

## Modeling of the lock-in thermography process through finite element method for measuring of the thermal diffusivity

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**ABSTRACT:** In industry, especially in the high technology sector such as aerospace, we produce and we use increasingly new materials for the construction of new structures that have good thermal and mechanical properties. The characterization of these materials requires knowledge of their thermo-physical properties. Thermal diffusivity is an important parameter in the materials characterization. Lock-in thermography is widely used in the materials thermal characterization. It involves applying on the front face sample a heater in the form of a sine wave and analyzing the phase difference or the amplitude difference between the incident thermal wave and the transmitted thermal wave. Indeed, the passage of the thermal wave through a material is influenced by its thermal diffusivity. We used the finite element method, in three dimensions, to calculate the instantaneous temperatures of the front and rear faces of the inspected sample, and deduct their phase shifts and therefore the sample thermal diffusivity. Our contribution in the lock-in thermography technique is the development of a new model for the thermal diffusivity evaluation with good precision. The results for polystyrene are very satisfactory. Indeed, the thermal diffusivity calculated by our new model is very close to the value reported in the literature. The proposed new model can be used in the characterization of new materials.

**KEYWORDS:** Lock-in thermography, Finite element method, Thermal diffusivity, Modeling, Simulation.

### 1 INTRODUCTION

The thermal diffusivity is one of the most important parameters which affects the efficiency of heat transfer. Several techniques have been developed for measuring the thermal diffusivity. The use of infrared thermography as a nondestructive evaluation technique is becoming increasingly attractive for measuring the thermal diffusivity [1], [2], [3], [4], [5]. Thermography offers several advantages over other non-destructive techniques in that it is non-contact, able to inspect wide areas and produce easily interpreted results.

Lock-in thermography is based on analyzing the response of the inspected sample which has been subjected to a variable excitation. The principle is to apply a heat flow or a temperature which varies sinusoidally in time. The passage of the transmitted thermal wave by conduction is influenced by the thermal properties of the characterized sample. This technique is used in determining the material thermal diffusivity.

When the considered material is uniform of thickness  $d$  and heated over wide plane surface ( $z=0$ ) by applying at  $z=0$  a temperature  $T_m \exp(i\omega t)$  varying at angular frequency  $\omega$  and assuming that heat flux is zero at  $z=d$  and providing  $d > \mu$ , the phase difference  $\Delta\phi$  between the temperature variation at  $z = d$  and at  $z = 0$  is simply as [6]:

$$\Delta\phi \approx -\frac{d}{\mu} \quad (1)$$

The thermal length  $\mu$  is related to the material thermal diffusivity by the following equation:

$$\mu = \sqrt{\frac{2\alpha}{\omega}} \quad (2)$$

Using a 3D finite element simulation, we are going to highlight in the first part of this work the difference between the actual value of the thermal diffusivity and its estimated value by the relation (1).

In the second part, we propose a model for estimating the thermal diffusivity with better accuracy.

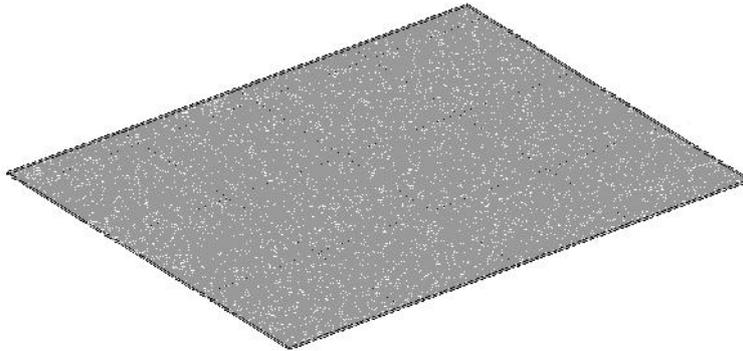
## 2 FINITE ELEMENT MODELING

The general equation of heat diffusion is given by:

$$\nabla \cdot [k \nabla T(r, t)] - \rho c_v \frac{\partial T(r, t)}{\partial t} = -Q(r, t) \quad (3)$$

Where  $T(r, t)$  is the temperature (K), at the position  $r(m)$  and at the moment  $t(s)$ ,  $k$  is the thermal conductivity (W/m.K),  $\rho$  is the density ( $\text{kg/m}^3$ ),  $c_v$  is the specific heat capacity (J/kg.K), and  $Q(r, t)$  is the density of input energy.

