

## Modeling Selection Criteria for Reinforcement Steel Bars with Stochastic Carbon Equivalent distribution

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**ABSTRACT:** Deciding the functional suitability of recycled steel using carbon equivalent (CE) and strength can be misleading since the formulae used to determine CE do not capture many of the elements that play a decisive role in establishing steel values. In this study, a mathematical model is developed to optimize the selection decision from a steel manufacturer considering a stochastic CE distribution. In the given model, a building/fabrication contractor intends to select one of two manufacturers of recycled steel bars basing on CE as determined by the IIW formula and strength values selected in equal monthly intervals. A Markov decision process approach is adopted where three states of a Markov chain represent possible states of CE of steel bars. The ultimate strength,  $\sigma_u$ , of steel is maximized for minimum CE where the decision to select the best steel is made using dynamic programming over a finite period planning horizon. A numerical example demonstrates the existence of an optimal state-dependent selection decision and strength of steel over the planning horizon.

**KEYWORDS:** Carbon equivalent, recycled steel, modeling, strength of steel, stochastic.

### 1 INTRODUCTION

Plain carbon steel is mainly an alloy of iron and carbon. The carbon content of steel is responsible for not only its strength, but also its ductility and many application based properties like weldability and hardenability. Reinforcement bars, projected to improve on the ductility of concrete, are principally made from plain carbon steel and derive much of their efficiency from its properties.

The nature of the influence of carbon on steel, however, is affected by other alloying elements which, present as impurities, modify the effect of carbon although some of them exercise a direct effect on steel properties. These alloying elements have individual effects that are additive and increase with particular alloying element content [15]. The total effect of these elements has been expressed by the carbon equivalent (CE) value with a view to convert the percentage of alloying elements other than carbon to an equivalent carbon percentage because the iron-carbon phases are better understood than other iron-alloy phases [8]. The widespread use of CE value in making important decisions in welding, casting, rolling and heat treatment of steel has made it an indispensable component of all steel standards world over [11].

The CE value tends to denote the susceptibility of steel to weld cracking, the machinability of castings, the hardenability of steel and cast iron during heat treatment [20] and the eligibility in concrete reinforcement use where the low CE values for high strength values is an optimum combination for high strength steel bar of high ductility, weldability and formability [13]. These features underlie the structural integrity of the steel-concrete composite in the event of cracking and enable the steel bars to reliably be joined and bent into small radii during their placement [1]. The same features result essential for fatigue

performance so as to enable the structure to endure cyclic loading [2, 21] rife in the event of wind and earthquake laden conditions [12] and influence the performance of the steel bars even when subjected to static bending moments which in the presence of ribs on their surfaces, increase their susceptibility to stress concentration [23].

When the weldability and other properties of steel are specified in terms of the chemical composition and in particular the CE however, only a limited number of alloying elements is taken into account. These elements are however, only a small fraction of the chemical content when it comes to recycled steel, the major resource for concrete reinforcing bars. Because of this, when the strength of steel is found sufficient and the accompanying CE assessed equally suitable for a specific purpose, chances are high that the CE value of the steel is under estimated in view of the fact that many of the tramp elements not captured in the CE formula have substantial influence on its real value and ultimately on vital functional steel properties. A salient example of such elements is Boron which, being a common tramp element in recycled steel, will all other factors being constant, increase the hardenability of steel by up to 1.5 times when present in less than 0.003% by weight [6]. The globally prevalent thermo-mechanically treated (TMT) high strength reinforcement bars owe their properties to heat treatment which relies on steel hardenability.

The fact that the tramp element content of steel cannot be economically decreased in industrial manufacturing and that the dwindling availability of steel scrap has caused a decisive and continually growing reduction in the quality of the scrap input over the years [17] makes it necessary that a method be devised to predict the steel with a CE value that corresponds to the lowest effect of the un captured tramp element content for the maximum acceptable tensile strength.

