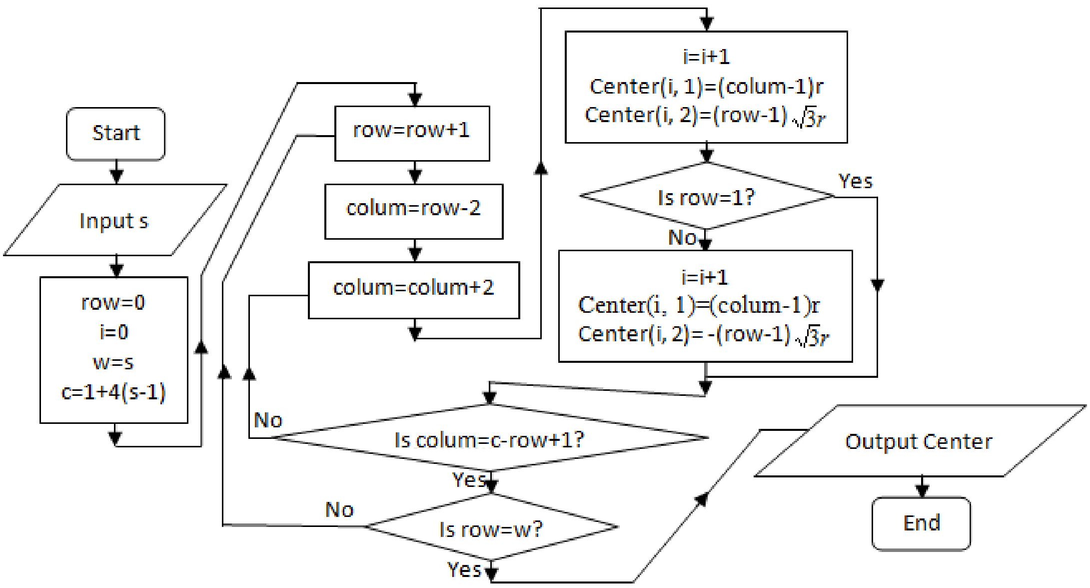
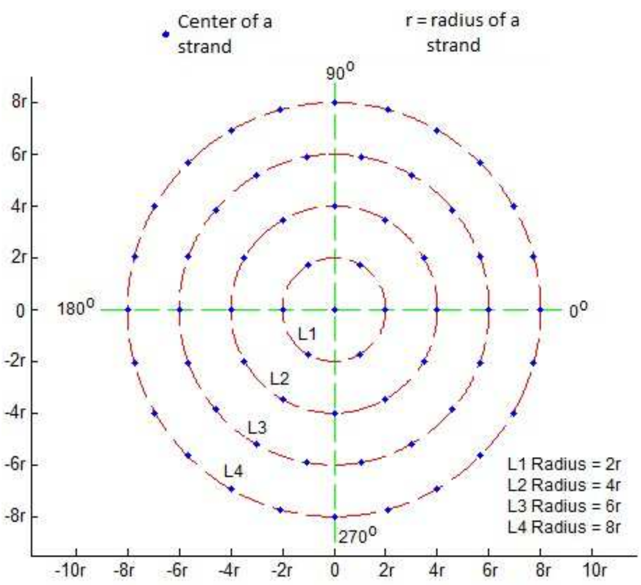


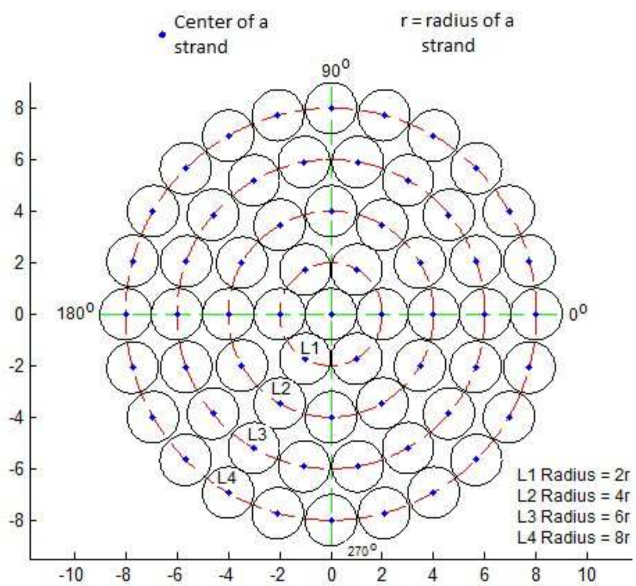
Fig. 6. Flowchart for the Arithmetic and Logical Model of Hexagonal Shaped Stranded Conductor

