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A Study on the concentration of Rural Taluk Population in Tamil Nadu State

***Dr. B. Renganathan*[1] *and Mrs. R. Selvam*[2]**

[1]Reader and Head (Rtd),
Department of Statistics,
Salem Sowdeswari College, Salem-10,
Tamil Nadu, India

[2]Assistant Professor of Statistics,
Government Arts College (Autonomous), Salem-7,
Tamil Nadu, India

ABSTRACT: This study presents the concentration of rural Taluk population using probabilistic model. Taluk size Distribution and the concentration of rural population in the Taluk are related. Taluk sizes are increased in both ways, population and area. Pareto and Exponential probabilistic model were used to describe the Taluk size Distribution. Gini's concentration ratio and Pietra index were used to study the amount of concentration of rural population. This study includes all Taluks of Tamil Nadu and their population as per 2011 census for an empirical analysis. As per this study, the concentration of the rural male population in the rural Taluk area is less than the concentration of the rural female population and the Exponential model is more suitable than the Pareto model for the study of variability in Taluk size Distribution using Gini's concentration ratio and Pietra index.

KEYWORDS: Exponential Model, Gini's coefficient, Lorenz Curve, Maximum Likelihood Estimation, Pareto model, Pietra Index.

1 INTRODUCTION

Rural area is the complement form of urban area. Taluk size is the population of the Taluk. Taluk size distribution is the distribution of Taluk with respect to their population. Taluk size undergoes changes over time and space. The growth of rural population attracts Demographers, planners, etc to study the changes in size and the structure of the rural population. Variation in the sizes of rural population influences the changes in size and structure of the rural population. Atkinson (1970) showed that rules for ordering risk prospects are written in terms of Lorenz curves. Sen (1976) used the Lorenz curves in the field of finance. Lorenz curves of cumulative electricity consumption has been studied by Jacoboson and kammen (2005). Sadras and Bongiovanni (2004) used Lorenz curve and Gini index to assess the yield inequality. Arnold, B.C (2008) used the Gini index and Pietra index as the measures of inequality in income distribution. Sarabia, J.M (2008) used Lorenz curve and Gini index to measure the inequality in income distribution. No one has done the **s**tudy on the concentration of rural Taluk Population in Tamil Nadu State. An attempt has been made to study the concentration of rural Taluk population using Pareto and Exponential probabilistic model. Parameters of these models can be estimated by using the Method of Maximum Likelihood. The concentration of rural population in all taluks of Tamil Nadu State is to be studied using Gini's concentration ratio and Pietra index.

The Lorenz curve, is the graph based on cumulative proportion (or percentage) of total population of Taluks against the cumulative proportion (or percentage) of number of Taluks having its population. The Gini Index(GI) named after its originator Gini(1912) is twice the area between the L(p) and the diagonal segment joining the points (0,0) and (100,100).The line segment joining the two points (0,0) and (100,100) is known as line of equal distribution or egalitarian line. The alternative names for Gini index are Lorenz concentration, Gini coefficient and Gini concentration ratio. Pietra index is twice the area of the largest triangle which can be inscribed in the area of concentration. Here, the Gini coefficient can be interpreted as twice the area of concentration between the Lorenz curve and the 45 degree line of perfect equality and the Pietra index is twice the area of the largest triangle which can be inscribed in the area of concentration.

2 OBJECTIVE

To study the concentration of rural population in all Taluks of Tamil Nadu state by developing Gini's concentration ratio and Pietra index using probabilistic model.

3 METHOD OF ESTIMATING GINI'S CONCENTRATION RATIO

3.1 LORENZ CURVE

Lorenz Curve is a powerful tool for analyzing the Taluk size concentration. The graph of the function q = f(p) is called Lorenz curve, where p and q are defined as

$$p_x = P[X \leq x]$$

$$= \int_{(0,x)} f(x)dx$$

$$= \frac{Number\ of\ Taluks\ size \leq x}{N}$$

$$q_x = \frac{\int_{(0,x)} xf(x)dx}{\int_{(0,\infty)} xf(x)dx}$$

$$= \frac{Total\ Population\ of\ the\ Taluks\ with\ size\ \leq x}{No.of\ Taluks}$$

The Lorenz function q = f (p) was graphed by taking values of p on x axis and the values of q on y-axis with suitable scale. The resulting graph was represented in figures. The line of equal distribution indicates the equality in the distribution of Taluks with respect to their populations. When the Lorenz curve deviates from the line of equal distribution, there will be observed the variations in the TSD. Gini's concentration ratio is a technique used to measure the degree of variation in the TSD.

3.2 GINI'S CONCENTRATION RATIO

Dispersion in the Taluk size distribution is measured by using co-efficient of variation, Relative mean deviation and Gini's concentration ratio. Gini's concentration ratio is an ideal measure for studying the variation in the TSD. A method of developing Gini's concentration ratio is described with probabilistic models.

3.2.1 PARETO MODEL

The Taluk size x is assumed to follow Pareto distribution f(x:a), where f(x:a) is described as

$$f(x:a) = \frac{ax^{*a}}{x^{a+1}}\ ,\ a>0,\ x \geq x^*$$

Where x^* is the threshold Taluk size. Parameters a and x^* are estimated by using the Method of Maximum Likely hood (Rao C.R. 1973)

$$\hat{a} = \left[\frac{\sum \log xi}{n} - \log x^*\right]^{-1}$$

$$\hat{x} = \min_{i \leq 1}(xi)$$

The expressions for p and q are obtained as follows.

$$P_x = F_x(x)$$

$$= \int_{x^*}^{x} \frac{ax^{*a}}{t^{a+1}} dt$$

$$P_x = 1 - (x^*/x)^a \qquad (1)$$

$q_x = \int_{x^*}^{x} t\, f(t,a)dt \;/\; \int_{x^*}^{\infty} t\, f(t,a)dt$.

