International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 57 No. 2 Nov. 2021, pp. 169-183

http://www.ijisr.issr-journals.org/

Optimal reducing of harmonics in output voltage of multilevel single phase and three phase inverters

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ABSTRACT: In this paper, we are interested to reduce harmonics odd H_k, with $k > 1$, of the signal in output voltage of five levels single phase and three phase cascade multilevel inverters. This consists to determine optimal switching angles that reduce certain harmonics to zero. As it is usually done, we first mathematically formulate the problem as an algebraic system of equations. After, we transform the problem as an unconstrained optimization problem. Indeed, we optimize the nonlinear functions say F_m and F_t, such that by applying the first order necessary optimality condition, we obtain solutions of the previously considered algebraic system of equations. To solve the problem, we apply conjugate gradient algorithm with both Armijo and Wolfe type line search. With respect to different values of the modulation index, we find optimal angles that eliminate odd harmonics H_k with rank $k = 2n \pm 1, 1 \leq n \leq 5$ for single phase inverters and harmonics H_k with rank $k = 6n \pm 1, 1 \leq n \leq 3$ for three-phase inverters.

KEYWORDS: Inverters, Harmonics, Optimization, Conjugate gradient, line search method.

1 INTRODUCTION

The output voltage issued from an inverter supplied by a battery or an output of a controlled rectifier contains several harmonics. In view to reduce such harmonics several solutions have been proposed. One of these solutions consists to increase the number of inverter levels. This contributes to improve significantly the output voltage signal. Note that the output signal can be square wave, quasi square wave or nearly sinusoidal wave. We distinguish single phase and tree phase inverters. Single-phase inverters convert a direct voltage into a single-phase alternating voltage. These types of inverters are widely used in back-up equipment in computer networks. They make it possible to ensure the continuity of the power supply in the event of an outage on the network, and to filter any faults in the network voltage (interference or overvoltage). They are also found in solar panels, refrigerators, televisions and other household appliances. These inverters deliver a voltage with a fixed frequency. Three-phase inverters convert a direct voltage into three phase alternating voltages. These types of inverters are widely used in power plants, in industry for the speed variation of three phase machines. An inverter is said multilevel when it generates a cut-out voltage composed of at least three levels (shown in Fig.1). The traditional two or three levels inverter does not completely eliminate the unwanted harmonics in the output waveform. When the number of levels increases, the Total Harmonic Distortion (THD) decreases significantly [2].

In literature, many research studies has been conducted on the subject that consists to improve inverters output voltage signal. In [3] to improve inverters output voltage signal, authors resort to the selective harmonic eliminated pulse width modulation technique (SHE PWM) for the control of single- phase and three-phase full bridge three-level inverters. They explain the resolution method procedure of the nonlinear equation

systems in order to achieve the appropriate switching angles. To solve nonlinear equation systems, they apply Newton Raphson algorithm devised to solve unconstrained optimization problems. The method of obtaining the best starting point for Newton algorithm is also described. Note that such a problem is to a difficult task.

In [4] authors present an adapted carrier-based PWM modulation technique applied on both Neutral Point-Clamped (NPC) and Flying Capacitor (FC) structures of multilevel inverters.

The proposed modulation leads to an improved performance on the output voltage Total Harmonics Distortion (THD). It also reduces the total loss dissipation across the semiconductors devices compared to other conventional carrier-based PWM modulation techniques, such as Phase-Shifted PWM (PSPWM) and Level-Shifted PWM (LSPWM). In order to demonstrate the effectiveness of the proposed technique, they developed a three-phase, three-level, 6kW prototype for both NPC and FC topologies.

Chinnathambi and Kaliaperumal (2010) (see [5]) present a power loss minimization technique for a cascaded multilevel inverter using hybrid carrier based space vector modulation (SVM). They combine the features of carrier based space vector modulation and the fundamental frequency modulation strategy. The main characteristic of this modulation is the reduction of switching loss and energy efficiency improvements with better harmonic performance. In [6] authors deal with the removal of few lower order harmonics that are presented in the output of multilevel inverters output voltage to produce better quality output. The removal of lower order harmonics minimizes the Total Harmonic Distortion (TDH) and hence the quality of the output waveform will be increased. The optimum switching angles are obtained by genetic algorithm concept to eliminate few lower order harmonics.

To reduce harmonics of the output voltage signal, Daniel Depernet (1995) consider the three levels inverter. From Fourier signal series expansion, the author formulates the problem that consists to reduce harmonics to an algebraic system of equations. After, Daniel Depernet (1995) (see [1]) reformulates the problem as an unconstrained optimization problem. For solving, he applies conjugate gradient algorithm. The purpose of this paper is to reduce harmonics H_k, with $k > 1$, of the signal in output voltage of five levels single phase and three phase inverters. This consists to determine optimal commutation angles that reduce harmonics to zero. As it is usually done, we first mathematically formulate the problem as an algebraic system of equations. After, we transform the problem as an unconstrained optimization problem. Indeed, we optimize no linear functions as F_m and F_t, such that by applying the first order necessary optimality conditions, we obtain solutions of the previously considered algebraic systems of equations. To solve the problems as in [1], for five levels inverters, we apply gradient conjugate algorithm with both Armijo and Wolfe type line search. With respect to different values of the modulation index, we find optimal angles that eliminate odd harmonics H_k with rank $k = 2n \pm 1, 1 \leq n \leq 5$ for single phase inverters and harmonics H_k with rank $k = 6n \pm 1, 1 \leq n \leq 3$ for three phase inverters, respectively (see result tables). As one can observe, computational results, reported on different tables, show that in few number of iterations, the conjugate gradient algorithm with Wolfe type line search generate stationary solutions of the objective functions F_m and F_t, comparatively to the one combined with Armijo type line search. This confirms the theoretical result. Moreover, Wolfe type line search ensures the fact that the Total Harmonic Distortion (THD) is significantly lower.