2.3 CIRCULAR SHAPED STRANDED CONDUCTOR

A careful study of circular shaped stranded conductors reveals some pattern with regard to location of centers of strands as illustrated in Fig. 7 and Table 3. The center of one strand is located at the point (0, 0). The centers of other strands are located at points along circumference of C concentric circles. The concentric circles L1, L2, L3, ... , LC with radii 2r, 4r, 6r, ... , 2Cr respectively have a common center at (0, 0) and are drawn with broken red lines in Fig. 7. 6, 12, 18, ... , 6C centers are on concentric circles L1, L2, L3, ... , LC respectively.



(a) Centers of Strands



(b) Strands

Fig. 7. Circular Shaped Stranded Conductor with s = 5, N = 61 and C = 4

Table 3. Circular Shaped Stranded Conductors data

Serial Number (s)	Number of Strands (N)	Number of Concentric Circles (C)
1	1	0
2	7	1
3	19	2
4	37	3
5	61	4

Based on the sequence in Table 3, N and s are found to be related as in Eqns. (11) and (12) which are also applicable to hexagonal shaped stranded conductor. C is found to be related to s as in Eqn. (15). Generally, the centers on concentric circle Ln are located at the intersection of the circumference of concentric circle Ln and 6n lines drawn from (0, 0) with angle $(360/6n)^{o}$ between the 6n lines. Fig. 7 is drawn for s=5, N=61 and C=4. All these observations are used to develop the flowchart of Fig. 8 to compute the Cartesian coordinates of the centers of the strands. The output, Center is an N by 2 matrix containing x and y coordinates of the centers. The N by 2 matrix Center facilitates both the graphical drawing of the stranded conductor and the computation of the GMR based on Eqns. (5) and (10).