$= I_1 / I_2$

where $I_1 = \int_{x^*}^{x} t\, f(t,a)dt$

$= \int_{x^*}^{x} \frac{a x^{*a}}{t^{a+1}}\, dt$

$= \left(\frac{a}{a-1}\right) x^* \left[\left(\frac{x^*}{x}\right)^{a-1} - 1\right]$

where $I_2 = \int_{x^*}^{\infty} t\, f(t,a)dt$

$= \int_{x^*}^{\infty} \frac{a x^{*a}}{t^{a+1}}\, dt$

$= \left(\frac{a}{a-1}\right)(-x^*)$

therefore $q_x = I_1 / I_2$

$$= \frac{\left(\frac{a}{a-1}\right) x^* \left[\left(\frac{x^*}{x}\right)^{a-1} - 1\right]}{\left(\frac{a}{a-1}\right)(-x^*)}$$

$$q_x = 1 - \left(\frac{x^*}{x}\right)^{a-1} \qquad (2)$$

The function q = f(q) has been obtained by using (1) & (2)

$1 - p_x = \left(\frac{x^*}{x}\right)^{a}$

$$\frac{x^*}{x} = (1 - p_x)^{\frac{1}{a}} \qquad (3)$$

$1 - q_x = \left(\frac{x^*}{x}\right)^{a-1}$

$$\frac{x^*}{x} = (1 - q_x)^{\frac{1}{a-1}} \qquad (4)$$

(3) =(4) => $(1 - q_x)^{\frac{1}{a-1}} = (1 - p_x)^{\frac{1}{a}}$

i.e., $1 - q_x = (1 - p_x)^{(a-1)/a}$

Since the TSD is known, q= f(p) is obtained by eliminating x on both sides

i.e., $q = 1 - (1-p)^{(a-1)/a}$, $0 \le p \le 1$

The graph of the function q = f(p) is Lorenz curve representing the variation in the size of the Taluks. Gini's concentration ratio is stated as,

$$\rho = \frac{Area\ between\ the\ Lorenz\ curve\ and\ the\ line\ of\ equal\ distribution}{Total\ area\ under\ diagonal}$$

ρ = 1-2A, where A is the area under the Lorenz curve

$$A = \int_0^1 f(p)dp$$

$$A = \int_0^1 [1 - (1-p)]^{(a-1)/a}\, dp$$

$$A = \frac{a-1}{2a-1}$$

$$\rho = 1 - \frac{2(a-1)}{(2a-1)}$$

$$\rho = \frac{1}{(2a-1)}$$

Gini's concentration for Taluk size using Pareto model was developed as

$$\rho = \frac{1}{(2a-1)}$$

3.2.2 EXPONENTIAL MODEL

Let x be the Taluk size and it follows an exponential distribution f(x, λ) is described as

$f(x,\lambda) = \lambda e^{-\lambda x}$, x>0.

The parameter λ is estimated by using the Method of Maximum Likely hood

$i.e., \hat{\lambda} = \frac{1}{\bar{x}}$

where,

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

Expression for p and q are obtained as follows

$$P_x = F_X(x)$$

$$p_x = \int_0^x \lambda e^{-\lambda t} dt$$

$$p_x = 1 - e^{-\lambda x} \qquad (1)$$

$$q_x = \frac{\int_0^x t\lambda e^{-\lambda t} dt}{\int_0^\infty t\lambda e^{-\lambda t} dt}$$

$$q_x = \frac{I_1}{I_2}$$

$$where, I_1 = \int_0^x t\lambda e^{-\lambda t}\, dt$$

$$I_1 = \frac{\lambda\left[\frac{te^{-\lambda t}}{-\lambda}\right]_0^x}{-\lambda} + \frac{1}{\lambda}\int_0^x e^{-\lambda t} dt$$

$$I_1 = \frac{1}{\lambda} - \frac{1}{\lambda} e^{-\lambda x} - xe^{-\lambda x}$$

where I_2 $= \int_0^\alpha t\,\lambda e^{-\lambda t}\, dt$

$= \lambda[te^{-\lambda t}/(-\lambda)]_0^\alpha + \frac{1}{\lambda}\int_0^\alpha e^{-\lambda t}\, dt$

$= \frac{1}{\lambda}$

Therefore, $q_x = I_1/I_2$

$= [\frac{1}{\lambda} - \frac{1}{\lambda} e^{-\lambda x} - xe^{-\lambda x}] / \frac{1}{\lambda}$

$q_x = 1 - e^{-\lambda x} - \lambda xe^{-\lambda x}$

$q_x = p_x - \lambda xe^{-\lambda x}$ because of (5)

Since the distribution of x is known q = f(p) is obtained by eliminating x

i.e., $q = p-\lambda e^{-\lambda}$

Gini's concentration ratio, based on the function $q = p-\lambda e^{-\lambda}$, is obtained as follows.

i.e., ρ = 1-2A where A is the area under Lorenz curve.

$A = \int_0^1 f(p)dp$

$=\int_0^1 [p - \lambda e^{-\lambda}]$ dp

A = 1/2 - $\lambda e^{-\lambda}$

Therefore, ρ = 1-2 (1/2-$\lambda e^{-\lambda}$)

$\rho = 2\lambda e^{-\lambda}$

Using exponential model, Gini's concentration ratio for rural size was obtained as

$\rho = 2\lambda e^{-\lambda}$

3.3 PIETRA INDEX

The Pietra index which can be defined as

$$P_X = \frac{E[|X - E(X)|]}{2E(X)}$$

$$P_X = \frac{M.D}{2E(x)}$$

also has a Lorenz curve interpretation(Pietra,1932). It can be viewed either as the maximal vertical deviation between the Lorenz curve and the egalitarian line or, equivalently, as two times the area of the maximal triangle that can be inscribed between Lorenz curve and the egalitarian line. The Gini coefficient can be interpreted as twice the area of concentration between the Lorenz curve and 45 degree line of perfect equality. Thus, Pietra Index is less than or equal to Gini Index or Gini Coefficient.

