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A Comparative Study on Heat Transfer in Straight Triangular Fin and Porous Pin Fin under Natural Convection

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ABSTRACT: The present work is a study on the efficiency and performance parameters of straight triangular fins and porous pin fins in natural convection. The study is based on a straight triangular fin and a general porous pin fin profile. To formulate heat transfer equation in straight triangular fin MODIFIED BESSEL'S EQUATION is used. Similarly to formulate heat transfer equation for porous fin ADOMIAN DECOMPOSITION METHOD (ADM) is used. General differential equations of different orders are used for formulation of both fins. On the basis of efficiency and effectiveness the two fins are compared and an approximate study is done.

KEYWORDS: Straight triangular fins, porous fins, pin fins.

1 INTRODUCTION

In this age of technology with need of small, intricate and light parts fins play an important role in heat exchanging. Variety of fins have been invented and studied so that they can be made immaculate and more efficient. Electronic gadgets and appliances are becoming smaller in size day by day and hence increasing the demand for small but efficient parts. Porous fins are compact and more efficient than other fins in case of heat transfer in electronic gadgets. Similarly straight triangular fin is more efficient than fin with rectangular cross section. These fins are also used widely for their varied use. They too transmit heat at a better rate as compared to straight fins. For the above discussed needs, research is going on for optimum use of material in fin for maximum heat transfer.

NOMENCLATURE

A	Adomian polynomials, i=0,1,2…
C_p	specific heat of the fluid passing through porous fin(J/kg-K)
Da	Darcy number
g	gravity constant(m/s2)
h	heat transfer coefficient over the fin surface(W/m2K)
k	thermal conductivity(W/mK)
kr	thermal conductivity ratio
K	permeability of the porous fin(m2)
M	mass flow rate of fluid passing through porous fin(kg/s)
Nu	Nusselt number
P	perimeter of the fin(m)
q	actual heat transfer rate per unit area of the porous fin(W/m2)
Q	dimensionless actual heat transfer rate per unit area(W/m2)
qi	ideal heat transfer rate per unit area(W/m2)
Qi	dimensionless ideal heat transfer rate per unit area(W/m2)
Ra	Rayleigh number
R	radius of the fin(m)
Ta	ambient temperature(K)
T_0	base temperature(K)
T	fin surface temperature(K)
v	average velocity of fluid passing through porous fin(m/s)
x	axial length measured from fin tip(m)
X	dimensionless axial distance

GREEK SYMBOLS

β	coefficient of thermal expansion(K-1)
ε	effectiveness of the fin
η	efficiency of the fin
ξ	porosity or void ratio
ψ	radius to length ratio
θ	dimensionless temperature
θ_0	dimensionless tip temperature
ϒ	kinematic viscosity(m2/s)
ρ	density of the fluid(kg/m3)

2 STRAIGHT TRIANGULAR FIN

It has been observed that straight fins with rectangular cross section transfers heat at a good rate. But it was also observed that the heat transfer rate decreases as the thickness increases which implies unnecessary use of thick material. For this reason straight triangular fins came into use. The tapered fin is of paramount practical importance since it yields the maximum heat flow per unit weight.

Let l=length of fin , b=width of the fin , y= thickness at base of the fin.It is assumed that the fin is sufficiently thin i.e. (y << l).

Applying energy balance on the small element dx, we have

$$Q_x = Q_{(x+dx)} + Q_{conv} \qquad (1)$$

On further solving and writing the heat transfer equations in differential form we have

$$\frac{d^2\theta}{dx^2} + \frac{d\theta}{xdx} - \frac{2hl\theta}{xky} = 0 \qquad (2)$$