The paper is organized as follow. In the second section, we present and describe the mathematical models that formulate the harmonics reducing problem. Such models are based on Fourier series expansion. With respect to single phase and three phase inverters, we consider the algebraic systems of equations resulting from the output voltage signal Fourier series expansions. After, we reformulate the problems as an unconstrained optimization problems. In section 3, we discuss the conjugate gradient algorithm combining with the Wolfe and Armijo type line searches used to solve optimization problems. In the fourth section, we report the computational results obtained applying the conjugate gradient algorithm considering different values of the modulation index.

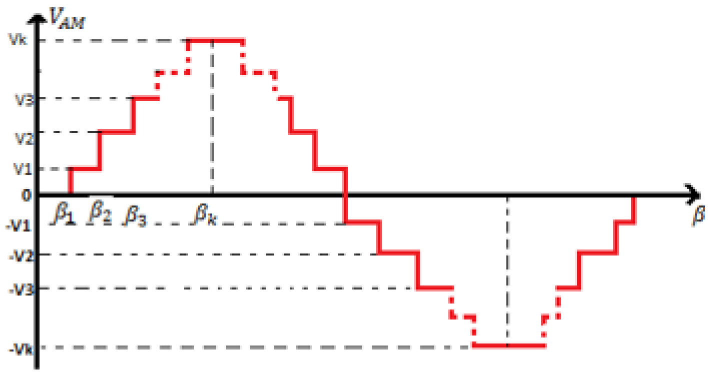

Fig. 1. Example of multilevel inverters output voltage

2 MATHEMATICAL MODELS

The output voltage of the multilevel inverter takes several values on a switching period as shown in Fig.1. The voltage VAM is double symmetrical with respect to the quarter ($\pi/4$) and at the half of period ($\pi/2$). Because of this feature, the study will be limited to a quarter-period only. Fourier series expansion which shows the existence of harmonics of odd orders only, is given by:

$$V_{AM}(\alpha) = \sum_{n=1}^{+\infty} a_n \sin\ (n\alpha)\ \ (1)$$

$$a_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} V_{AM}\ (\alpha)\ \sin(n\alpha)\, d\alpha\ (2)$$

After integration, we get:

$$H_n = \frac{4E}{n\pi}[S_1\cos(n\alpha_1) + S_2\cos(n\alpha_2) + \cdots + S_c\cos(n\alpha_c)]\ (3)$$

Where

n: is an odd number for single-phase inverter and non-multiple of 3 odd number for three-phase inverter;
E: is a DC voltage at the input of the inverter;
H_n: represents an harmonic of order n of the output voltage VAM;
H_1: is the fundamental of the output voltage VAM (n=1);
α_i: is a switching angles;
S_i: the sign of cos ($S_i = 1$ for the angle at which there is a passage from a lower level to a higher level and $S_i = -1$, otherwise [7]);
C: is the number of switching angles for quarter period.

2.1 Mathematical Model For Single Phase Inverters

For single-phase inverters, even harmonics cancel each other out naturally [1]. We thus obtain that equations of harmonics of order $2n \pm 1$ for $n \geq 1$. In our study, these are harmonics H_1, H_3, H_5, H_7, H_9 and H_{11}. The first non-zero harmonic is of rank 13. The system to be solved is:

$$\begin{cases} H_1 = \frac{4E}{\pi}[S_1\cos(\alpha_1) + S_2\cos(\alpha_2) + \cdots + S_c\cos(\alpha_c)] \\ H_3 = \frac{4E}{3\pi}[S_1\cos(3\alpha_1) + S_2\cos(3\alpha_2) + \cdots + S_c\cos(3\alpha_c)] \\ \cdot \\ \cdot \\ \cdot \\ H_{11} = \frac{4E}{11\pi}[S_1\cos(11\alpha_1) + S_2\cos(11\alpha_2) + \cdots + S_c\cos(11\alpha_c)] \end{cases} \quad (4)$$

By setting $r = \frac{H_1}{2E}$ (with $0 < r < 1$) the nonlinear system (4) can be written in the form:

$$\begin{cases} f_1(\alpha_1, \alpha_2, \dots, \alpha_6) = \frac{\pi r}{2} \\ f_3(\alpha_1, \alpha_2, \dots, \alpha_6) = 0 \\ \cdot \\ \cdot \\ \cdot \\ f_{11}(\alpha_1, \alpha_2, \dots, \alpha_6) = 0 \end{cases} \quad (5)$$

r is the modulation index. The objective function to be minimized to resolve the nonlinear system (5) is:

$$F_m = \left[(f_1 - \frac{\pi r}{2})^2 + f_2{}^2 + \cdots + f_{11}{}^2 \right]\ (6)$$

The solutions must satisfy the following condition:

$$\alpha_1 < \alpha_2 < \cdots < \alpha_6 < \frac{\pi}{2} \quad (7)$$

2.2 MATHEMATICAL MODEL FOR THREE PHASE INVERTERS

For a three-phase inverter, even and odd harmonics multiple of 3 cancel each other out naturally [1]. We thus obtain equations of harmonics of order $6n \pm 1$ for $n \geq 1$, in this study they are harmonics $H_1, H_5, H_7, H_{11}, H_{13}$ and H_{17}. The first non-zero harmonic is of rank 19. The system to solve is therefore the following:

