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A Direct Approach of Searching for an Optimal solution for Transportation Problems

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ABSTRACT: In this paper, a new method (ASM-Method) of finding an optimal solution for transportation problems is implemented. The method finds an optimal solution without requiring an initial basic feasible solution. A numerical illustration of the method is implemented and the result obtained is compared with that of a designed LINGO computer program for transportation problems.The most attractive feature of this method is that it does not require complex arithmetic and logical calculations which makes it easy to understand and use. The method can be of good for decision makers who are dealing with problems of supply chains and inventory. The method can thus be adopted on existing transportation problems.

KEYWORDS: ASM-Method, Optimality, Lingo, Transportation problems.

1 INTRODUCTION

A transportation problem is one of the most important and earliest applications of linear programming. In transportation problem, the goal is to minimize the total cost of shipping a single commodity from m origins to n destinations subject to supply and demand constraints [4],[1]. The mathematical formulation of transportation problem based on [1],[2],[3] is described as;

$$Min\sum_{i=1}^{m}\sum_{j=1}^{n}C_{ij}X_{ij} \tag{1}$$

$$\sum_{j=1}^{n}X_{ij} = a_i, i = 1,...,m \tag{2}$$

$$\sum_{i=1}^{m}X_{ij} = b_j, j = 1,...,n \tag{3}$$

$$Xij \geq 0, i = 1,...m; j = 1,...,n \tag{4}$$

- C_{ij} : Cost to transport one unit of commodity from origin i to destination j.
- X_{ij} : Amount to be shipped from origin i to destination j.
- a_i : Supply available at origin i.
- b_j : Demand requested at destination j.

In 1941, Hitchcock [2] developed the basic transportation problem together with constructive method of solution. And later in 1949, a scientist Koopmans [7] discussed the problem in a more expanded form. Dantzig [3] in 1951 formulated the transportation problem as a linear programming problem and predicted the solution.

For obtaining an optimal solution for transportation problem, the problem is required to be solved in two stages. The first stage is to find an initial basic feasible solution obtained by any of the well known methods such as the "Northwest corner rule", "vogel's approximation, "Least cost method". The next stage is to apply the "MODI (modified distribution) and the stepping stone method to obtain an optimal solution for the transportation problem.

In this, research, we are going to apply the ASM-Method to find an optimal solution for a transportation problem without finding an initial basic feasible solution using the Vogel's, Northwest corner rule or the minimum cost method. A designed lingo computer program is used for optimality check and verification of the method.

2 ASM-METHOD

The iteration algorithm for the proposed method os described below;

Algorithm 1

Step 1 : Construct the transportation tableau.

Step 2 : Subtract the the minimum cost of each row i and each column j from the the entries of the row and column $(maxC_{ij} - minC_{ij})$.

Step 3 : Select the first zero (row-wise) occurring in the cost matrix as there will be at least one zero in each column and each row of the cost matrix. Suppose (i^{th}, j^{th}) zero is selected, count the total number of zeros (excluding the selected one in both the i^{th} row and j^{th} column respectively. Then select the next zero and count the total number of zeros in the corresponding row and column. Do this procedure for all the zeros in the cost matrix.

Step 3 : select a zero in which the number of zeros counted in step 3 is minimum and allocate maximum possible amount to the cell. If there is tie for some zeros, then choose a $(K, I)^{th}$ zero breaking tie such that the sum of all the cost the cost elements in the K^{th} row and I^{th} column is maximum. Allocate as much as possible to the cell.

Step 4 : Delete the row and column where demand and supply is satisfied.

Step 5 : Check to see if the resultant matrix has atleast one zero in each row and column, if not, repeat step 2, else goto step 6.

Step 6 : Repeat step 3 to 6 and stop when both demands and supply are satisfied.