3.3.1 PIETRA INDEX FOR PARETO MODEL

Consider the mean of pareto distribution,

$$Mean, E(x) = \int x\, f(x) dx$$

$$= \int_{x^*}^{\infty} x \frac{a x^{*a}}{x^{a+1}} dx$$

$$= a x^{*a} \int_{x^*}^{\infty} \frac{x}{x^{a+1}} dx$$

$$= a x^{*a} \int_{x^*}^{\infty} x^{-a} dx$$

(i.e). Mean $= \frac{ax^*}{a-1}$, $a > 1$ (1)

$$Mean\ deviation\ about\ mean(M.D) = \int_{x^*}^{\infty} \left| x - \frac{ax^*}{a-1} \right| f(x)\, dx$$

$$Mean\ deviation\ about\ mean(M.D) = \int_{x^*}^{\infty} \left| x - \frac{ax^*}{a-1} \right| \frac{ax^{*a}}{x^{a+1}} dx$$

$$= 2 \int_{\frac{ax^*}{a-1}}^{\infty} \left[x - \frac{ax^*}{a-1} \right] \frac{ax^{*a}}{x^{a+1}} dx$$

$$= 2ax^{*a} \int_0^{\infty} \frac{t}{\left[t + \frac{ax^*}{a-1} \right]^{a+1}} dt$$

$$= 2ax^{*a} \int_0^{\infty} \frac{t}{\left[\frac{ax^*}{a-1} \right]^{a+1} \left[\frac{t(a-1)}{ax^*} + 1 \right]^{a+1}} dt$$

$$= \frac{2ax^*}{\left[\frac{ax^*}{a-1}\right]^{a+1}} \int_0^\infty \frac{t}{\left[1+\frac{t(a-1)}{ax^*}\right]^{a+1}} dt$$

$$= \frac{2ax^*}{\left[\frac{ax^*}{a-1}\right]^{a}} \int_0^\infty \frac{t\left(\frac{a-1}{ax^*}\right)}{\left[1+\frac{t(a-1)}{ax^*}\right]^{a+1}} dt$$

$$= 2a^{(1-a)}(a-1)^a \int_0^\infty \frac{t\left(\frac{a-1}{ax^*}\right)}{\left[1+\frac{t(a-1)}{ax^*}\right]^{a+1}} dt$$

$$i.e., M.D = 2a^{(1-a)}(a-1)^a \int_0^\infty \frac{x}{[1+x]^{a+1}} dx\left(\frac{ax^*}{a-1}\right)$$

$$i.e., M.D = 2ax^* a^{(1-a)}(a-1)^{a-1}\beta(2, a-1)$$

$$i.e., M.D = 2ax^*\left[\frac{a-1}{a}\right]^{a-1} \frac{\Gamma a-1\, \Gamma 2}{\Gamma a-1+2}$$

$$i.e., M.D = \frac{2x^*}{a-1}\left[\frac{a-1}{a}\right]^{a-1}$$

$$i.e., M.D = 2x^*(a-1)^{-1}\left(1-\frac{1}{a}\right)^{a-1} \qquad (2)$$

$$Pietra\ index, (p_x) = \frac{Meamdeviation}{2[E(x)]}$$

Using equations (1) and (2) , we get,

$$p_x = \frac{2x^*(a-1)^{-1}\left(1-\frac{1}{a}\right)^{a-1}}{\left[2\frac{ax^*}{a-1}\right]}$$

(i.e.) $p_{x} = \frac{(a-1)^{a-1}}{a^a}$

3.3.2 PIETRA INDEX FOR EXPONENTIAL MODEL

Consider the mean of exponential distribution,

$$Mean, E(x) = \int x\, f(x) dx$$

$$= \int_0^\infty x\, \lambda e^{-\lambda x}\, dx = \frac{1}{\lambda}$$

$$Mean\ deviation\ about\ mean(M.D) = \int_0^\infty \left|x - \frac{1}{\lambda}\right| f(x)\, dx$$

$$Mean\ deviation\ about\ mean(M.D) = \int_0^\infty \left|x - \frac{1}{\lambda}\right| \lambda e^{-\lambda x}\, dx$$

$$= 2\int_{\frac{1}{\lambda}}^\infty \left[x - \frac{1}{\lambda}\right] \lambda e^{-\lambda x} dx$$

$$= 2\lambda \int_0^\infty t\, e^{-\lambda(t+\frac{1}{\lambda})} dx$$

$$= \frac{2}{\lambda e} = \frac{2}{e} S.D$$

$$(i.e.). Mean\ deviation\ about\ mean(M.D) = \frac{2}{e} S.D$$

$Pietra\ index, (p_x) = \frac{Meamdeviation}{2[E(x)]}$ (i.e.). $p_x = \frac{[\frac{2}{\lambda e}]}{[\frac{2}{\lambda}]} = 1/e = e^{-1}$

EMPIRICAL RESULTS

The populations of all Taluks in Tamil Nadu state as per 2011 census were used for estimating Gini's concentration ratio. Taluks are polytomized into ten classes as per 2011 census classes are I: 0 -28616; II: 28616 – 57232; III: 57232-85848; IV:85848-114464; V:114464-143080; VI: 143080-171696; VII: 171696-200312; VIII: 200312-228928; IX: 228928-257544; X: 257544 – 286160.The TSD for rural male population in Tamil Nadu state has been formulated and presented in table (1)

Table 1. Taluk size distribution of Rural Male Population

Taluk size interval	No. of Taluks
0-28616	15
28616-57232	48
57232-85848	63
85848-114464	40
114464-143080	19
143080-171696	16
171696-200312	10
200312-228928	2
228928-257544	1
257544-286160	1

Since TSD for rural male population has the skew nature, Skew models were used to present the TSD. The Pareto distribution is one of the skew models. It has been proposed to measure the Gini's concentration ratio using the TSD in given table(1).

The estimates of the parameters and Gini's concentration ratio are obtained as follows:

The estimates of the parameter is,

$$\hat{a} = \left[\frac{\sum flogxi}{\sum f} - \log x^*\right]^{-1},\ \text{where } x^* = \min(x) \text{ and } x = \text{mid value}$$

$$\widehat{a} = \left[\frac{1044.98}{215} - 4.15\right]^{-1} = 1.41$$

Gini's concentration ratio, $\rho = \frac{1}{2a-1} = 0.556$

Pietra index, $p_{x=\frac{(a-1)^{a-1}}{a^a}}$

(i.e). $p_x = 0.4274$

Since ρ = 0.556 and $p_x = 0.4274$, this indicates the concentration of the male rural population in Taluk size distribution. The Lorenz curve of the function q = f(p) are drawn by using the cumulative values of p and q. The observed values of p and q and estimated values of p and q are obtained and presented in table (2) and table (3).