The system to be solved is:

$$\begin{cases} H_1 = \frac{4E}{\pi}[S_1\cos(\alpha_1) + S_2\cos(\alpha_2) + \cdots + S_c\cos(\alpha_c)] \\ H_5 = \frac{4E}{5\pi}[S_1\cos(5\alpha_1) + S_2\cos(5\alpha_2) + \cdots + S_c\cos(5\alpha_c)] \\ \cdot \\ \cdot \\ \cdot \\ H_{17} = \frac{4E}{17\pi}[S_1\cos(17\alpha_1) + S_2\cos(17\alpha_2) + \cdots + S_c\cos(17\alpha_c)] \end{cases} \quad (8)$$

The system (8) becomes:

$$\begin{cases} f_1(\alpha_1, \alpha_2, \ldots, \alpha_6) = \frac{\pi r}{2} \\ f_5(\alpha_1, \alpha_2, \ldots, \alpha_6) = 0 \\ \cdot \\ \cdot \\ \cdot \\ f_{17}(\alpha_1, \alpha_2, \ldots, \alpha_6) = 0 \end{cases} \quad (9)$$

The objective function to be minimized to resolve the nonlinear system (8) is:

$$F_t = \left[(f_1 - \frac{\pi r}{2})^2 + f_5{}^2 + \cdots + f_{17}{}^2 \right] \quad (10)$$

The solutions must satisfy the condition (7). Note that by expressing the first order necessary condition on functions F_m and F_t respectively, we determine the switching angles which eliminate the first harmonics of orders 3, 5, 7, 9 and 11 for the single-phase inverter and 5, 7, 11, 13 and 17 for three-phase which are the most troublesome for the loads (with $Si = [1 \ -1 \ 1 \ 1 - 1 \ 1]$).

In the following section, we briefly describe the iterative algorithm and solve system (4) and (8) for single phase and three phase inverters, respectively.

3 THE CONJUGATE GRADIENT ALGORITHM AND LINEAR SEARCH

Even if the conjugate gradient algorithm has been first devised to solve unconstrained optimization, in this paper, we resort to it, in view to solve systems (4) and (8) defined in the previous section. For this reason, applying the first order necessary condition on F_m and F_t is sufficient. Linear search consists in determining the step θ_k which ensures a significantly decrease of the objective function. This amounts to minimizing the one-dimensional function $f\ (x_k + \theta_k d_k)$. There are several methods to determine this step.

In this paper we will focus on *Wolfe*'s and *Armijo*'s methods.

- We say that the step θ_k is *Armijo* step (noted θ_k *)* if it satisfies the so-called *Armijo*'s condition:

$$f(x_k + \theta_k d_k) \leq f(x_k) + \theta_k \beta_1 \nabla f(x_k)^T d_k \quad (11)$$

With $0 < \beta_1 < 1$

- We say that the step θ'_k is *Wolfe* step (noted θ'_k) if it satisfies the so-called *Wolf*'s conditions:

$$\begin{cases} f(x_k + \theta'_k d_k) \leq f(x_k) + \theta'_k m_1 \nabla f\ (x_k)^{\ T} d_k \\ \nabla f\ (x_k + \theta'_k d_k)^{\ T} \geq m_2 \nabla f\ (x_k)^{\ T} d_k \end{cases} \quad (12)$$

with $0 < m1 < m2 < 1$

In the following section, consider different values of the modulation index r, we report on results obtained by solving systems of no linear equations (4) and (8), defined in Section 2 using the conjugate gradient algorithm.

Algorithm 1: Conjugate gradient algorithm (Fletcher & Reeves (1964))

Data: A function F and an initial x^0
Result: The stationnary point x^k of F

```
begin
    ε ← 10^-6
    k ← 1, u_0 ← −∇F(x^0)
    θ ← LinearSearch(0)
    x^1 ← x^0 + θ_0 u_0
    while ||∇F(x^k)|| > ε do
        β_k = ||∇f(x_k)||^2 / ||∇f(x_{k-1})||^2 .
        u_k ← −∇F(x^k) + β_{k-1} * u_{k-1}
        β_k ← LinearSearch(k)
        x^{k+1} ← x^k + θ_k u_k
        k ← k + 1
    end
    return x^k
end
```

4 Computational Results

The computational results of the different objective functions giving the optimal angles for different values of the modulation index *r* are given in Tables 1 to 5 for five levels single-phase inverter and Tables 6 to 9 for five levels three phase inverter. The THD is calculated by the following formula:

$$THD\% = 100 * \frac{\sqrt{\sum_k^{\infty}\ \left(\frac{1}{k}\sum_{i=1}^{c}\cos\ (n\alpha_i)\right)^{\ 2}}}{\sum_{i=1}^{c}\cos\ (n\alpha_i)} \quad (13)$$

In our study, the infinite sum was limited. $k = 3, 5, 7, 9, 11$ and 13 for single-phase inverter and

$k = 5, 7, 11, 13, 17$ and 19 for three phase inverter.

In what follows, Table 1 summarizes entries of Tables 2 to 9 presented below.