Let, $B^2 = \frac{2hl}{ky}$

Then the equation (2) becomes

$$X^2\frac{d^2\theta}{dx^2} + X\frac{d\theta}{dx} - B^2X\theta = 0 \qquad (3)$$

Again assuming z to be new independent variable such that

$$Z = 2B\sqrt{x} \quad \text{then } \frac{dz}{dx} = BX^{-1/2} \qquad (4)$$

Thus after further calculation the equation (3) reduces to(in terms of z)

$$\frac{d^2\theta}{dz^2} + \frac{d\theta}{zdz} - \theta = 0 \qquad (5)$$

2.1 BESSEL'S EQUATION

The equation is perfectly identical to the modified Bessel's equation of zero order(n=0) and its general solution is given by

$$\theta = C_1 I_0 (z) + C_2 K_0 (z)$$

putting the value of z we have

$$\theta = C_1 I_0 (2B\sqrt{x}) + C_2 K_0 (2B\sqrt{x}) \qquad (6)$$

Where I_0 and k_0 are modified zero order Bessel's function of first and second kind respectively.[2]

The constants of integration C_1 and C_2 can be found out by applying boundary condition.

At x=l , $\theta=\theta_0$;

At x=0 , θ=finite ; (7)

Solving the equation (6) we get

$$C_2 = 0 \text{ and } C_1 = \frac{\theta_0}{(2B\sqrt{l})I_0} \qquad (8)$$

Putting the value of C_1 and C_2 in the general equation we get the heat transfer in straight triangular fin as:

$$Q_{fin} = b\sqrt{2hky}\ .\ \theta_0 .\frac{I_1(2B\sqrt{l})}{I_0(2B\sqrt{l})} \qquad (9)$$

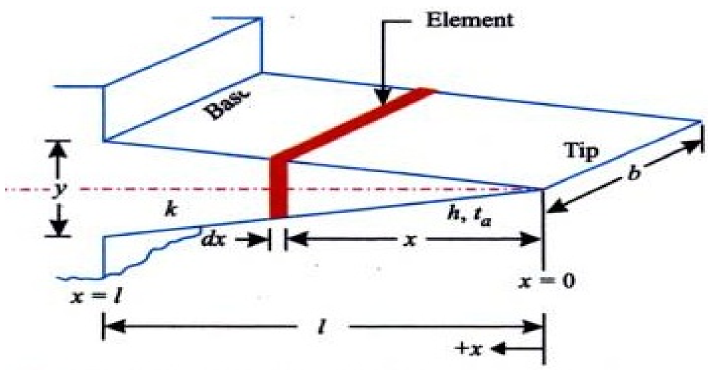

Fig. 2.1.1.Straight Triangular Fin

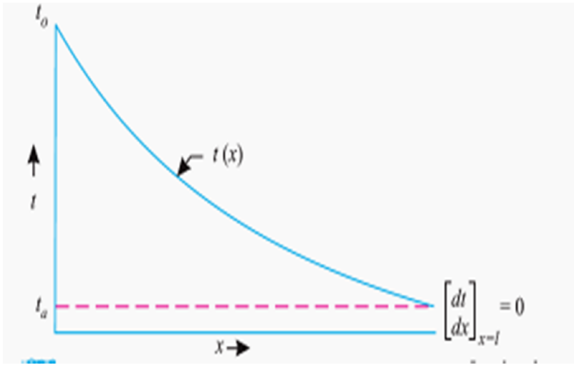

Fig 2.1.2 Temperature variation with distance

As discussed above, there are two Bessel's function I_0 and I_1 , which can be evaluated by using different values of 'z'. These values of some of the Bessel's function are mentioned in the table 2.1 below.[3]

Table 2.1 Typical values of Bessel's function

Z	$I_0(z)$	$I_1(z)$	$\frac{2}{\pi}K_0(z)$	$\frac{2}{\pi}K_1(z)$
0.0	1.000	0.000	∞	∞
0.2	1.010	0.1005	1.116	3.040
0.4	1.040	0.02040	0.7095	1.391
0.6	1.092	0.314	0.4956	0.829
0.8	1.166	0.433	0.360	0.5486
1.0	1.266	0.565	0.2680	0.383
2.0	2.279	1.591	0.0725	0.0890
3.0	4.881	3.953	0.0221	0.0256
4.0	11.302	9.799	0.0071	0.00795
5.0	27.240	24.336	0.00235	0.00257
6.0	67.2348	61.342	0.00027	0.000688
7.0	168.6	156.04	0.000093	0.000289
8.0	427.6	399.9	0.000032	0.000099
9.0	1093.6	1040.9	0.000032	0.000034
10.0	-	-	0.000011	0.000011