Table 2. P and q value based on empirical TSD of rural male population

Taluk size interval	Taluk size X	% cumulative value of x (P)	Observed No of Taluks	% of Cumulative Observed No of Taluks (q)
0-28616	14308	1	15	6.97
28616-57232	42924	4	48	29.3
57232-85848	71540	9	63	58.6
85848-114464	100156	16	40	72.21
114464-143080	128772	25	19	86.05
143080-171696	157388	36	16	93.49
171696-200312	186004	49	10	98.14
200312-228928	214620	64	2	99.07
228928-257544	243236	81	1	99.53
257544-286160	271852	100	1	100

The distribution function of the pareto variable x is stated as follows:

$F_X(x) = 1 - \left[\frac{x^*}{x}\right]^{\hat{a}}$, Where, $\hat{a} = 1.4$; $x^* = 14308$

The value of F(x) are obtained and presented in table (3)

Table 3. P and q value based on expected TSD of rural male population (pareto)

Taluk size X	Distribution function of x (p)	Cumulative No of Taluks Expected	% of cumulative No of Taluks Expected by Pareto Model(q)
14308	0	0	0
42924	0.7852	168.8	79.8
71540	0.8949	192.4	90.96
100156	0.9344	200.8	94.94
128772	0.9539	205.08	96.96
157388	0.9652	207.5	98.10
186004	0.9724	209.06	98.84
214620	0.9774	210.141	99.35
243236	0.9811	210.9	99.71
271852	0.9838	211.5	100

The TSD for rural female population in Tamil Nadu state has been formulated and presented in table (4).

Table 4. Taluk size distribution of Rural Female Population

Taluk size	No.of Taluks
0-28616	14
28616-57232	45
57232-85848	63
85848-114464	43
114464-143080	18
143080-171696	17
171696-200312	12
200312-228928	1
228928-257544	1
257544-286160	1

The estimates of the parameters and Gini's concentration ratio for Taluk size distribution of rural female population are obtained as follows:

The estimates of the parameter is,

$$\hat{a} = \left[\frac{\sum f logxi}{\sum f} - \log x^*\right]^{-1}$$ where, x^* = min(x) and x=mid value

$$\hat{a} = \left[\frac{1047.227}{215} - 4.1556\right]^{-1} = 1.38$$

Gini's concentration ratio, $\rho = \frac{1}{2a-1}$ = 0.5682

Pietra index, $p_{x=\frac{(a-1)^{a-1}}{a^a}}$

(i.e) $p_x = 0.4439$

The Gini's concentration ratio, ρ = 0.5682 and Pietra index, $p_x = 0.4439$ shows the concentration of the female rural population in Taluk size distribution. The Lorenz curve of the function q = f(p) are drawn by using the cumulative values of p and q.The observed values of p and q and estimated values of p and q are obtained and presented in table(5) and table (6).

Table 5. P and q value based on empirical TSD of rural female population

Taluk size interval	Taluk size X	% cumulative value of x (p)	Observed No of Taluks	% of cumulative Observed No of Taluks (q)
0-28616	14308	1	14	6.51
28616-57232	42924	4	45	27.44
57232-85848	71540	9	63	56.74
85848-114464	100156	16	43	76.74
114464-143080	128772	25	18	85.11
143080-171696	157388	36	17	93.02
171696-200312	186004	49	12	98.60
200312-228928	214620	64	1	99.06
228928-257544	243236	81	1	99.53
257544-286160	271852	100	1	100

The distribution function of the pareto variable x is stated as follows:

$$F_X(x) = 1 - \left[\frac{x^*}{x}\right]^{\hat{a}}$$, Where, $\hat{a}$= 1.38, x^* = 14308

The value of F(x) are obtained and presented in table (6)

Table 6. P and q value based on expected TSD of rural female population (Pareto Model)

Taluk size X	Distribution function of x (p)	Cumulative No of Taluks Expected	% of cumulative No of Taluks Expected by pareto (q)
14308	0	0	0
42924	0.7804	167.7860	79.4065
71540	0.8915	191.6725	90.7111
100156	0.9318	200.3370	94.8116
128772	0.9518	204.6370	96.8467
157388	0.9636	207.1525	98.0372
186004	0.9718	208.7650	98.7993
214620	0.9762	209.8830	99.3294
243236	0.98	210.7	99.7160
271852	0.9828	211.3	100

EXPONENTIAL MODEL

The estimates of parameter for rural male population can be obtained as,

λ=0.0000115

Gini's concentration ratio, ρ = 0.000023

Pietra index,,$P_x = (2.7174)^{-1}$ = 0.3680

The distribution function of the exponential variable X is stated as follows

$F_X(x) = 1- e^{-\lambda x}$, where, λ = 0.0000115

The value of F(x) are obtained and presented in table (7)

Table 7. P and q value based on expected TSD of rural Male Population (Exponential Model)

Taluk size	Distribution function(p)	Cumulative No of Taluks expected	% of cumulative No of Taluks Expected by Exponential Model
14308	0.1456	31.3040	15.3312
42924	0.3763	80.9045	39.6230
71540	0.5448	117.1320	57.3655
100156	0.6677	143.5555	70.3064
128772	0.7574	162.8410	79.7515
157388	0.8229	176.9235	86.6484
186004	0.8708	187.222	91.6921
214620	0.9057	194.7255	95.367
243236	0.9311	200.1865	98.0415
271852	0.9497	204.1855	100