Table 1. Meaning of entries of Tables 2 to 9

Column	Description
r	The modulation index
α_i $(i = 1,2\ldots6)$	Switching angles
H_k	Harmonics order k
F_m and F_t	Objective functions depending on the type of inverter
T DH (%)	Total Harmonic Distorsion

Table 2. Computational angles (in radians) related to F_m with Wolfe line search

r	*α1*	*α2*	*α3*	*α4*	*α5*	*α6*	*Fm*
0,614	0,30289	0,47767	0,61599	1,32733	1,41743	1,57724	0
0,662	0,27685	0,43293	0,55998	1,17157	1,25745	1,51281	0
0,665	0,27497	0,42958	0,55627	1,16647	1,25434	1,5109	0
0,666	0,27433	0,42846	0,55504	1,16485	1,25338	1,51026	0
0,667	0,27363	0,42719	0,55369	1,1631	1,25229	1,50964	0
0,668	0,27301	0,4261	0,55249	1,16157	1,25145	1,50901	0
0,669	0,27234	0,42491	0,55122	1,15998	1,25053	1,50839	0
0,67	0,27163	0,42362	0,54987	1,15833	1,24954	1,50776	0
0,671	0,27094	0,4224	0,54858	1,15677	1,24867	1,50715	0
0,672	0,27025	0,42117	0,54728	1,15523	1,24783	1,50653	0
0,673	0,26955	0,41992	0,54598	1,15372	1,247	1,50592	0
0,674	0,26884	0,41866	0,54468	1,15222	1,2462	1,50531	0
0,675	0,26828	0,41772	0,54365	1,15103	1,24572	1,50466	0
0,676	0,26756	0,41643	0,54234	1,14956	1,24495	1,50405	0
0,677	0,26682	0,41512	0,54101	1,14811	1,2442	1,50345	0
0,678	0,26608	0,41381	0,53969	1,14667	1,24348	1,50284	0
0,679	0,26533	0,41247	0,53835	1,14525	1,24277	1,50224	0
0,68	0,26457	0,41112	0,53701	1,14385	1,24208	1,50164	0
0,681	0,2638	0,40975	0,53566	1,14246	1,24141	1,50105	0
0,682	0,26302	0,40837	0,53431	1,14108	1,24075	1,50045	0
0,683	0,26222	0,40697	0,53294	1,13972	1,24011	1,49986	0
0,684	0,26142	0,40555	0,53157	1,13837	1,23949	1,49926	0
0,685	0,2606	0,40411	0,5302	1,13703	1,23888	1,49867	0
0,688	0,25808	0,39969	0,52602	1,1331	1,23715	1,4969	0
0,691	0,25543	0,39509	0,52179	1,12927	1,23554	1,49514	0
0,694	0,25266	0,39031	0,51748	1,12554	1,23404	1,49338	0
0,697	0,24973	0,38532	0,51311	1,12189	1,23265	1,49163	0
0,7	0,24659	0,38	0,50858	1,11828	1,2313	1,48989	0
0,76	0,03777	0,17551	0,40822	1,05699	1,21722	1,45446	0

Table 3. Harmonics and THD related to (Fm) with Wolfe line search

r	*H1*	*H3*	*H5*	*H7*	*H9*	*H11*	*H13*	THD %
0,614	1,228	0,00000	0,00006	0,00009	0,00008	0,00004	0,28102	22,88427
0,662	1,324	0,00008	0,0001	0,00003	0,00001	0,00002	0,05366	4,05281
0,665	1,33	0,00005	0,00009	0,00005	0,00004	0,00005	0,04090	3,07535
0,666	1,332	0,00002	0,00004	0,00003	0,00003	0,00003	0,03671	2,75564
0,667	1,334	0,0001	0,00011	0,00004	0,00001	0,00003	0,0321	2,40601
0,668	1,336	0,00003	0,00006	0,00004	0,00003	0,00004	0,02805	2,09982
0,669	1,338	0,00003	0,00006	0,00004	0,00004	0,00004	0,02376	1,77585
0,67	1,34	0,00009	0,0001	0,00002	0,00001	0,00003	0,01919	1,43197
0,671	1,342	0,00009	0,0001	0,00002	0,00001	0,00003	0,01487	1,10827
0,672	1,344	0,00009	0,00009	0,00002	0,00002	0,00003	0,01055	0,78476
0,673	1,346	0,00009	0,00009	0,00002	0,00002	0,00003	0,00623	0,46265
0,674	1,348	0,00009	0,00009	0,00002	0,00002	0,00003	0,00189	0,14053
0,675	1,35	0,00017	0,00015	0,00003	0,00003	0,00006	0,00154	0,11548
0,676	1,352	0,00017	0,00015	0,00003	0,00003	0,00006	0,00589	0,43565
0,677	1,354	0,00017	0,00015	0,00002	0,00003	0,00006	0,01023	0,75595
0,678	1,356	0,00017	0,00015	0,00002	0,00003	0,00006	0,01459	1,07584
0,679	1,358	0,00017	0,00015	0,00002	0,00003	0,00007	0,01895	1,39531
0,68	1,36	0,00017	0,00015	0,00002	0,00003	0,00007	0,02332	1,71483
0,681	1,362	0,00017	0,00015	0,00002	0,00002	0,00007	0,02771	2,03425
0,682	1,364	0,00017	0,00015	0,00002	0,00002	0,00007	0,03209	2,35247
0,683	1,366	0,00017	0,00015	0,00002	0,00002	0,00007	0,03649	2,67125
0,684	1,368	0,00017	0,00015	0,00002	0,00002	0,00007	0,04090	2,98963
0,685	1,37	0,00017	0,00015	0,00001	0,00002	0,00007	0,04532	3,30802
0,688	1,376	0,00018	0,00015	0,00001	0,00002	0,00008	0,05864	4,26148
0,691	1,382	0,00018	0,00014	0,00001	0,00002	0,00008	0,07207	5,21479
0,694	1,388	0,00018	0,00015	0,00000	0,00002	0,00008	0,08560	6,16735
0,697	1,394	0,00018	0,00014	0,00000	0,00001	0,00008	0,09928	7,12168
0,7	1,4	0,00012	0,00009	0,00000	0,00001	0,00006	0,11332	8,09418
0,76	1,52	0,00023	0,00015	0,00005	0,00005	0,00010	0,43800	28,81557