3 Porous Fins

Although many innovative ideas have been used in heat transfer in electronic gadgets , porous fins have become an excellent passive means to provide high heat transfer rate for electronic components in a small, light weight, low maintenance and energy free package. Pop and Ingham [4], Nield and Bejan [5]. Enlightened the above discussion quite effectively. A simple method has developed by Kiwan [6] to analyze the performance of porous fins in a natural convection environment. Kiwan and Alnimr [7] numerically investigated the effect of using porous fins to enhance the heat transfer from a given surface.

Similarly Bhanja and Kundu [8] established an analytical model to analyse T shaped porous fins.

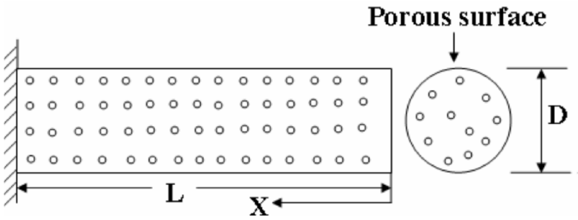

Figure 3: shows a porous fin of length L having pores in it.

Fin is attached to a vertical isothermal wall from which heat has to be dissipated through natural convection. As the fin is porous, it allows fluid to penetrate through it. The porous fin increases the effective surface area of the fin through which the fin convects heat to the working fluid. In order to simplify the solution, the following assumptions are made

- Porous medium is homogeneous, isotropic and saturated with a single phase fluid
- Physical properties of solid as well as fluid are considered as constant except density variation of liquid, which may affect the buoyancy term where Boussinesq approximation is employed.
- Darcy formulation is used to simulate the interaction between the porous medium and fluid.
- The temperature inside the fin is only function of x.
- There are no heat sources in the fin itself and no contact resistance at the fin base.
- The fin tip is adiabatic type.

The total convective heat transfer from the porous fin can be expressed as the sum of convection due to motion of the fluid passing through the fin pores and that from the solid surface. By energy balance considering only convection,

$$q(x) - q(x+dx) = mc_p(T-T_a)+hp(dx)(1-\xi)(T-T_a) \qquad (10)$$

where 'm' is the mass flow rate of the fluid passing through the pores i.e. $m=\rho v.dx.P$

The fluid velocity v can be estimated from Darcy's equation i.e. $v=gK\beta(T-T_a)/\Upsilon$ [4]

Using the above results the general Fourier's equation can be written as

$$\frac{d^2T}{dX^2} - \frac{4\rho c_p gK\beta(T-T_a)^2}{\gamma D k_{eff}} - \frac{4h(1-\xi)(T-T_a)}{Dk_{eff}} = 0 \qquad (11)$$

The dimensionless form of the equation can be written as

$$(X;\psi;k_r;\theta) = \left(\frac{X}{L};\frac{R}{L};\frac{k_s}{k_f};\frac{T-T_a}{T_b-T_a}\right);$$

$$(Nu;Ra;Da) = \left(\frac{hD}{k_f};\frac{\rho c_p g\beta(T_b-T_a)D^3}{k_f\gamma}\frac{K}{D^2}\right); \qquad (12)$$

$$(\omega_1;\omega_2) = \left(\frac{RaDa}{\Omega\psi^2};\frac{Nu(1-\xi)}{\Omega\psi^2}\right);$$

Where,

$$\Omega = \frac{k_{eff}}{k_f} = \xi + (1-\xi)k_r$$

So the equation reduces to

$$\frac{d^2\theta}{dX^2} = \omega_1\theta^2 + \omega_2\theta \qquad (13)$$

The above equation is a differential equation of second degree.