The estimates of parameter for rural female population can be obtained as,

λ= 0.0000118

Gini's concentration ratio, ρ = 0.000024

Pietra index, $P_x = (2.7174)^{-1}$ = 0.3680

The distribution function of the exponential variable X is stated as follows

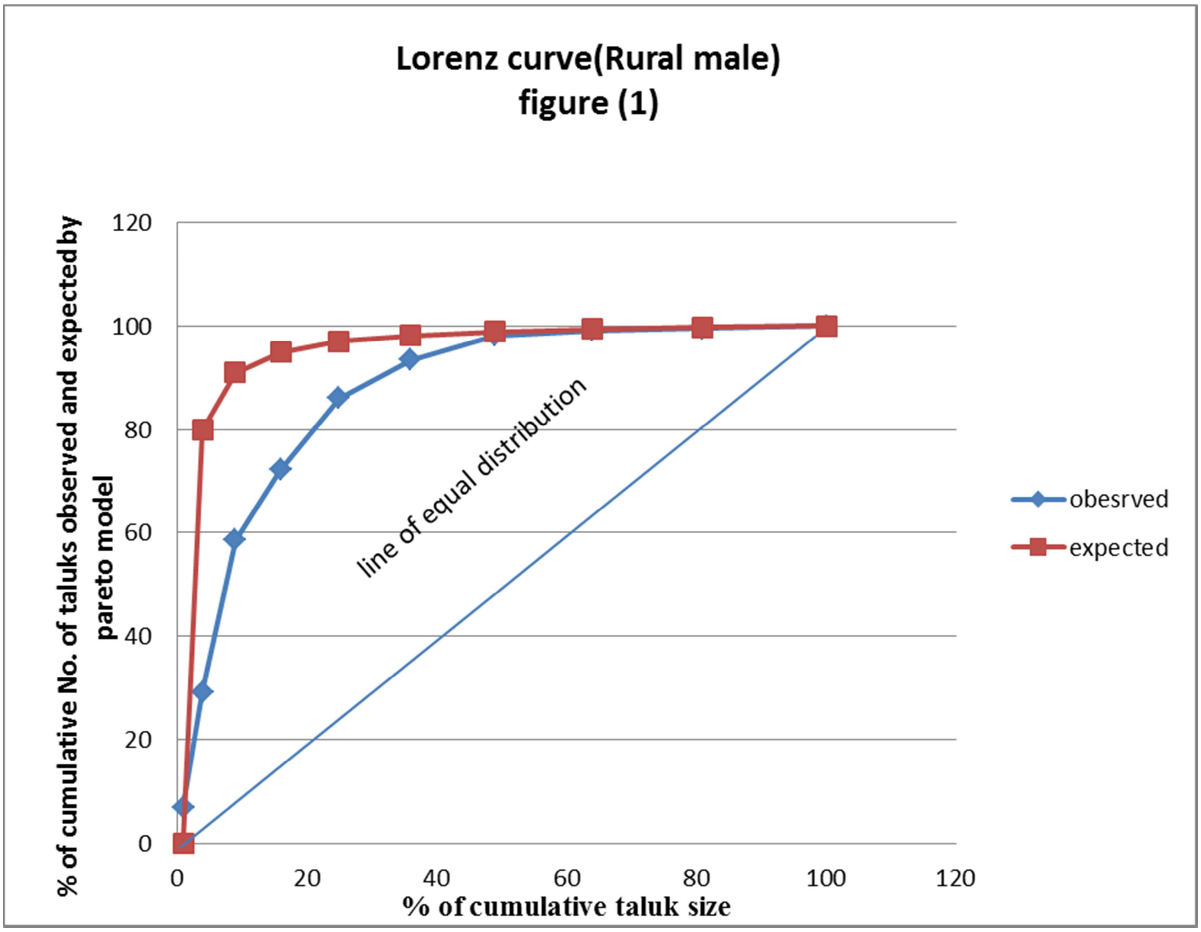