Table 4. Computational angles (in radians) related to (Fm) with Armijo line search

r	α1	α2	α3	α4	α5	α6	Fm
0,614	0,30289	0,47767	0,61599	1,32734	1,41745	1,57725	0
0,662	0,27697	0,43321	0,56022	1,17192	1,25781	1,51278	0
0,665	0,27507	0,42982	0,55648	1,16676	1,25462	1,51086	0
0,666	0,27443	0,42867	0,55522	1,16509	1,25361	1,51023	0
0,667	0,27377	0,42751	0,55396	1,16345	1,25264	1,50959	0
0,668	0,27311	0,42633	0,55269	1,16182	1,25168	1,50897	0
0,669	0,27245	0,42514	0,55142	1,16022	1,25076	1,50834	0
0,67	0,27177	0,42394	0,55014	1,15864	1,24986	1,50772	0
0,671	0,27109	0,42272	0,54886	1,15708	1,24899	1,50711	0
0,672	0,27040	0,42149	0,54756	1,15554	1,24814	1,50649	0
0,673	0,26970	0,42025	0,54627	1,15402	1,24731	1,50588	0
0,674	0,26900	0,41899	0,54496	1,15252	1,2465	1,50527	0
0,675	0,26816	0,41744	0,54342	1,15078	1,24549	1,50472	0
0,676	0,26739	0,41607	0,54204	1,14925	1,24467	1,50412	0
0,677	0,26663	0,41472	0,54067	1,14777	1,24388	1,50352	0
0,678	0,26590	0,41343	0,53937	1,14637	1,24318	1,5029	0
0,679	0,26516	0,41211	0,53804	1,14497	1,24249	1,50229	0
0,68	0,26439	0,41076	0,5367	1,14357	1,24181	1,50169	0
0,681	0,26362	0,40938	0,53535	1,14218	1,24114	1,5011	0
0,682	0,262830	0,40799	0,53399	1,14081	1,24049	1,5005	0
0,683	0,262030	0,40658	0,53262	1,13945	1,23985	1,49991	0
0,684	0,261230	0,40516	0,53125	1,13811	1,23923	1,49931	0
0,685	0,260400	0,40370	0,52986	1,13676	1,23862	1,49872	0
0,688	0,258030	0,39959	0,52595	1,13304	1,23709	1,49691	0
0,691	0,255380	0,3950	0,52171	1,12922	1,23548	1,49515	0
0,694	0,252600	0,39021	0,517400	1,12548	1,23399	1,49339	0
0,697	0,249680	0,38521	0,513030	1,12184	1,2326	1,49164	0
0,7	0,246650	0,38011	0,508670	1,11833	1,23135	1,48988	0
0,76	0,035940	0,17494	0,40810	1,05695	1,21719	1,45447	0

Table 5. Harmonics and THD related to (Fm) with Armijo line search

r	*H1*	*H3*	*H5*	*H7*	*H9*	*H11*	*H13*	THD %
0,614	1,228	0,00000	0,00005	0,00009	0,00008	0,00003	0,28102	22,88444
0,662	1,324	0,00015	0,00016	0,00005	0,00003	0,00004	0,05451	4,11724
0,665	1,330	0,00016	0,00016	0,00004	0,00003	0,00004	0,04162	3,12907
0,666	1,332	0,00016	0,00015	0,00004	0,00003	0,00004	0,03731	2,80099
0,667	1,334	0,00016	0,00016	0,00004	0,00003	0,00005	0,03301	2,47467
0,668	1,336	0,00016	0,00015	0,00004	0,00003	0,00005	0,02870	2,14828
0,669	1,338	0,00016	0,00015	0,00004	0,00003	0,00005	0,02439	1,82305
0,67	1,340	0,00016	0,00015	0,00004	0,00003	0,00005	0,02008	1,49867
0,671	1,342	0,00016	0,00015	0,00003	0,00003	0,00005	0,01577	1,17548
0,672	1,344	0,00016	0,00015	0,00003	0,00003	0,00005	0,01145	0,85208
0,673	1,346	0,00016	0,00015	0,00003	0,00003	0,00006	0,00713	0,52977
0,674	1,348	0,00017	0,00015	0,00003	0,00003	0,00006	0,0028	0,20815
0,675	1,35	0,00005	0,00009	0,00007	0,00005	0,00006	0,00225	0,16722
0,676	1,352	0,00011	0,00015	0,00007	0,00004	0,00008	0,0068	0,50358
0,677	1,354	0,00014	0,00016	0,00006	0,00002	0,00008	0,01128	0,8333
0,678	1,356	0,00011	0,00012	0,00003	0,00000	0,00005	0,01556	1,14773
0,679	1,358	0,00010	0,0001	0,00002	0,00001	0,00004	0,01989	1,46468
0,68	1,360	0,00009	0,00009	0,00001	0,00001	0,00004	0,02425	1,78330
0,681	1,362	0,00010	0,00009	0,00001	0,00001	0,00004	0,02865	2,10324
0,682	1,364	0,00009	0,00009	0,00001	0,00001	0,00004	0,03303	2,42192
0,683	1,366	0,00010	0,00009	0,00001	0,00001	0,00004	0,03744	2,74107
0,684	1,368	0,00010	0,00009	0,00001	0,00001	0,00004	0,04185	3,05948
0,685	1,370	0,00011	0,0001	0,00001	0,00001	0,00005	0,04631	3,38048
0,688	1,376	0,00011	0,00009	0,00001	0,00001	0,00005	0,05886	4,27798
0,691	1,382	0,00012	0,00009	0,00001	0,00001	0,00005	0,07229	5,23066
0,694	1,388	0,00012	0,00009	0,00000	0,00001	0,00005	0,08583	6,18365
0,697	1,394	0,00012	0,00009	0,00000	0,00001	0,00006	0,09951	7,13824
0,7	1,400	0,00019	0,00014	0,00000	0,00001	0,00009	0,11309	8,07772
0,76	1,520	0,00016	0,00010	0,00003	0,00003	0,00007	0,43844	28,84504