The boundary condition are as follows:

$$\frac{d\theta}{dX} = \mathbf{0} \qquad \text{when } X=0$$

$$\theta = \mathbf{1} \qquad \text{when } X=l \qquad (14)$$

The above equation cannot be solved by general analytical method and hence a method suggested by Adomian G. is used.

3.1 ADOMIAN DECOMPOSITION METHOD

The equation (13) with the boundary condition as in equation (14) is very complicated to solve using general analytical method. Hence, Adomian Decomposition Method is used to solve it as developed by Adomian.[9]

As per ADM the equation can be reduced to

$$L_x\theta = \omega_1\theta^2 + \omega_2\theta \qquad (15)$$

Where L_x is the second order differential. Assuming that inverse of the second order differential exists we can write

$$L_x^{-1}(.) = \iint_{\mathbf{0}}^{X} (.)\, dX\, dX \qquad (16)$$

Applying this inverse operator to equation

$$\theta = \theta(0) + \frac{d\theta(0)}{dX} + \omega \mathbf{1}\ Lx - \mathbf{1}\ (\theta 2) + \omega_2 L_x^{-1}(\theta) \qquad (17)$$

θ(0) can be decomposed in terms of a new variable θ_i which is as below

$$\theta_0 = \sum_{i}^{\infty} \theta_i$$

Hence the equation can be written as

$$\sum_{i}^{\infty} \theta_i = \theta_0 + \frac{d\theta(0)}{dX} + \omega_1 L_x^{-1} \sum_{\mathbf{0}}^{\infty} Ai + \omega_2 L_x^{1} \sum_{i}^{\infty} \theta \qquad (18)$$

Where A_i is the Adomian polynomial which corresponds to a nonlinear term θ_i.

This adomian polynomial can be expressed in matrix form as below:

$(A_0;A_1;A_2;A_3;.....)=(\theta_0^2;2\,\theta_1\,\theta_2\,;2\,\theta_2\,\theta_3\,;......)$

Using the above equation we get the temperature excess expression as

$$\theta = \theta_0 + (\omega_1\,\theta_0^2 + \omega_2\,\theta_0)\frac{X^2}{2!} + (2\omega_1^2\,\theta_0^3 + 3\omega_1\,\omega_2\,\theta_0^2 + \omega_2\,\theta_0)\frac{x^4}{4!} + \dots\dots\dots \qquad (19)$$

Applying the boundary condition we have the equation re written as below

$$1 - \theta_0 + (\omega_1\,\theta_0^2 + \omega_2\,\theta_0)\frac{1}{2!} + (2\omega_1^2\,\theta_0^3 + 3\omega_1\,\omega_2\,\theta_0^2 + \omega_2\,\theta_0)\frac{1}{4!} + \dots\dots\dots \qquad (20)$$

Now expressing the expression as Fourier equation we have

$$Q = \frac{\frac{q}{k_{f(T0-Ta)}}}{R} = \Omega\,\psi\left(\frac{d\theta}{dx}\right) \qquad (21)$$