$F_X(x) = 1- e^{-\lambda x}$

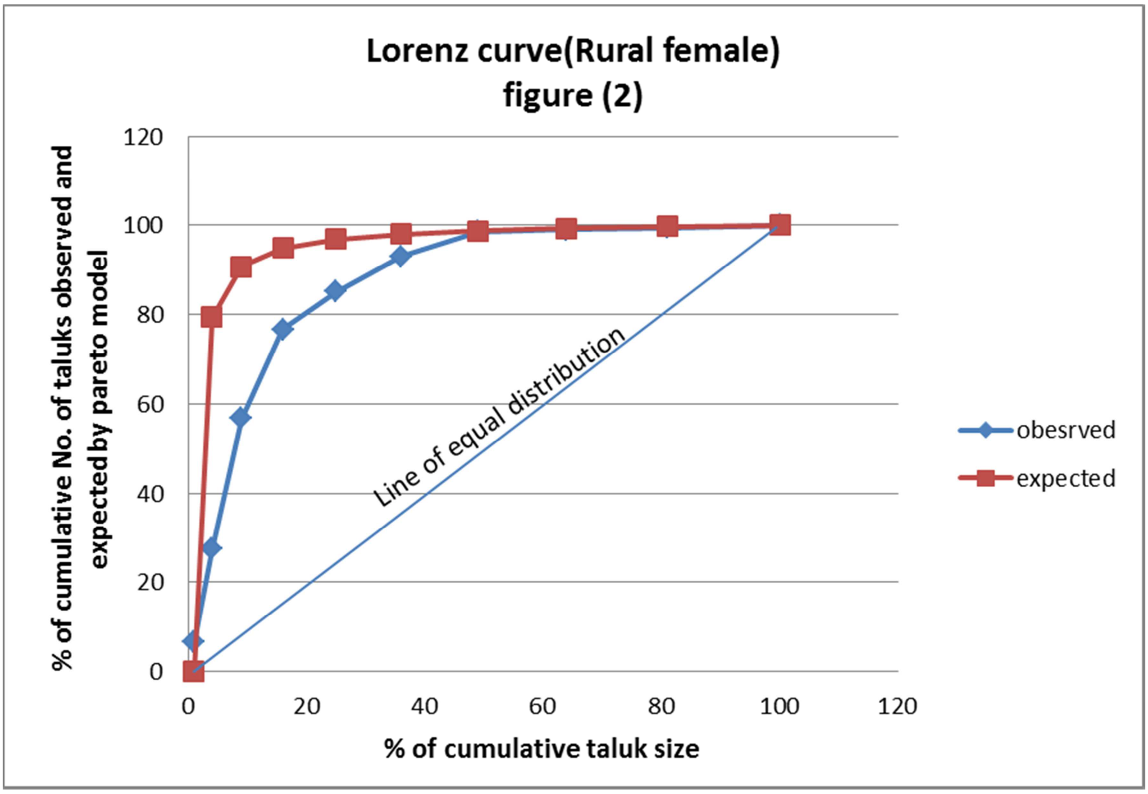
where, λ = 0.0000118

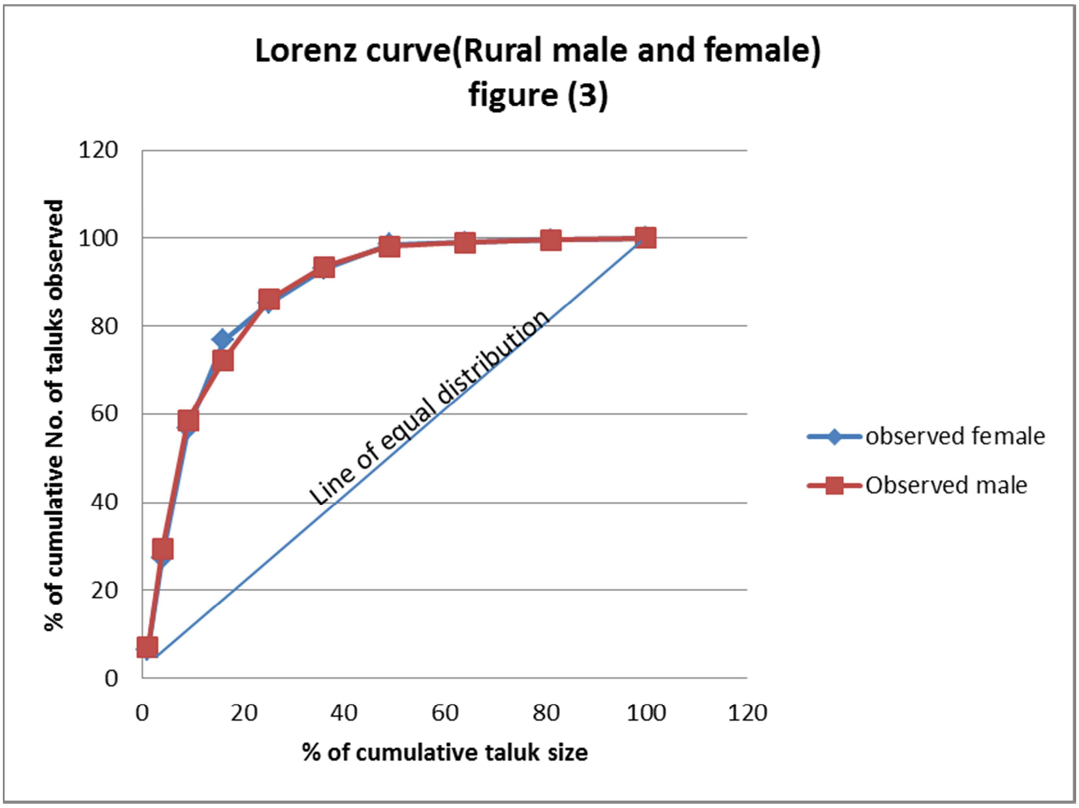
The value of F(x) are obtained and presented in table (8)

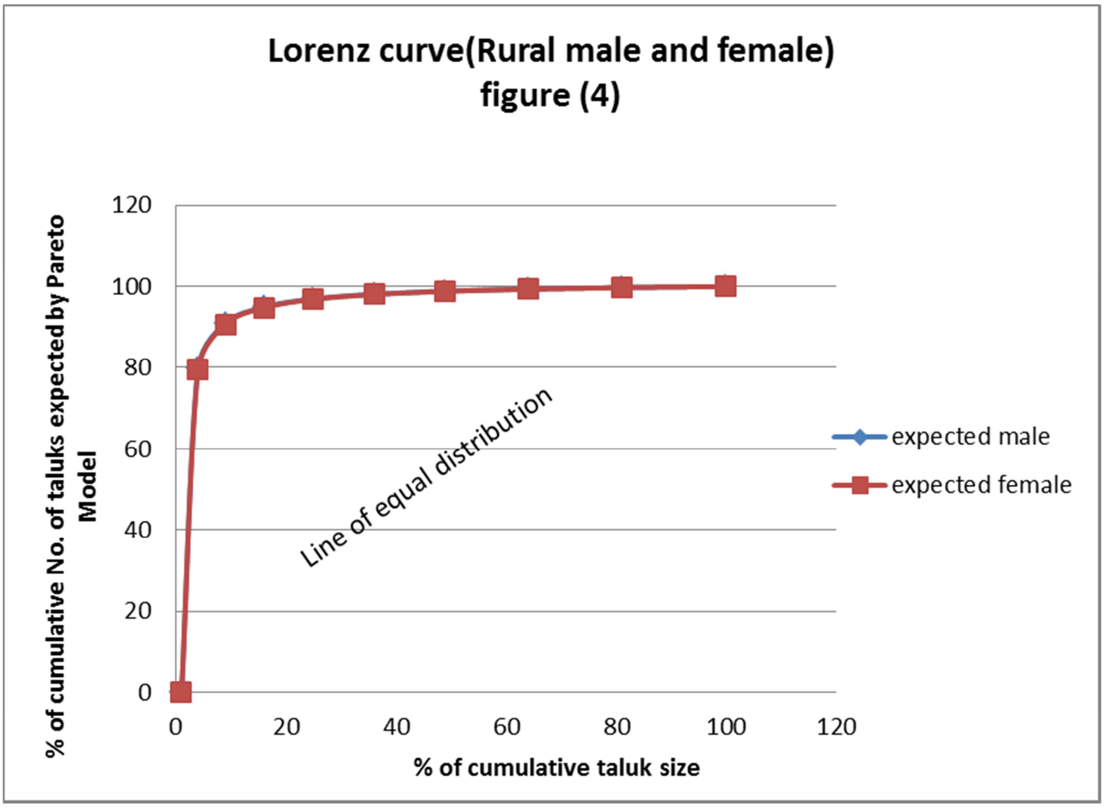
Table 8. P and q value based on expected TSD of rural female Population (Exponential Model)