Solving equation (3) is generally inaccessible, an approximate solution is obtained by a finite element calculation. Our considered model is a thin plate of dimensions (40mm x 32 mm) and of a variable thickness  $d$  (Figure 1).



**Fig. 1: 3D model of a thin plate**

We applied on the front face of the considered sample a sinusoidal temperature assuming that the flux through the rear surface is zero and that the sides are thermally insulated.

The thermal properties of the used materials are as follows:

**Table 1: Thermal properties of materials at 25 °C**

	Density $\text{kg/m}^3$	Thermal conductivity $\text{W/mK}$	Specific heat $\text{J/kg K}$	Thermal diffusivity $\text{m}^2/\text{s}$
<b>Polymethyl Methacrylate PMMA[7]</b>	1190	0.193	1420	$1.1421 \cdot 10^{-7}$
<b>Polypropylene PP[8]</b>	900	0.1	2000	$5.5556 \cdot 10^{-8}$

The transient thermal analysis was performed with a step of 0.01s for each sample and each value of excitation frequency.

3 RESULTS AND DISCUSSION

For each considered material and for each value of excitation frequency, we calculated, by the finite element method, the phase difference  $\Delta\phi$ . The thickness of PMMA sample is  $d=1.93\text{mm}$  and the thickness of polypropylene (PP) sample is  $d=1.95\text{mm}$ . Figures 2-a and 2-b show the evolution of the phase difference opposite  $-\Delta\phi$  as a function of  $d/\mu$ .

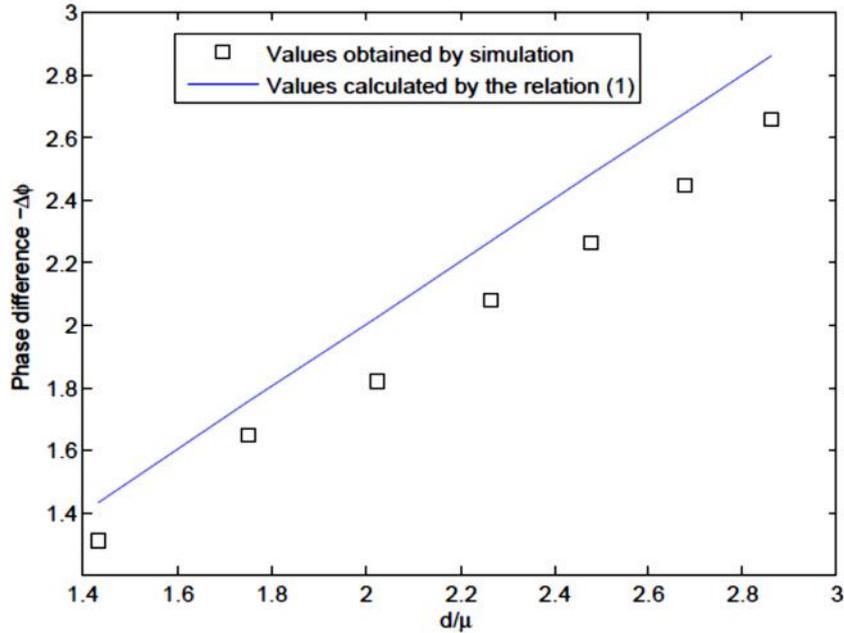


Fig. 2-a: Variation of phase difference opposite  $-\Delta\phi$  versus  $d/\mu$ , Case of PMMA, Maximal gap = 0.2354 rad