Also heat transfer rate per unit area in ideal porous fin can be expressed as

$$Q_i = Ra\ Da + Nu(1-\xi)/\psi$$

Similarly for porous unfinned we have

$$Q_w = 0.5\ Nu$$

Thus efficiency of the porous pin fin can be expressed as

$$\eta = Q / Q_i$$

The fin effectiveness can be expressed as

$$\varepsilon = Q / Q_w \tag{22}$$

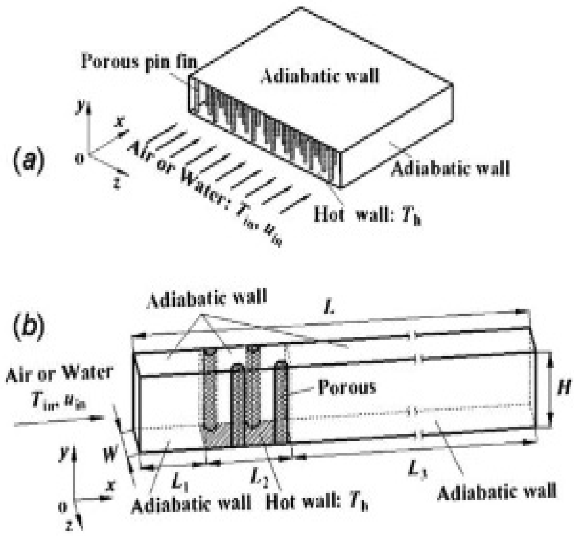

Fig 3.1.1 Porous pin fins as heat exchangers

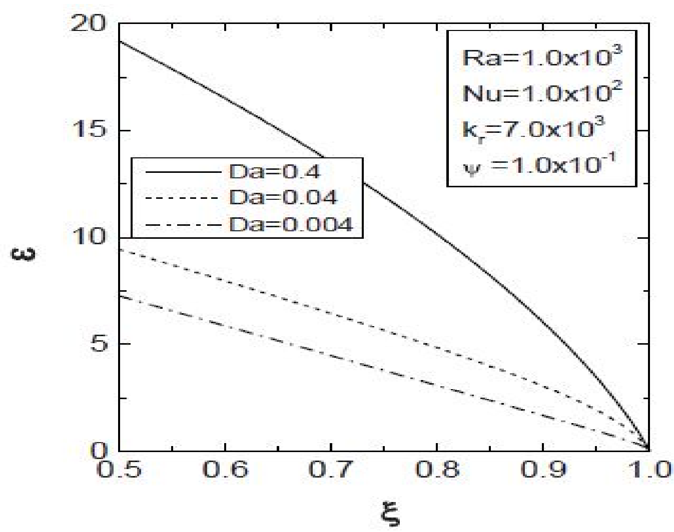

Fig.3.1.2 Plot showing effectiveness Vs void ratio

4 RESULTS AND CONCLUSION

As discussed earlier we observed that the electronic gadgets work much efficiently in lower temperature range. The electronic products are mainly composed of semiconductors which are very sensitive to temperature. Hence they are temperature sensitive. It was a common thinking among the human race that lowering the temperature increases the reliability to an approximate 10 degree celcius lower in temperature so as to double the reliability. A typical electronic gadget works well or near perfectly under 100 degree celcius.

With increase in technology gadgets are getting smaller and quicker. This throws light on the increasing workload on the electronic gadgets but small size; and hence increases in heat produced during operation in the gadgets. Alternatives for this problem were investigated on a wider scale and different types of cooling fans were introduced. But although these were invented porous pin fins emerged as a better option in heat transfer in electronics gadgets.

4.1 STRAIGHT TRIANGULAR FIN VERSUS POROUS PIN FIN

Straight triangular fins are fins which yields the maximum heat flow per unit weight. But porous fins yields maximum heat flow than straight triangular fin because of huge surface area as pores and outer surface. As the fluid pass through the numerous pores they transmit a much larger amount of heat than general profile of straight triangular fins.

As plotted in graph it can be seen that the porous fin has a wide range of variation in relation with Darcy number and distance from the tip. The analytical method or ADM or Adomian Decomposition Method is used because the general analytical method cannot be used for its study. Other than ADM numerical methods are also used. Numerical method use Gauss – Siedel iteration to find out the approximate values of heat transfer in porous pin fins.

Graphs have been plotted between Darcy number , Nusselt number,Rayleigh number and distance from tip of the porous fin.The graphs are plotted in Fig 3.1.2, Fig 4.1.1 and Fig 4.1.2.The value of I_0 and I_1 is shown in table 2.1 for different values of B.