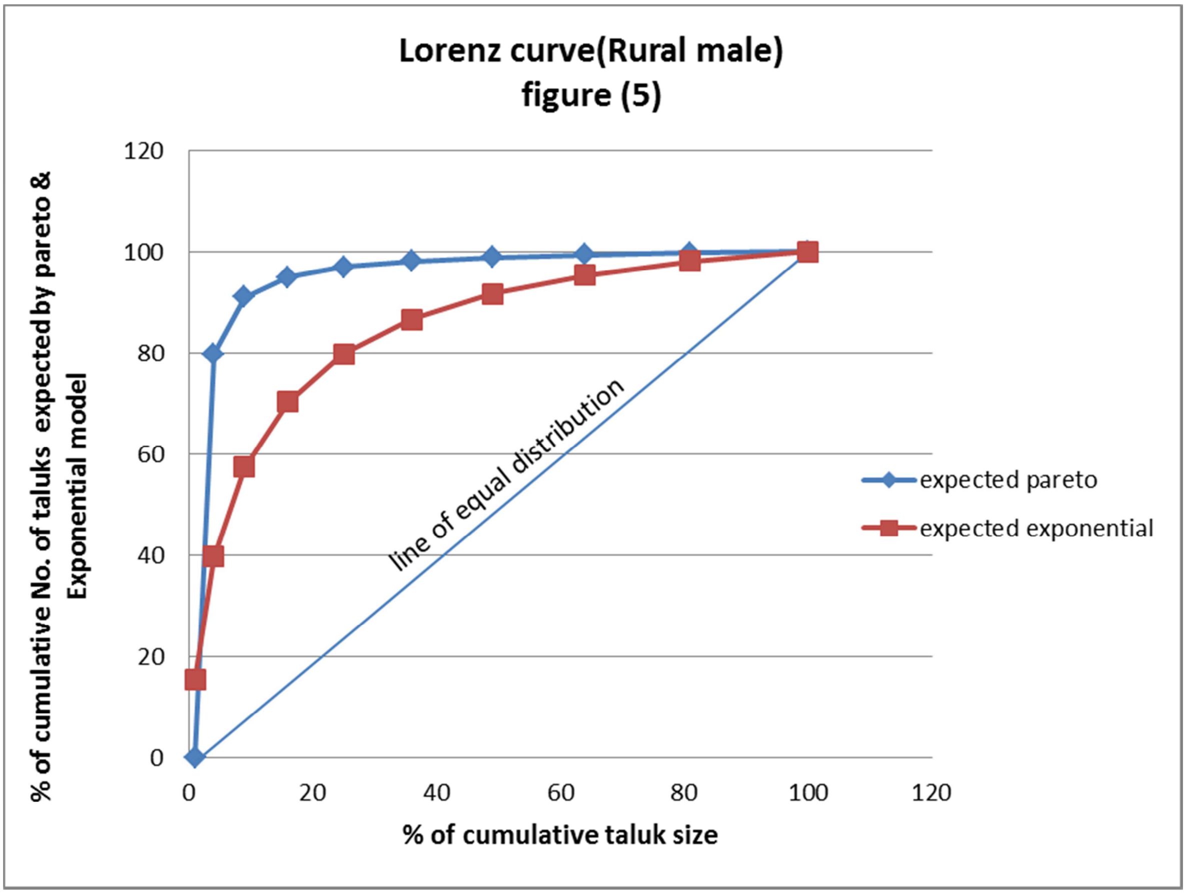
Taluk size	Distribution function (p)	Cumulative No of Taluks expected	% of cumulative No of Taluks Expected by Exponential Model (q)
14308	0.1578	33.927	16.4084
42924	0.4026	86.559	41.8634
71540	0.5770	124.055	59.998
100156	0.6994	150.371	72.7254
128772	0.7867	169.1405	81.8031
157388	0.8487	182.4705	88.250
186004	0.8927	191.9305	92.8252
214620	0.9239	198.6385	96.0695
243236	0.9460	203.39	98.3675
271852	0.9617	206.7655	100