Research has shown that the occurrence of the tramp elements in recycled steel exhibits a random (stochastic) incidence [16]. The purpose of this paper therefore, is to resource a probabilistic approach adept at attaching the real value of the CE corresponding to the ultimate strength of a selected consignment of steel bars through a Markov decision process approach using dynamic programming for optimality tests.

## 2 LITERATURE REVIEW

Carbon equivalent (CE) formulae were first developed to provide a numerical value for a steel composition that would give an indication of a carbon content which would cause an equivalent level of hardenability as that of the steel in question [4]. These formulae were later extended to represent the contribution of the composition to the hydrogen cracking tendency of steel in welding operations and are now also linked to other properties that may be related to hardness, such as toughness and strength [9]. Their aim was to convert the percentage of alloying elements other than carbon to an equivalent carbon percentage. The resulting figure is important in the decision making that lays precursor to casting, welding, heat treatment and many other mechanical engineering manufacturing projects.

In physical terms, while in welding, CE is used to understand how the different alloying elements affect the hardness of the steel being welded ; a feature which is directly related to hydrogen-induced cold cracking [14], in the foundry practice, the CE concept is used to understand how alloying elements will affect casting behavior and as a predictor of the strength of cast irons. This is achieved by giving an approximation of the austenite and graphite contents in the final structure and features related to critical cooling time as indicator of how easily a steel or a cast iron undergoes martensitic transformation [18]. It also provides an idea of the cooling temperature range and graphitization or carbide forming potential [7]. In the heat treatment of steel, the CE expresses the critical cooling time required for a steel to change into 100% martensite which then has a direct relation with the ideal critical diameter [20].

Determining the carbon equivalent of steel is therefore an issue of substantial importance. In the past several equations have been used to determine the CE value basing on their chemical composition. These have taken many forms.

In earlier times, Dearden and O'Neill first proposed a formula for steel strength, hardenability and HAZ hardness which took a simplified version used by the International Institute for Welding (IIW) as:

$$C_{eq} = C + \frac{Mn}{6} + \frac{(Cr + V + Mo)}{5} + \frac{(Cu + Ni)}{15} \dots \dots \dots i)$$

and has generally been used to measure weldability [8].

Ito and Bessyo [10] developed another formula based on a wider range of steels than the IIW formula

$$P_{cm} = C + \frac{Si}{30} + \frac{Mn + Cu + Cr}{20} + \frac{Ni}{60} + \frac{Mo}{15} + \frac{V}{10} + 5B \dots \dots ii)$$

While Düren [9] came up with a similar version

$$C_{Eq} = C + \frac{Si}{25} + \frac{Mn + Cu}{16} + \frac{Ni}{40} + \frac{Cr}{10} + \frac{Mo}{15} + \frac{V}{10} \dots \dots \dots iii)$$

also for low alloy steel.

The Yurioka [22] formula:

$$CEN = C + A(C) \times \left( \frac{Si}{24} + \frac{Mn}{6} + \frac{Cu}{15} + \frac{Ni}{60} + \frac{Cr + Mo + Nb + V}{5} + 5B \right) \dots iv)$$

where  $A(C) = 0.75 + 0.25 \tanh\{20 \times (C - 0.12)\}$

also came up for a still wider variety of steel, including low-alloy and carbon structural steels.

Cotrell [7] later originated the relation;

$$CE_w = C + \frac{Mn}{14} + \frac{Ni}{30} + \frac{Cr}{10} + \frac{Mo}{10} + \frac{V}{6} + \frac{Nb}{2.5} + \frac{Cu}{30} + 3N + 20B \dots \dots \dots v)$$

to improve on the prediction of weld cracking.

It can be noted that equation *i)*, the most widely used CE, does not capture many of the elements that form part of the composition of steel bars made from recycled steel. A major concern is the case of Boron which is known to be introduced into induction furnace steel by their scrap origin and the boric acid binder often used in induction furnace ramming mass [17].