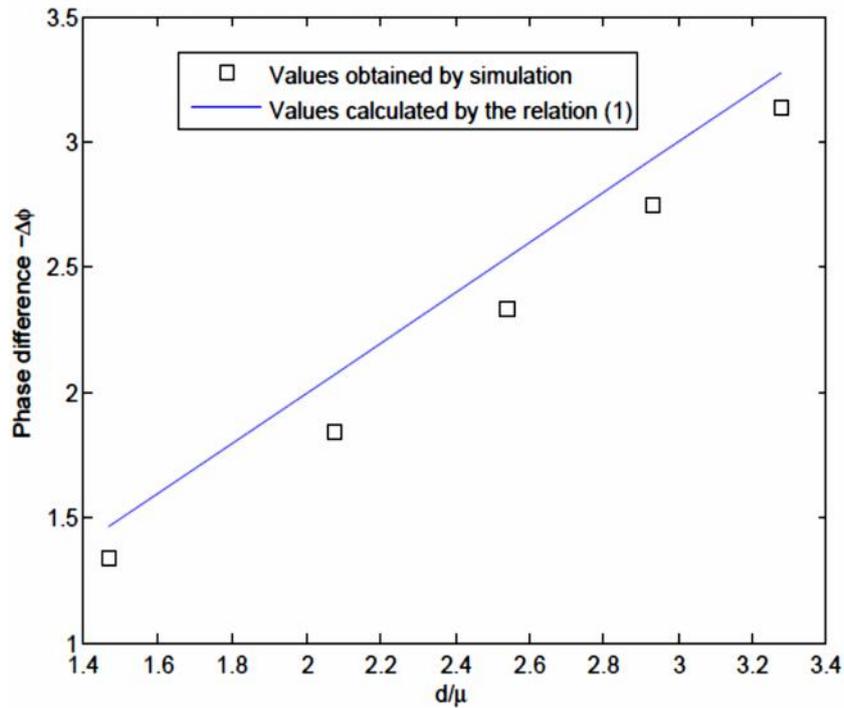


Fig. 2-b: Variation of phase difference opposite  $-\Delta\phi$  versus  $d/\mu$ , Case of polypropylene (PP), Maximal gap = 0.2298 rad

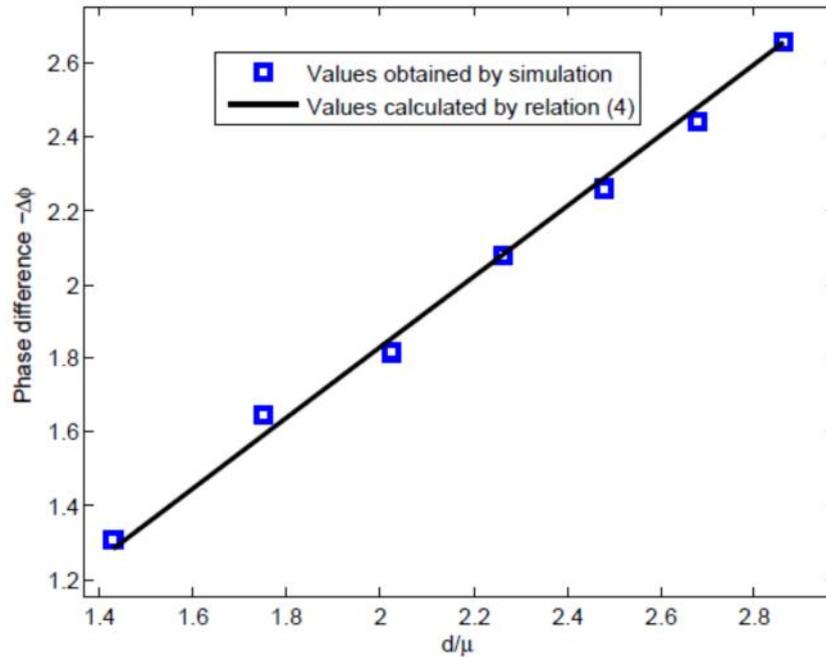
We observe the difference between the values calculated by finite elements method and their estimated by relation (1), this difference is reflected in the estimation of the thermal diffusivity. A correction of the relation (1) is necessary to

approximate the actual values. In order to have a better accuracy in estimating the thermal diffusivity, we propose a new model that allows estimating of phase difference from sample thickness and thermal length.