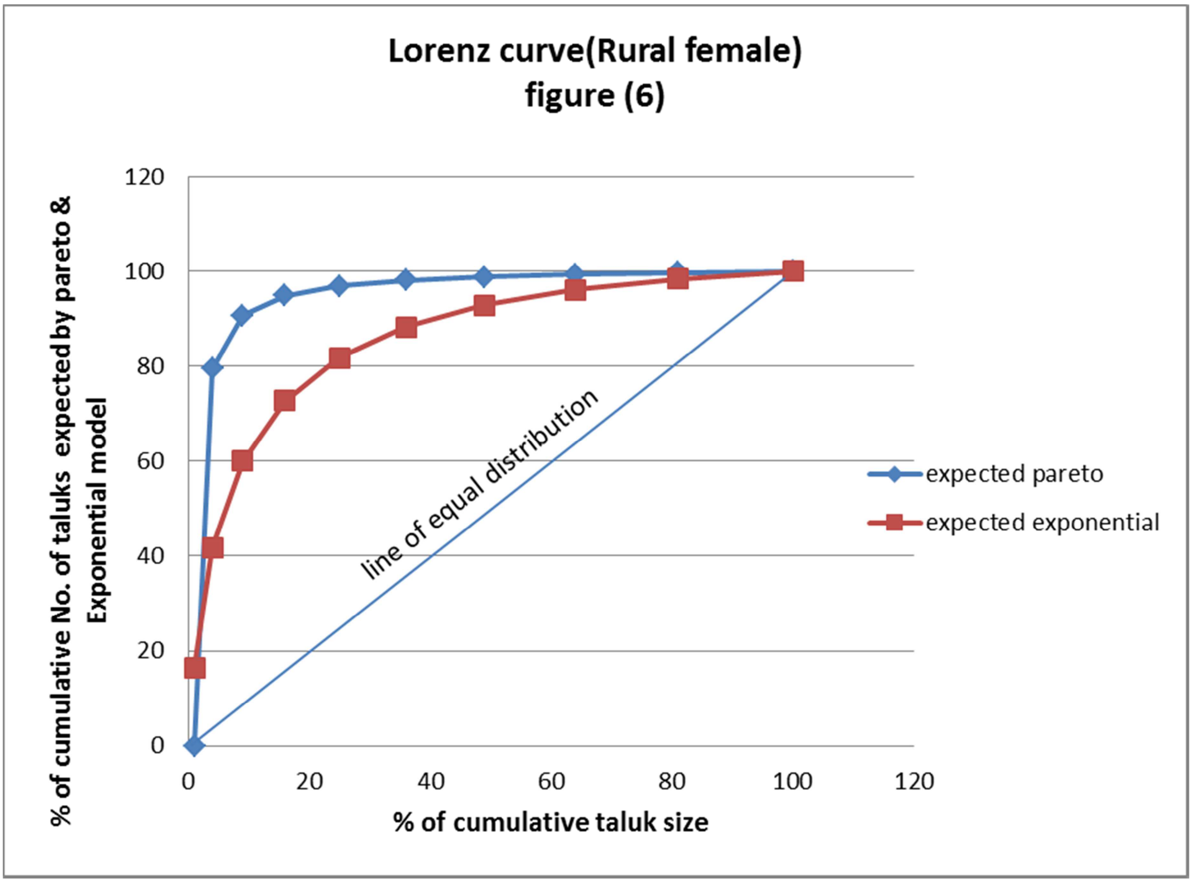



Lorenz curve(Rural female)
figure (2)
% of cumulative No. of taluks observed and expected by pareto model
120
100
80
60
40
20
0
0
20
40
60
80
100
120
% of cumulative taluk size
Line of equal distribution
obesrved
expected


Lorenz curve(Rural male and female)
figure (3)
% of cumulative No. of taluks observed
120
100
80
60
40
20
0
0
20
40
60
80
100
120
% of cumulative taluk size
Line of equal distribution
observed female
Observed male


Lorenz curve(Rural male and female)
figure (4)
% of cumulative No. of taluks expected by Pareto Model
% of cumulative taluk size
Line of equal distribution
expected male
expected female
0
20
40
60
80
100
120


Lorenz curve(Rural male)
figure (5)
% of cumulative No. of taluks expected by pareto & Exponential model
% of cumulative taluk size
line of equal distribution
expected pareto
expected exponential
0
20
40
60
80
100
120



4 DISCUSSION

The size of rural Taluk population is gradually increasing in nature. It creates the disequilibrium between the existing resources and rural population. Variation in the Taluk size influences the changes in the size and structure of the rural population. Pareto model and Exponential Model were applied to analyse the concentration of Taluk size of rural population using Gini's concentration ratio and Pietra index.

All the revenue Taluks as per 2011 census of India were considered for an empirical study. The results of the study are described as follows.

4.1 ESTIMATES OF THE PARAMETERS-PARETO MODEL

Parameter	Male	Female	Interpretation
x^*	14308	14308	Thresholds level in both the population same.
$\hat{a}$	1.41	1.38	The estimate based on male population is more than the estimate based on female.
Gini's concentration ratio, ρ	0.556	0.5682	The value of ρ for rural female population is greater than the value of ρ for rural male population. It shows TSD for female population has more concentrated than the TSD for male population.
Pietra Index,(P_x)	0.4274	0.4439	The value of P_x for rural female population is greater than the value of P_x for rural male population. It shows TSD for female population has more concentrated than the TSD for male population.

4.2 ESTIMATES OF THE PARAMETERS- EXPONENTIAL MODEL

Parameter	Male	Female	Interpretation
Λ	0.0000115	0.0000118	The estimate based on male population is more than the estimate based on female population.
Gini's concentration ratio, ρ	0.000023	0.000024	The value of ρ for rural female population is greater than the value of ρ for rural male population. It shows TSD for female population has more concentrated than the TSD for male population.
Pietra Index, (P_x)	0.3680	0.3680	The value of P_x for rural male population is equal to the value of P_x for rural female population. It shows that both male and female populations have eqally concentrated in TSD.

4.3 BASED ON GRAPH

Figure (1) has been drawn based on p and q values given in table (2) and table (3). It shows that the observed Lorenz curve and the expected Lorenz curve by Pareto are deviated from the line of equal distribution in the case of male population. It indicates the variability in the Taluk size distribution of male population. But the differences between the observed and expected Lorenz curves existed visible level.

The value of p and q given in table (5) and table (6), the figure (2) has been drawn. It shows that the observed Lorenz curve and the expected Lorenz curve by Pareto are deviated from the line of equal distribution in the case of female population. It indicates the variability in the Taluk size distribution of female population. But the differences between the observed and expected Lorenz curves existed visibly.

Firure (3) has been drawn based on p values given in table (2) and table (5).It shows that the observed Lorenz curve for the empirical distribution of both male and female are deviated from the line of equal distribution. It indicates the variability in the Taluk size distribution of male and female population. But there is a little differences between the observed Lorenz curves existed visibly for both male and female population.

Figure (4) has been drawn based on q values given in table (3) and table (6).It shows that the expected Lorenz curves of TSD for male and female population are deviated from the line of equal distribution. But the difference between the two Lorenz curves approaches to zero level.

Figure (5) has been drawn based on q values given in table (3) and table (7).It shows that the expected Lorenz curve of TSD for male population by both pareto and exponential model are deviated from the line of equal distribution. But expected Lorenz curve by exponential model is close to the line of equal distribution than pareto model. It indicates the variability in the Taluk size distribution of male population existed visibly and exponential model is more suitable than the pareto model for the study of variability in Taluk size distribution.

The value of q given in table (6) and table (8), the figure (6) has been drawn. It shows that the expected Lorenz curve of TSD for female population by both pareto and exponential model are deviated from the line of equal distribution. But expected Lorenz curve by exponential model is close to the line of equal distribution than pareto model. It indicates the variability in the Taluk size distribution of female population existed visibly and exponential model is more suitable than the pareto model for the study of variability in Taluk size distribution.

5 CONCLUSION

A study on the concentration of Rural Taluk population of Tamil Nadu State has been studied using Pareto and Exponential model by considering 2011census.In this study, Exponential model is more suitable than the Pareto model for the study on concentration of rural Taluk size distribution in Tamil Nadu state using Gini's concentration ratio and Pietra index. The concentration of the rural male population in the rural Taluk area is less than the concentration of the rural female population. Because the rural male populations migrate to the nearest cities/urban areas for their employment. The migratory moment of male population to the urban area may be minimized by adapting certain remedial measures such as creating employment opportunity and motivating agricultural activities in rural area.

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