Formula *ii)* due to Ito and Bessyo [5] and formula *iv)* by Yurioka [22] both indicate the significance of the Boron content while the Cotrell formula (*v)*, [7] captures Nitrogen, an interstitial element effective in minute inclusion levels with high strengthening capacity.

The use of formula *i)* therefore leaves out a large measure of elemental influence and used for recycled steel, it could be substantially inaccurate in many circumstances.

For a given bunch of steel bars, the steel strengths corresponding to calculated CE values of selected bars can be optimized so that only certain CE value levels match pre-decided steel bar strengths using probabilistic projections to minimize the inclusion of bars with the unwanted residual element effect. Ideally, this would mean low CE values to correspond to high ultimate strength inside the standardized strength bracket. For a maximum carbon content of 0.27%C for thermo-mechanically treated bars [5], the CE value of not exceeding 0.3% would correspond to the ultimate strength ( $\sigma_u$ ) range whose upper limit would be 710Mpa.

### 3 MODEL FORMULATION

#### 3.1 NOTATION AND ASSUMPTIONS

$i, j$	=	States of demand
H	=	High state
A	=	Average state
L	=	Low state
$n, N$	=	Stages
Z	=	Selection decision
$N_{ij}^z$	=	Number of transitions
$C_{ij}^z$	=	Transition matrix for carbon equivalence
$C_{ij}^z$	=	Carbon equivalence transition probability
$S^z$	=	Matrix for strength of steel
$S_{ij}^z$	=	Strength of steel due to carbon equivalence transition
$e_i^z$	=	Expected strength of steel
$a_i^z$	=	Accumulated strength of steel
m	=	Manufacturer
$i, j \in \{H, A, L\}$	$m \in \{1, 2\}$	$Z \in \{1, 2\}$ $n=1, 2, \dots, N$

Consider a production system consisting of two manufacturing plants producing recycled steel bars in batches for a designated number of customers. The CE of steel bars during each time period over a fixed planning horizon is classified as *High* (denoted by state H), *Average*, (denoted by state A) or *Low* (denoted by state L). The transition probabilities for carbon equivalence over the planning horizon from one state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to select bars from manufacturer 1(a decision denoted by Z=1) or to select bars from manufacturer 2 (a decision denoted by Z=2) during each time period over the planning horizon. Optimality is defined such that the expected strength of steel is accumulated at the end of N consecutive time periods spanning the planning horizon under consideration. In this paper, a two-period (N=2) planning horizon is considered.

**3.2 FINITE - PERIOD DYNAMIC PROGRAMMING PROBLEM FORMULATION**

Recalling that CE can be in state H, state A, or in state L, the problem of finding an optimal selection decision among the manufacturers may be expressed as a finite period dynamic programming model.

Let  $T_n(i)$  denote the optimal expected strength of steel accumulated during the periods  $n, n+1, \dots, N$  given that the state of the system at the beginning of period  $n$  is  $i \in \{H, A, L\}$ . The recursive equation relating  $T_n$  and  $T_{n+1}$  is:

$$T_n(i) = \max_Z [C_{iH}^Z(m)S_{iH}^Z(m) + S_{n+1}(H), C_{iA}^Z(m)S_{iA}^Z(m) + S_{n+1}(A), C_{iL}^Z(m)S_{iL}^Z(m) + S_{n+1}(L)] \tag{1}$$

i.e.  $\{H, A, L\}$ ,  $m = \{1, 2\}$ ,  $n = 1, 2, \dots, N$

together with the final conditions

$$T_{N+1}(H) = T_{N+1}(A) = T_{N+1}(L) = 0$$

This recursive relationship may be justified by noting that the cumulative strength of steel  $S_{ij}^Z(m) + S_{N+1}(j)$

resulting from reaching state  $j \in \{H, A, L\}$  at the start of period  $n+1$  from state  $i \in \{H, A, L\}$  at the start of period  $n$  occurs with probability  $C_{ij}^Z(m)$ .