Table 6. Computational angles (in radians) related to (Ft) with Wolfe line search

r	α1	α2	α3	α4	α5	α6	Ft
0,632	0,23319	0,34115	0,40786	1,16431	1,21155	1,57041	0
0,711	0,26297	0,3437	0,64439	1,11391	1,22256	1,37623	0
0,712	0,26379	0,34489	0,64358	1,11354	1,22312	1,37619	0
0,713	0,26459	0,34605	0,64279	1,11315	1,22366	1,37613	0
0,714	0,26538	0,34717	0,64202	1,11276	1,22418	1,37604	0
0,715	0,26615	0,34827	0,64127	1,11236	1,22469	1,37592	0
0,738	0,28183	0,36903	0,62734	1,10225	1,23442	1,36917	0
0,749	0,28862	0,37749	0,62189	1,09711	1,23825	1,36435	0
0,761	0,29574	0,38631	0,61634	1,09143	1,24203	1,35845	0
0,772	0,30216	0,39419	0,61159	1,08615	1,24509	1,35245	0
0,784	0,30900	0,40269	0,60661	1,08036	1,24798	1,34538	0
0,798	0,31681	0,41258	0,60104	1,07356	1,25061	1,33631	0
0,814	0,32549	0,42396	0,59496	1,06574	1,25224	1,32456	0
0,87	0,35217	0,46485	0,57471	1,03668	1,21999	1,24814	0
0,881	0,35504	0,47148	0,56924	1,02911	1,18635	1,20865	0
0,888	0,35516	0,47352	0,56395	1,02205	1,14887	1,17018	0
0,892	0,35404	0,47297	0,55968	1,01627	1,12143	1,14416	0
0,896	0,35191	0,47084	0,55430	1,00804	1,09059	1,11734	0
0,899	0,34969	0,46819	0,54956	0,99941	1,06648	1,09875	0
0,903	0,34599	0,46341	0,54244	1,07869	1,03530	0,98416	0
0,907	0,34179	0,45755	0,53456	1,06564	1,00800	0,96574	0
0,911	0,33697	0,45083	0,52633	1,05630	0,98414	0,94696	0
0,916	0,33049	0,44159	0,51562	1,04863	0,96002	0,92505	0
0,921	0,32352	0,43138	0,50439	1,04362	0,94216	0,90657	0
0,922	0,3221	0,4294	0,50233	1,04251	0,93846	0,90279	0
0,923	0,32068	0,42752	0,50039	1,04128	0,93453	0,89885	0
0,924	0,31922	0,42551	0,49827	1,04013	0,93089	0,8952	0
0,925	0,3177	0,42323	0,49592	1,03949	0,92857	0,89235	0
0,926	0,31613	0,42086	0,49351	1,03894	0,92656	0,88972	0
0,934	0,30512	0,40731	0,47975	1,03001	0,89948	0,86111	0

Table 7. Harmonics and THD related to (Ft) with Wolfe line search

r	*H*1	*H*5	*H7*	*H*11	*H*13	*H*17	*H*19	THD %
0,632	1,264	0,00016	0,00010	0,00010	0,00005	0,00009	0,10967	8,67669
0,711	1,422	0,00013	0,00002	0,00002	0,00010	0,00008	0,02922	2,05467
0,712	1,424	0,00014	0,00002	0,00002	0,00010	0,00008	0,02999	2,10608
0,713	1,426	0,00014	0,00002	0,00001	0,00010	0,00008	0,03081	2,16078
0,714	1,428	0,00014	0,00002	0,00001	0,00010	0,00008	0,03168	2,21827
0,715	1,43	0,00014	0,00002	0,00001	0,00010	0,00008	0,03258	2,27835
0,738	1,476	0,00012	0,00000	0,00001	0,00008	0,00006	0,06000	4,06487
0,749	1,498	0,00009	0,00001	0,00002	0,00004	0,00004	0,07543	5,03513
0,761	1,522	0,0001	0,00001	0,00005	0,00006	0,00003	0,0926	6,08425
0,772	1,544	0,00006	0,00001	0,00005	0,00003	0,00000	0,10861	7,03437
0,784	1,568	0,00005	0,00001	0,00005	0,00002	0,00001	0,12559	8,00985
0,798	1,596	0,00005	0,00002	0,00004	0,00002	0,00001	0,14432	9,04282
0,814	1,628	0,00006	0,00003	0,00005	0,00003	0,00001	0,16353	10,04469
0,87	1,74	0,00016	0,00016	0,00012	0,00012	0,00000	0,19155	11,00884
0,881	1,762	0,00013	0,00014	0,00009	0,00011	0,00001	0,17836	10,12264
0,888	1,776	0,00001	0,00001	0,00000	0,00001	0,00000	0,15984	8,99983
0,892	1,784	0,00001	0,00000	0,00000	0,00001	0,00000	0,1446	8,10522
0,896	1,792	0,00003	0,00004	0,00002	0,00003	0,00002	0,12646	7,05669
0,899	1,798	0,00002	0,00005	0,00002	0,00003	0,00002	0,11166	6,21033
0,903	1,806	0,00003	0,00003	0,00001	0,00000	0,00002	0,09172	5,07841
0,907	1,814	0,00022	0,00047	0,00026	0,00029	0,00019	0,07191	3,96459
0,911	1,822	0,00017	0,00049	0,00023	0,00027	0,00022	0,05472	3,00340
0,916	1,832	0,00012	0,00059	0,00025	0,00026	0,00031	0,03541	1,93304
0,921	1,842	0,00008	0,00079	0,00032	0,00028	0,00045	0,01741	0,94650
0,922	1,844	0,00011	0,00063	0,00025	0,00029	0,00034	0,01468	0,79710
0,923	1,846	0,00013	0,00037	0,00013	0,00027	0,00017	0,01248	0,67671
0,924	1,848	0,00001	0,00015	0,00006	0,00005	0,00009	0,01031	0,55788
0,925	1,85	0,00001	0,00018	0,00007	0,00005	0,00011	0,00687	0,37143
0,926	1,852	0,00000	0,00022	0,00009	0,00006	0,00013	0,00326	0,17683
0,934	1,868	0,0002	0,00255	0,00109	0,00045	0,00159	0,00258	0,22137