3 NUMERICAL EXAMPLE

Consider the following balanced transportation problem in table 1 below;

Table 1. Cost,supply and demand of water from 3 reservoirs to 4 cities in Jordanian Dinar (JD)

FROM/TO	CITY 1	CITY 2	CITY 3	CITY 4	SUPPLY
Reservoir A	13	18	30	8	8
Reservoir B	55	20	25	40	10
Reservoir C	30	6	50	10	11
DEMAND	4	7	6	12	

Table 2. Optimal solution obtained from the ASM-Method

FROM/TO	CITY 1	CITY 2	CITY 3	CITY 4	SUPPLY
Reservoir A	13 **[4]**	18	30	8 **[4]**	8
Reservoir B	55	20 **[4]**	25 **[6]**	40	10
Reservoir C	30	6 **[3]**	50	10 **[8]**	11
DEMAND	4	7	6	12	

The Total Optimal cost associated with these allocations is

Min Z=4*13+8*4+20*4+25*6+6*3+10*8 = **412JD**

4 OPTIMALITY CHECK AND METHOD VERIFICATION

To verify the the validity of the ASM-Method, a program of the above transportation problem example is designed in LINGO linear programming software [6]. The result of the program is shown the section 4.1, the optimal result obtained from the ASM-Method corresponds to that of the program.

4.1 THE PROGRAM RESULT

Global optimal solution found.

Objective value: **412.0000**

Infeasibilities: 0.000000

Total solver iterations: 6

Elapsed runtime seconds: 0.09

Model Class: LP Total variables: 12

Nonlinear variables: 0

Integer variables: 0

Total constraints: 8

Nonlinear constraints: 0

Total nonzeros: 36

Nonlinear nonzeros: 0

Variable Value Reduced Cost

SUPPLY(RESERVOIR1) 8.000000 0.000000

SUPPLY(RESERVOIR2) 10.00000 0.000000

SUPPLY(RESERVOIR3) 11.00000 0.000000

DEMAND(CITY1) 4.000000 0.000000

DEMAND(CITY2) 7.000000 0.000000

DEMAND(CITY3) 6.000000 0.000000

DEMAND(CITY4) 12.00000 0.000000

COST(RESERVOIR1, CITY1) 13.00000 0.000000 COST(RESERVOIR1, CITY2) 18.00000 0.000000 COST(RESERVOIR1, CITY3) 30.00000 0.000000 COST(RESERVOIR1, CITY4) 8.000000 0.000000 COST(RESERVOIR2, CITY1) 55.00000 0.000000 COST(RESERVOIR2, CITY2) 20.00000 0.000000 COST(RESERVOIR2, CITY3) 25.00000 0.000000 COST(RESERVOIR2, CITY4) 40.00000 0.000000 COST(RESERVOIR3, CITY1) 30.00000 0.000000 COST(RESERVOIR3, CITY2) 6.000000 0.000000 COST(RESERVOIR3, CITY3) 50.00000 0.000000 COST(RESERVOIR3, CITY4) 10.00000 0.000000 SHIP(RESERVOIR1, CITY1) 4.000000 0.000000 SHIP(RESERVOIR1, CITY2) 0.000000 14.00000 SHIP(RESERVOIR1, CITY3) 0.000000 21.00000 SHIP(RESERVOIR1, CITY4) 4.000000 0.000000 SHIP(RESERVOIR2, CITY1) 0.000000 26.00000 SHIP(RESERVOIR2, CITY2) 4.000000 0.000000 SHIP(RESERVOIR2, CITY3) 6.000000 0.000000 SHIP(RESERVOIR2, CITY4) 0.000000 16.00000 SHIP(RESERVOIR3, CITY1) 0.000000 15.00000 SHIP(RESERVOIR3, CITY2) 3.000000 0.000000 SHIP(RESERVOIR3, CITY3) 0.000000 39.00000 SHIP(RESERVOIR3, CITY4) 8.000000 0.000000

Row Slack or Surplus Dual Price 1 412.0000 -1.000000 2 0.000000 -29.00000 3 0.000000 -20.00000 4 0.000000 -25.00000 5 0.000000 -24.00000 6 0.000000 16.00000 7 0.000000 0.000000 8 0.000000 14.00000

5 Conclusion

It can be concluded that the ASM-Method indeed provides an optimal solution for transportation problems. One can straight use apply the method to obtain an optimal solution as it does not require a basic feasible solution to attain optimality. As such, the proposed method is better than the MODI method and the stepping stone method that require a basic feasible solution to attain optimal solution. The method is therefore applicable to all categories of transportation problems.As this method consumes less time and is very easy to understand and apply, it will be very helpful for decision makers who are dealing with logistic, inventory and supply chain problems.

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