$$\text{Clearly, } e^Z(m) = [C_{ij}^Z(m)] [S_{ij}^Z(m)]^T, \quad Z \in \{1, 2\}, \quad m \in \{1, 2\} \tag{2}$$

where 'T' denotes matrix transposition, and hence the dynamic programming recursive equations

$$T_N(i) = \max_Z [e_i^Z(m) + C_{iH}^Z(m)T_{N+1}(H) + C_{iA}^Z(m)T_{N+1}(A) + C_{iL}^Z(m)T_{N+1}(L)] \tag{3}$$

$$T_N(i, m) = \max_Z [e_i^Z(m)] \tag{4}$$

result where (4) represents the Markov chain stable state.

**3.2.1 COMPUTING  $C^Z(m)$**

The transition probability for CE from state  $i \in \{H, A, L\}$  to state  $j \in \{H, A, L\}$ , given selection decision  $Z \in \{1, 2\}$  may be taken as the number of state transitions observed at manufacturing plant  $m$  with CE initially in state  $i$  and later with CE changing to state  $j$ , divided by the sum of transitions over all states. That is,

$$C_{ij}^Z(m) = N_{ij}^Z(m) / [N_{iH}^Z(m) + N_{iA}^Z(m) + N_{iL}^Z(m)] \tag{5}$$

i.e.  $\{H, A, L\}$ ,  $Z \in \{1, 2\}$ ,  $m = \{1, 2\}$

**4 OPTIMIZATION**

The optimal selection decision and strength of steel are found in this section for each period separately.

4.1 OPTIMIZATION DURING PERIOD 1

When CE is High (i.e. in state H), the optimal selection decision during period 1 is

$$Z = \begin{cases} 1 & \text{if } e_H^1(m) > e_H^2(m) \\ 2 & \text{if } e_H^1(m) \leq e_H^2(m) \end{cases}$$

The associated strength of steel is then:

$$T_1(H, m) = \begin{cases} e_H^1(m) & \text{if } Z = 1 \\ e_H^2(m) & \text{if } Z = 2 \end{cases}$$

Similarly, when CE is Average (i.e. in state A), the optimal selection decision during period 1 is

$$Z = \begin{cases} 1 & \text{if } e_A^1(m) > e_A^2(m) \\ 2 & \text{if } e_A^1(m) \leq e_A^2(m) \end{cases}$$

The associated strength of steel is then:

$$T_1(A, m) = \begin{cases} e_A^1(m) & \text{if } Z = 1 \\ e_A^2(m) & \text{if } Z = 2 \end{cases}$$

When CE is low (i.e. in state L), the optimal selection decision during period 1 is:

$$Z = \begin{cases} 1 & \text{if } e_L^1(m) > e_L^2(m) \\ 2 & \text{if } e_L^1(m) \leq e_L^2(m) \end{cases}$$

The associated strength of steel is then:

$$T_1(L, m) = \begin{cases} e_L^1(m) & \text{if } Z = 1 \\ e_L^2(m) & \text{if } Z = 2 \end{cases}$$

4.2 OPTIMIZATION DURING PERIOD 2

Using dynamic programming recursive equation (1) and recalling that  $a^z_i(m,2)$  denotes the already accumulated strength of steel at the end of period 1 as a result of decisions made during that period, when CE is High (i.e. in state H), the optimal selection decision during period 2 is:

$$Z = \begin{cases} 1 & \text{if } a_H^1(m, 2) > a_H^2(m, 2) \\ 2 & \text{if } a_H^1(m, 2) \leq a_H^2(m, 2) \end{cases}$$

while the associated strength of steel is:

$$T_2(H, m) = \begin{cases} a_H^1(m, 2) & \text{if } Z = 1 \\ a_H^2(m, 2) & \text{if } Z = 2 \end{cases}$$