Table 8. Computational angles (in radians) related to (Ft) with Armijo line search

r	α1	α2	α3	α4	α5	α6	Ft
0,632	0,23308	0,34099	0,40777	1,16447	1,21167	1,57037	0
0,711	0,26295	0,34351	0,64456	1,11385	1,22247	1,37603	0
0,712	0,26377	0,34470	0,64375	1,11347	1,22303	1,37600	0
0,713	0,26458	0,34587	0,64295	1,11309	1,22357	1,37594	0
0,714	0,26537	0,34699	0,64219	1,11270	1,22409	1,37584	0
0,715	0,26614	0,34810	0,64143	1,11230	1,22461	1,37573	0
0,738	0,28187	0,36896	0,62744	1,10221	1,23436	1,36905	0
0,749	0,28866	0,37746	0,62195	1,09708	1,2382	1,36428	0
0,761	0,29580	0,38630	0,61640	1,09140	1,24198	1,35837	0
0,772	0,30220	0,39420	0,61162	1,08613	1,24505	1,35241	0
0,784	0,30904	0,40270	0,60663	1,08034	1,24795	1,34535	0
0,798	0,31684	0,41260	0,60106	1,07355	1,25058	1,33628	0
0,814	0,32555	0,42400	0,59499	1,06572	1,25219	1,3245	0
0,87	0,35241	0,46508	0,57485	1,03656	1,21923	1,24744	0
0,881	0,35521	0,47165	0,56933	1,02896	1,18552	1,20793	0
0,888	0,35505	0,47342	0,56391	1,02223	1,14962	1,17077	0
0,892	0,35395	0,4729	0,55966	1,01649	1,12208	1,14460	0
0,896	0,35186	0,47081	0,55431	1,00826	1,09098	1,11752	0
0,899	0,34964	0,46817	0,54958	0,99966	1,06673	1,09875	0
0,903	0,34586	0,46332	0,54244	1,07814	1,03507	0,98452	0
0,907	0,34169	0,45753	0,53467	1,06489	1,00729	0,96577	0
0,911	0,33689	0,45089	0,52651	1,05553	0,98301	0,94658	0
0,916	0,33046	0,44182	0,51598	1,04773	0,95820	0,92407	0
0,921	0,32360	0,43196	0,50509	1,04245	0,93935	0,90482	0
0,922	0,32217	0,42987	0,50285	1,0416	0,93628	0,90144	0
0,923	0,32071	0,42772	0,50058	1,04083	0,93346	0,89823	0
0,924	0,31928	0,42568	0,49851	1,04003	0,93055	0,89488	0
0,925	0,31788	0,4239	0,49669	1,03877	0,92656	0,8909	0
0,926	0,31620	0,42108	0,49379	1,03880	0,92612	0,88934	0
0,934	0,30177	0,39917	0,47329	1,03572	0,91650	0,87328	0