$$C = s - 1 \tag{15}$$

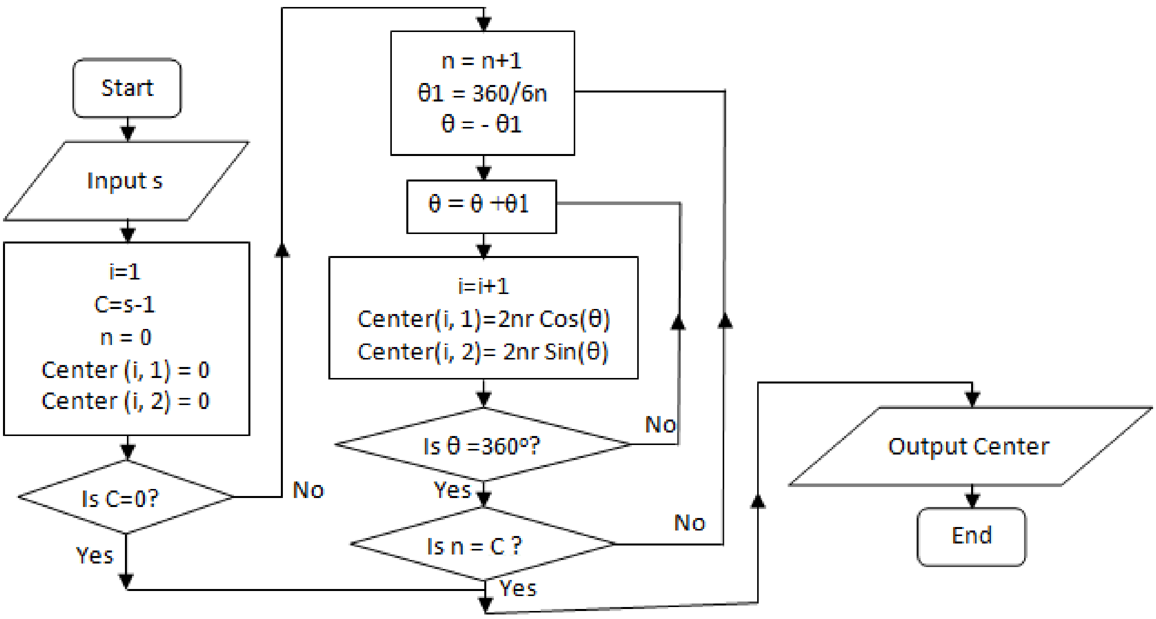


Fig. 8. Flowchart for the Arithmetic and Logical Model of Circular Shaped Stranded Conductor

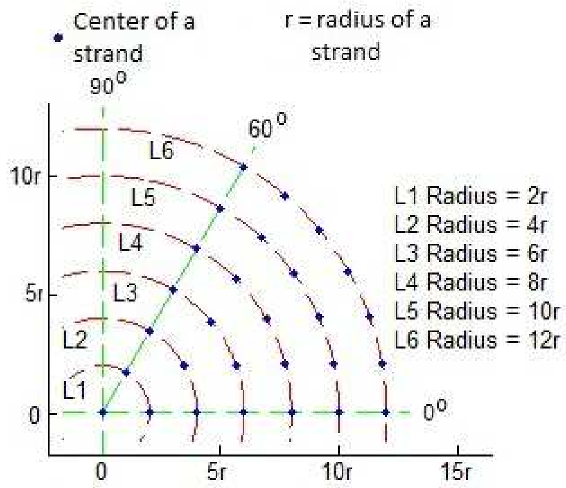
2.4 Sector Shaped Stranded Conductor

Sector shaped stranded conductor is derivable from circular shaped semiconductor. Strands are located between the lines 0^{o} and 60^{o} only. The two types are similar except that the strand with center at (0, 0) is included in type 1 but omitted in type 2 as shown in Fig. 9.

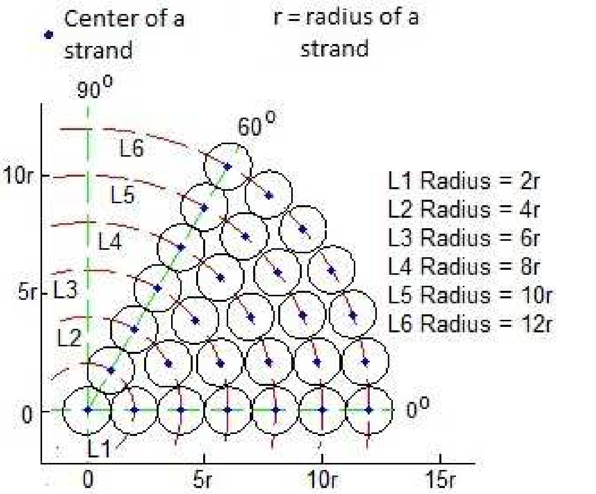
The arithmetic and logical models of Figs. 4, 6 and 8 are coded into computer programs.

3 Results

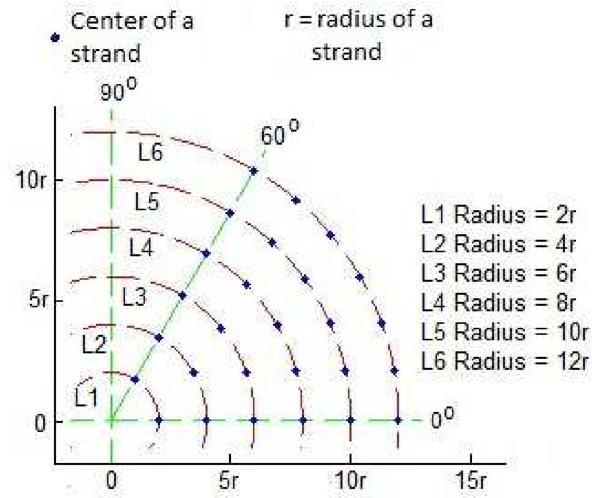
Reliability of the developed computer programs is verified. The prediction of centers of strands is found to be accurate as shown by accurate drawing of the stranded conductors some of which are shown in Figs 3(b), 5(b), 7(b), 9(b) and 9(d). The computation of GMR is also found to be accurate as some of the results are compared with manually calculated values as shown in Table 4. Using the developed computer programs GMR values are evaluated for different values of s for triangular, hexagonal, circular and sector shaped stranded conductors. The results are displayed in Table 5.