Similarly, when CE is Average (i.e. in state A), the optimal selection decision during period 2 is:

$$Z = \begin{cases} 1 & \text{if } a_A^1(m, 2) > a_A^2(m, 2) \\ 2 & \text{if } a_A^1(m, 2) \leq a_A^2(m, 2) \end{cases}$$

while the associated strength of steel is:

$$T_2(A, m) = \begin{cases} a_A^1(m, 2) & \text{if } Z = 1 \\ a_A^2(m, 2) & \text{if } Z = 2 \end{cases} \dots$$

When carbon equivalence is Low (i.e. in state L), the optimal selection decision during period 2 is:

$$Z = \begin{cases} 1 & \text{if } a_L^1(m, 2) > a_L^2(m, 2) \\ 2 & \text{if } a_L^1(m, 2) \leq a_L^2(m, 2) \end{cases}$$

and the associated strength of steel is:

$$T_2(L, m) = \begin{cases} a_L^1(m, 2) & \text{if } Z = 1 \\ a_L^2(m, 2) & \text{if } Z = 2 \end{cases}$$

## 5 CASE STUDY

In order to demonstrate use of the model in sections 3 to 4, real case applications from rolling mills 1 and 2 in Uganda are presented in this section. Steel bars are manufactured for fabrication shops and the degree of CE varies for the two manufacturers. The fabrication shop wants to avoid poor strength of steel when the state of CE is High (state H) or Average (state A) in order to utilize steel at lower levels of CE. Hence, decision support is sought for the fabrication shop in terms of an optimal selection decision and the associated strength of steel in a two-month planning period for the two competing manufacturers.

### 5.1 DATA COLLECTION

Past data revealed the following patterns of CE and strength of steel ( $\sigma_u$ ) over 30 days.

Table 1: Manufacturer 1

Days	CE, $C_{ij}^1(1)$	$\sigma_u, S_{ij}^1(1)$	Days	CE, $C_{ij}^1(1)$	$\sigma_u, S_{ij}^1(1)$
1	0.431	673	16	0.406	710
2	0.424	664	17	0.434	677
3	0.394	665	18	0.444	623
4	0.440	651	19	0.362	632
5	0.348	670	20	0.361	646
6	0.390	638	21	0.371	641
7	0.348	670	22	0.399	621
8	0.390	638	23	0.330	606
9	0.430	683	24	0.384	610
10	0.395	668	25	0.376	619
11	0.359	680	26	0.380	656
12	0.368	656	27	0.531	702
13	0.361	686	28	0.518	683
14	0.396	686	29	0.415	718
15	0.521	702	30	0.341	669

Table 2: Manufacturer 2

Days	CE, $C_{ij}^2(2)$	$\sigma_u, S_{ij}^2(2)$	Days	CE, $C_{ij}^2(2)$	$\sigma_u, S_{ij}^2(2)$
1	0.335	665	16	0.326	632
2	0.351	631	17	0.373	675
3	0.442	680	18	0.303	676
4	0.448	710	19	0.322	675
5	0.348	657	20	0.385	629
6	0.369	648	21	0.271	682
7	0.515	702	22	0.501	708
8	0.365	622	23	0.341	640
9	0.486	701	24	0.441	710
10	0.328	575	25	0.341	638
11	0.387	626	26	0.277	658
12	0.358	684	27	0.334	634
13	0.387	673	28	0.315	645
14	0.323	656	29	0.315	634
15	0.381	660	30	0.518	669

5.2 DETERMINING  $C^2(M)$  AND  $S^2(M)$

5.2.1 ESTIMATING ELEMENTS OF  $C^1(1)$  AND  $S^1(1)$

Table 3: Average state

State Transition	No. of Transitions	CE	$\sigma_u$	Transition Probability, CE	$\sigma_u$ Due to state transition
AA	7	0.424 0.394 0.440 0.430 0.394 0.434 0.444	664 665 651 683 668 677 629	$\frac{7}{13} = 0.5385$	$\frac{4637}{7} = 662.4$
AL	5	0.348 0.348 0.359 0.362 0.341	670 670 680 632 669	$\frac{5}{13} = 0.3846$	$\frac{3321}{5} = 664.2$
AH	1	0.521	702	$\frac{1}{13} = 0.0769$	$\frac{702}{1} = 702$
<b>TOTALS</b>	<b>13</b>			<b>1</b>	