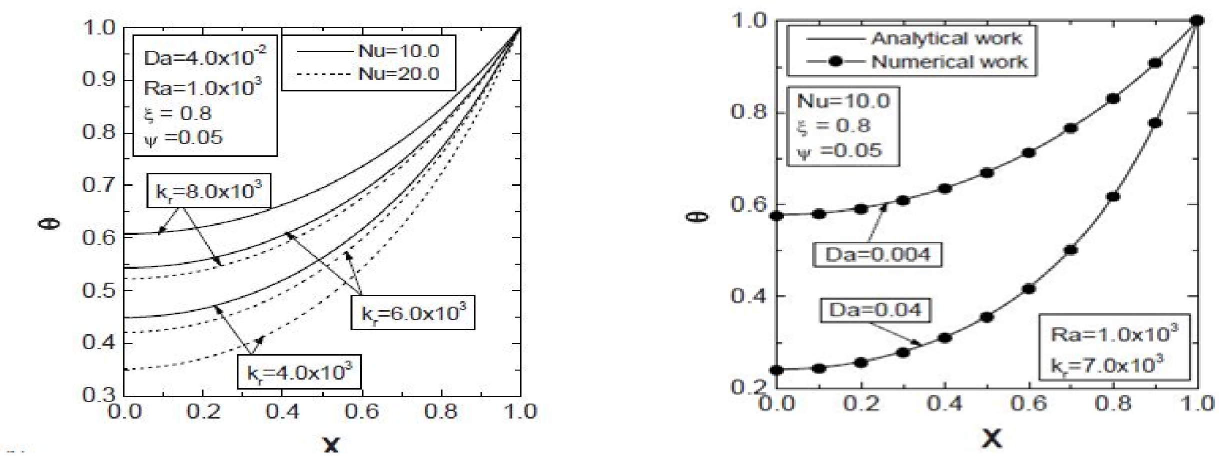

Fig.4.1.1 Plot showing variation of temperature excess to distance with constant darcy number and constant thermal conductivity

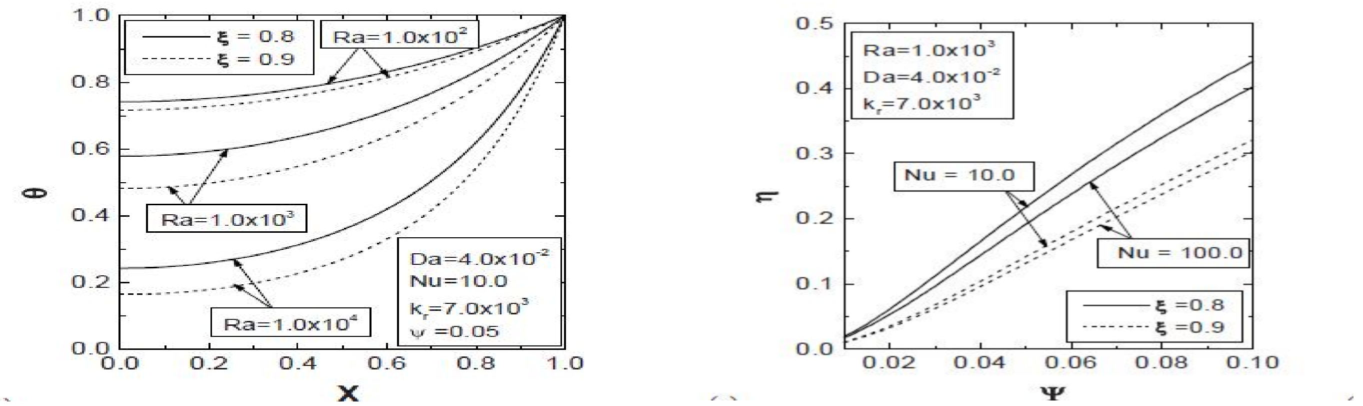

Fig.4.1.2 Variation of temperature excess to distance and efficiency to length to radius ratio

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