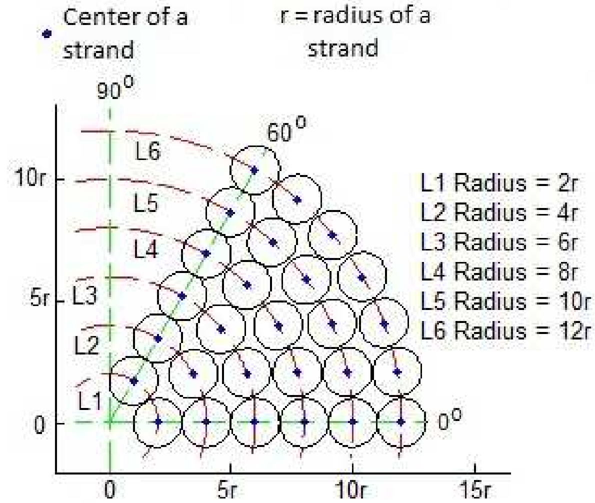
(a) Centers of Strands (Type 1)



(b) Strands (Type 1)



(c) Centers of Strands (Type 2)



(d) Strands (Type 2)

Fig. 9. Circular Shaped Stranded Conductor with s = 5, N = 61 and C = 4

Table 4. Comparison of GMR Values Obtained from the Arithmetic and Logical Models with those Manually Calculated

Triangular Shaped Stranded Conductors				**Circular Shaped Stranded Conductors**			
Serial Number (s)	**Number of Strands (N)**	**GMR (Manually Calculated)**	**GMR (Obtained from Program)**	**Serial Number (s)**	**Number of Strands (N)**	**GMR (Manually Calculated)**	**GMR (Obtained from Program)**
1	1	0.7788r	0.7788r	**1**	1	0.7788r	0.7788r
2	3	1.4605r	1.4605r	**2**	7	2.1767r	2.1767r
3	6	2.1023r	2.1023r	**3**	19	3.7883r	3.7882r
4	10	2.7323r	2.7323r	**4**	37	5.3745r	5.3744r
5	15	3.3573r	3.3573r	**5**	61	6.9499r	6.9488r

Table 5. GMR Values Obtained from the Arithmetic and Logical Models for Different Values of s

Triangular Shaped Stranded Conductor			Hexagonal Shaped Stranded Conductor			Circular Shaped Stranded Conductor			Sector Shaped Stranded Conductor (Type 1)			Sector Shaped Stranded Conductor (Type 2)		
Serial No	No of Strands	Geometric Mean Radius	Serial No	No of Strands	Geometric Mean Radius	Serial No	No of Strands	Geometric Mean Radius	Serial No	No of Strands	Geometric Mean Radius	Serial No	No of Strands	Geometric Mean Radius
s	**N**	**GMR**	**s**	**N**	**GMR**	**s**	**N**	**GMR**	**s**	**N**	**GMR**	**s**	**N**	**GMR**
1	1	0.7788r	**1**	1	0.7788r	**1**	1	0.7788r	**1**	1	0.7788r	**1**	0	-
2	3	1.4605r	**2**	7	2.1767r	**2**	7	2.1767r	**2**	3	1.4605r	**2**	2	1.2480r
3	6	2.1023r	**3**	19	3.5817r	**3**	19	3.7882r	**3**	6	2.1787r	**3**	5	1.9892r
4	10	2.7323r	**4**	37	4.9948r	**4**	37	5.3744r	**4**	10	2.8755r	**4**	9	2.6998r
5	15	3.3573r	**5**	61	6.4112r	**5**	61	6.9488r	**5**	15	3.5599r	**5**	14	3.3976r
6	21	3.9796r	**6**	91	7.8292r	**6**	91	8.5172r	**6**	21	4.2375r	**6**	20	4.2375r
7	28	4.6004r	**7**	127	9.2481r	**7**	127	10.0824r	**7**	28	4.9111r	**7**	27	4.9111r
8	36	5.2202r	**8**	169	10.6676r	**8**	169	11.6456r	**8**	36	5.5823r	**8**	35	5.5823r
9	45	5.8393r	**9**	217	12.0874r	**9**	217	13.2075r	**9**	45	6.2517r	**9**	44	6.2517r
10	55	6.4580r	**10**	271	13.5074r	**10**	271	14.7684r	**10**	55	6.9200r	**10**	54	6.9200r
20	210	12.6344r	**20**	1141	27.7131r	**20**	1141	30.3592r	**20**	210	13.5773r	**20**	209	13.5773r