**Table 4: Low state**

State Transition	No. of Transitions	CE	$\sigma_u$	Transition Probability, CE	$\sigma_u$ Due to state transition
LA	3	0.390 0.390 0.306	638 638 686	$\frac{3}{13} = 0.2308$	$\frac{1962}{3} = 654$
LH	1	0.531	702	$\frac{1}{3} = 0.0769$	$\frac{702}{1} = 702$
LL	9	0.368 0.361 0.361 0.371 0.399 0.330 0.381 0.376 0.380	658 686 646 641 621 606 610 619 658	$\frac{9}{13} = 0.6923$	$\frac{5743}{9} = 638$
<b>TOTALS</b>	<b>13</b>			<b>1</b>	

**Table 5: High state**

State Transition	No. of Transitions	CE	$\sigma_u$	Transition Probability, CE	$\sigma_u$ Due to state transition
HA	2	0.406 0.415	710 718	$\frac{2}{3} = 0.6667$	$\frac{1428}{2} = 714$
HL	0	0	0	0	0
HH	1	0.518	683	$\frac{1}{3} = 0.3333$	$\frac{683}{1} = 683$
<b>TOTALS</b>	<b>3</b>			<b>1</b>	

State Transition	No. of Transitions	CE	$\sigma_u$	Transition Probability, CE	$\sigma_u$ Due to state transition
HA	0	0	0	0	0
HL	2	0.365 0.341	622 640	$\frac{2}{2} = 1$	$\frac{1262}{2} = 631$
HH	0	0	0	0	0
<b>TOTALS</b>	<b>2</b>			<b>1</b>	

5.2.2 ESTIMATING ELEMENTS OF  $C^2(2)$  AND  $S^2(2)$

Table 6: Low state

State Transition	No. of Transitions	CE	$\sigma_u$	Transition Probability, CE	$\sigma_u$ Due to state transition
LA	3	0.442	710	$\frac{3}{23} = 0.1304$	$\frac{2,121}{3} = 707$
		0.486	701		
		0.441	710		
LL	17	0.351	631	$\frac{17}{23} = 0.7391$	$\frac{11,118}{17} = 654$
		0.369	648		
		0.387	626		
		0.358	684		
		0.387	673		
		0.323	656		
		0.381	660		
		0.326	632		
		0.373	675		
		0.303	676		
		0.322	675		
		0.385	629		
		0.271	682		
		0.277	658		
		0.334	634		
0.315	645				
0.315	634				
LH	3	0.515	702	$\frac{3}{23} = 0.1304$	$\frac{2,079}{3} = 693$
		0.501	708		
		0.518	669		
<b>TOTALS</b>	<b>23</b>			<b>1</b>	

Manufacturer 1:

$$S^1(1) = \begin{bmatrix} 662.4 & 664.2 & 702 \\ 654 & 702 & 638 \\ 714 & 0 & 683 \end{bmatrix}$$

$$C^1(1) = \begin{bmatrix} 0.5385 & 0.3846 & 0.0769 \\ 0.2308 & 0.0769 & 0.6923 \\ 0.6667 & 0 & 0.3333 \end{bmatrix}$$

Manufacturer 2:

$$S^2(2) = \begin{bmatrix} 710 & 623 & 0 \\ 707 & 654 & 694 \\ 0 & 631 & 0 \end{bmatrix}$$

$$C^2(2) = \begin{bmatrix} 0.2500 & 0.7500 & 0 \\ 0.1304 & 0.7391 & 0.1304 \\ 0 & 1 & 0 \end{bmatrix}$$