Table 9. Harmonics and THD related to (Ft) with Armijo line search

r	*H*1	*H*5	*H7*	*H*11	*H*13	*H*17	*H*19	THD %
0,632	1,264	0,00001	0,00001	0,00001	0,00000	0,00001	0,10988	8,69291
0,711	1,422	0,00004	0,00001	0,00000	0,00003	0,00002	0,02959	2,08072
0,712	1,424	0,00004	0,00001	0,00000	0,00003	0,00002	0,03036	2,13172
0,713	1,426	0,00004	0,00001	0,00000	0,00003	0,00002	0,03117	2,18611
0,714	1,428	0,00004	0,00001	0,00000	0,00003	0,00002	0,03205	2,24437
0,715	1,430	0,00004	0,00001	0,00000	0,00003	0,00002	0,03295	2,30393
0,738	1,476	0,00003	0,00001	0,00001	0,00002	0,00001	0,06031	4,08617
0,749	1,498	0,00003	0,00000	0,00001	0,00002	0,00001	0,07563	5,04853
0,761	1,522	0,00003	0,00000	0,00001	0,00002	0,00001	0,09288	6,10265
0,772	1,544	0,00002	0,00000	0,00001	0,00001	0,00000	0,10878	7,04521
0,784	1,568	0,00002	0,00000	0,00001	0,00001	0,00000	0,12573	8,01823
0,798	1,596	0,00002	0,00000	0,00001	0,00001	0,00000	0,14445	9,05065
0,814	1,628	0,00002	0,00001	0,00002	0,00002	0,00000	0,1637	10,05549
0,870	1,740	0,00002	0,00001	0,00001	0,00002	0,00000	0,19189	11,02836
0,881	1,762	0,00001	0,00001	0,00000	0,00002	0,00000	0,17838	10,12383
0,888	1,776	0,00011	0,00013	0,00008	0,00010	0,00002	0,16000	9,00894
0,892	1,784	0,00010	0,00013	0,00008	0,00010	0,00003	0,14484	8,11871
0,896	1,792	0,00010	0,00016	0,00009	0,00011	0,00005	0,12670	7,07030
0,899	1,798	0,00010	0,00019	0,00009	0,00013	0,00006	0,11195	6,22662
0,903	1,806	0,00026	0,00032	0,00027	0,00024	0,00015	0,09225	5,10788
0,907	1,814	0,00005	0,00008	0,00004	0,00004	0,00003	0,07272	4,00859
0,911	1,822	0,00003	0,00008	0,00003	0,00005	0,00003	0,05569	3,05636
0,916	1,832	0,00002	0,00008	0,00003	0,00004	0,00003	0,03699	2,01905
0,921	1,842	0,00001	0,00011	0,00004	0,00004	0,00006	0,02023	1,09835
0,922	1,844	0,00001	0,00012	0,00004	0,00005	0,00007	0,01696	0,91997
0,923	1,846	0,00001	0,00013	0,00005	0,00005	0,00008	0,01366	0,74018
0,924	1,848	0,00015	0,00007	0,00000	0,00023	0,00002	0,01065	0,57654
0,925	1,85	0,00016	0,00026	0,00014	0,00017	0,00022	0,00914	0,49456
0,926	1,852	0,00014	0,00012	0,00002	0,00023	0,00003	0,00377	0,20400
0,934	1,868	0,00003	0,00029	0,00013	0,00003	0,00018	0,02777	1,48669

Table 10. The lowest TDH obtained by the two methods

THD %	F_m	F_t
conjugate gradient with Wolfe line search	0.11548	0.17683
conjugate gradient with Armijo line search	0.16722	0.20400

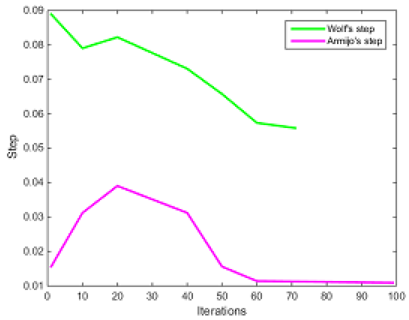

Fig. 2. Optimal steps according to the number of iterations for F_m (r = 0.675)

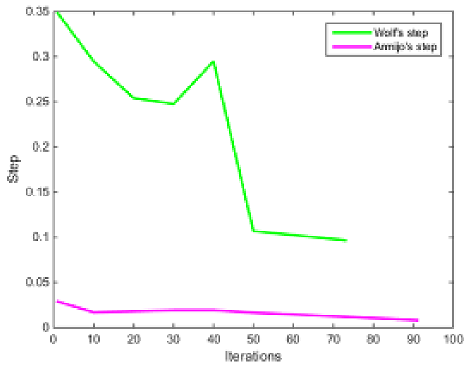

Fig. 3. Optimal steps according to the number of iterations for F_t (r = 0.926)

Tables 2 and 4 give the switching angles of the single-phase inverter obtained with applying the conjugate gradient method Wolfe and Armijo line searches. In other words, this corresponds to minimize harmonics voltage. These angles minimize the objective function F_m and represent solutions of the nonlinear system (4). The solutions are global minimums, because the minimum the objective function F_m is equal to zero for the values of the modulation index between 0, 614 and 0, 760.

Tables 2 to 5 show that the best value of the THD is obtained for a modulation index *r* = 0,675 for single phase inverters Tables 6 and 8 give the switching angles of the three-phase inverter obtained with applying the conjugate gradient method Wolfe line search and Armijo line search to minimize harmonics voltage. These angles minimize the objective function F_t and represent solutions of the nonlinear system (8). The solutions are global minimums, because the minimum the objective function F_t is equal to zero for the values of the modulation index between 0, 632 and 0,932. Tables 6 to 9 show that the best value of THD obtained for $r = 0{,}926$.

gives the lowest TDH obtained by the two methods for the objective functions. We also see that the TDH is lower when we minimize the harmonics by the conjugate gradient method with Wolfe line search.

Observing Tables 3 and 5 for single phase inverter and and 9 for three phase inverter, it follows that odd harmonics $k = 2n \pm 1$ (for single phase inverter) and $k = 6n \pm 1$ (for three phase inverter) are almost reduced to zero. Fig 2 and 3 give the shape of the Wolfe and Armijo optimal step of F_m and F_t objectives. We can see that at each iteration the optimal step of Wolfe is greater than Armijo's optimal step. So the conjugate gradient algorithm by Wolfe line search converges faster than Armijo line search. This is confirmed by comparing the number of iterations of the two methods.

5 Conclusion

In this paper we have minimized harmonics of five-level single-phase and three-phase inverters. The conjugate gradient algorithms with Wolfe and Armijo line searches have been applied for this purpose. Results obtained from the running of the conjugate gradient algorithm confirm that Wolfe line search converges faster than Armijo line search. With respect to different values of the modulation index, we find optimal angles that eliminate odd harmonics H_k with rank $k = 2n \pm 1, 1 \leq n \leq 5$ for single phase inverters and harmonics H_k with rank $k = 6n \pm 1, 1 \leq n \leq 3$ for three phase inverters. More ever Wolfe type line search ensures that the Total Harmonic Distortion (THD) to be lowered possible.

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