4 CONCLUSION

Arithmetic and Logical Models of Triangular, Hexagonal, Circular and Sector Shaped Conductors have been developed and coded into computer programs. The models predict accurately centers of strands; compute accurately the Geometric Mean Radius and provide graphical drawing of stranded conductors. The models are useful in evaluation of transmission line parameters and therefore useful for Voltage and Voltage-Drop Analysis of Electric Power Systems.

REFERENCES

[1] Kothari and Nagrath, *Power System Engineering*, 2nd Ed. McGraw-Hill Publishing Company Ltd New Delhi, India, 2008.

[2] Glover, Sarma, and Overbye, *Power System Analysis and Design*, 4th Ed. Cengage Learning, Stamford, USA, 2010.

[3] Stevenson, *Element of Power System Analysis, 2nd Ed*. McGraw - Hill Singapore, 1982.

[4] J. G. Andrews and A. J Shrapnel, ''Electric Field Distribution around an Isolated Stranded Conductor,'' Proc.IEE, vol. 119, no. 8, pp 1162-1166, 1972.

[5] P. N. Mikropoulos and V.N. Zagkanas, ''The effect of Stranded Conductor Geometry on DC Corona in Coaxial Cylindrical Electrode Arrangement in Air,'' *48th Universities' Power Engineering, Conference (UPEC), Dublin, Ireland*, paper No. 276, 2013.

[6] A. Zangeneh, A. Gholami and V. Zamani, '' A New Method for Calculation of Corona Inception Voltage in Stranded Conductors of Overhead Transmission Lines,'' *First International Power and Energy Conference PECon, Putrajaya, Malaysia*, pp. 571-574, 2006.

[7] W. I. Middleton and E. W. Davies ''Skin Effect in Large Stranded Conductors at Low Frequencies,'' *Journal of the American Institute of Electrical Engineers*, vol. 40, no. 9, pp 757-763, 1921.

[8] K. Preis, O. Biro, H. Reisinger, K. Papp and I. Ticar, ''Eddy Current Losses in Large Air Coils with Layered Stranded Conductors,'' *IEEE Transactions on Magnetics*, vol. 48, no. 6, pp 1318-1321, 2008.

[9] V. T. Morgan, ''The Radial Temperature Distribution and Effective Radial Thermal Conductivity in Bare Solid and Stranded Conductors,'' *IEE Trans. On Power Delivery*, vol. PWRD-5, pp 1443-1452, 1990.

[10] V. Filipovic-Gledja, V. T. Morgan and R. D. Findlay, ''A Unified Model for Prediction the Electrical, Mechanical and Thermal Characteristics of Stranded Overhead-Line Conductors,'' *Canadian Conference on Electrical and Computer Engineering*, pp 182-185, 1994.

[11] G. Gaba and M. Abou-Dakka, ''A Simplified and Accurate Calculation of Frequency Dependence Conductor Impedance,'' *8th IEEE International Conference on Harmonics and Quality of Power (ICHQP), Athens, Greece*, pp 939-945, 1998.

[12] E. C. M. Costa, S. Kurokawa, J. Pissolato, A. J. Prado, and L. F. Bovolato, ''A Model for Bundled Conductors Considering a Non-Homogeneous Distribution of the Current through Subconductors,'' IEEE Latin America Transactions, vol. 8, no. 3, pp 221-228, 2010.