5.3 CALCULATING  $E_z^1(M)$  AND  $A_z^1(M)$

When steel bars are selected from manufacturer 1 ( $m=1, Z=1$ ), the matrices  $C^1(1)$  and  $S^1(1)$  yield the following strength;

$$e_H^1(1) = (0.5385)(662.4) + (0.3846)(614.2) + (0.0769)(702) = 646.91$$

$$e_A^1(1) = (0.2308)(654) + (0.0769)(702) + (0.6923)(638) = 646.61$$

$$e_L^1(1) = (0.6667)(714) + (0)(0) + (0.3333)(683) = 703.67$$

When steel bars are selected from manufacturer 2 ( $m=2, Z=2$ ), the matrices  $C^2(2)$  and  $S^2(2)$  yield the following strength:

$$e_H^2(2) = (0.2500)(710) + (0.7500)(623) + (0)(0) = 644.25$$

$$e_A^2(2) = (0.1304)(707) + (0.7391)(654) + (0.1304)(694) = 666.06$$

$$e_L^2(2) = (0)(0) + (1)(631) + (0)(0) = 631$$

#### 5.4 THE OPTIMAL DECISION FOR STEEL SELECTION

Month 1:

Since  $646.91 > 644.25$ , it follows that  $Z=1$  is an optimal decision for steel selection in month 1 with associated total strength of 646.91 when CE is High. Since  $666.06 > 646.61$ , it follows that  $Z=2$  is an optimal decision for steel selection in month 1 with associated total strength of 666.06 when CE is Average.

Since  $703.67 > 631$ , it follows that  $Z=1$  is an optimal decision for steel selection in month 1 with associated total strength of 703.67 when CE is Low. Hence, optimality calls for selection of manufacturer 1 when CE is High or Low. Manufacturer 2 can be selected when CE is Average.

The accumulated strength of steel is computed for manufacturer 1 when CE is High, Average or Low and the following results are obtained:

$$a_H^1(1) = 646.91 + (0.5385)(646.91) + (0.3846)(646.61) + (0)(631) = 1243.96$$

$$a_A^1(1) = 646.61 + (0.2308)(646.91) + (0.0769)(646.91) + (0.6923)(631) = 1282.46$$

$$a_L^1(1) = 703.67 + (0.6667)(646.91) + (0)(646.61) + (0.3333)(631) = 1345.28$$

Similarly, the accumulated strength of steel is computed for manufacturer 2 when CE is High, Average or Low and the following results are obtained as;

$$a_H^2(2) = 644.25 + (0.2500)(646.91) + (0.7500)(646.61) + (0)(631) = 1290.94$$

$$a_A^2(2) = 666.06 + (0.1304)(646.91) + (0.7391)(646.61) + (0.1304)(631) = 1310.61$$

$$a_L^2(2) = 631 + (0)(646.91) + (1)(646.61) + (0)(631) = 1277.61$$

Month 2:

Since  $1290.94 > 1243.96$ , it follows that  $Z=2$  is an optimal decision for steel selection in month 2 with associated accumulated strength of 1290.94 when CE is High. Since  $1310.61 > 1282.46$ , it follows that  $Z=2$  is an optimal decision for steel selection in month 2 with associated accumulated strength of 1310.61 when CE is Average.

Since  $1345.28 > 1277.61$ , it follows that  $Z=1$  is an optimal decision for steel selection in month 2 with associated accumulated strength of 1345.28 when CE is Low. Hence in month 2, the optimal selection criterion is in favor of manufacturer 2 when CE is High or Average. Manufacturer 1 can be selected when CE is Low.

## 6 CONCLUSION

An optimization model for determining the selection criteria of steel bars under stochastic CE was presented in this paper. The decision of selecting better steel from two competing manufacturers is modeled as a multi-period decision problem using dynamic programming over a finite period planning horizon. The working of the model was demonstrated by means of a real case study as demonstrated in section 5 of the paper.

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