The model is as follows:

$$\Delta\varphi = -0.9593 \frac{d}{\mu} + 0.09 \quad (4)$$

In Figures 3-a and 3-b, we superimposed the results obtained by simulation and the results obtained by the proposed model in the relationship (4). We plotted the variations of the phase difference as a function of ratio of sample thickness and thermal length.



**Fig. 3-a: Results of the proposed model for PMMA sample, Maximal gap = 0.0541 rad.**

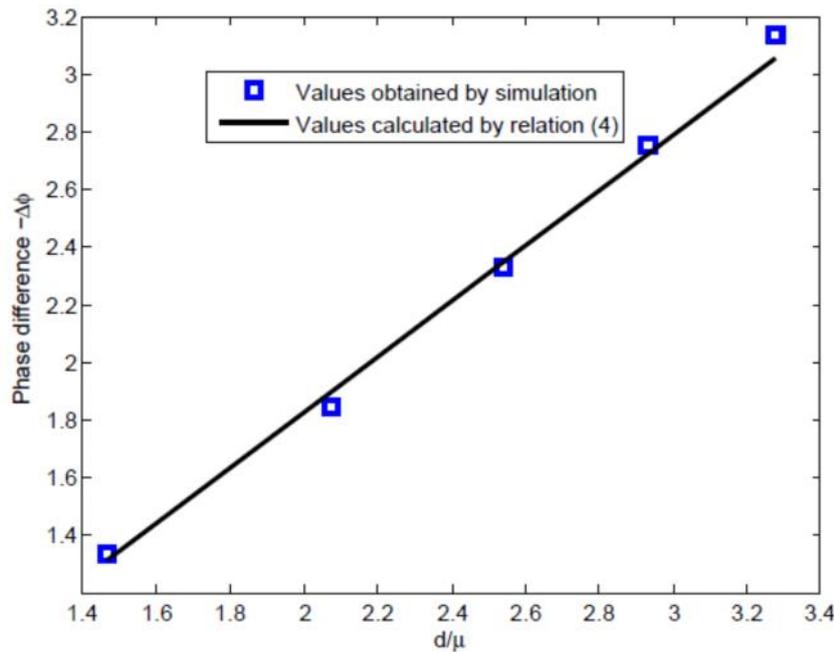


Fig. 3-b: Results of the proposed model for polypropylene sample, Maximal gap = 0.0844 rad.

The relation (4) gives a good approximation of the phase difference  $\Delta\phi$  for both materials, PMMA and polypropylene. Indeed, the obtained error values remain low for both tested samples.

From relations (2) and (4), the thermal diffusivity may be evaluated by equation (5) as follow:

$$\alpha = \frac{\omega}{2} \left( \frac{0.9593d}{\Delta\phi - 0.09} \right)^2 \quad (5)$$

To evaluate the proposed improvement in this work, we apply the formula (5) to the phase difference value obtained for a Polystyrene sample (Table 2):

Table 2: Improvement of accuracy estimation of polystyrene thermal diffusivity

	Frequency f (Hz)	Thickness d (mm)	phase difference $-\Delta\phi$ (rad)	Diffusivity without correction ( $10^{-7} \text{ m}^2/\text{s}$ )	Thermal diffusivity by relation (5) ( $10^{-7} \text{ m}^2/\text{s}$ )	Thermal diffusivity in literature ( $10^{-7} \text{ m}^2/\text{s}$ ) [7]
Polystyrene	0.05	1.94	2.4049	1.0222	0.8740	0.87

We note that the thermal diffusivity estimated by equation (5) is closer to the value cited in literature.

#### 4 CONCLUSION

In this work, the objective was to improve the estimation of thermal diffusivity of a sample by the use of lock-in thermography. In the first part, we have highlighted the need for a correction of the approximation formula of the diffusivity from the phase difference value. For this, a finite element simulations were performed and results show the necessity of correcting the estimation formula published in the literature. In this context, we have proposed a model for estimating the thermal diffusivity with better accuracy. In the end, we have applied the proposed model to determine the thermal diffusivity of a polystyrene sample. The obtained value is close to the actual value with a good accuracy.

## REFERENCES

- [1] Martin Rides, Junko Morikawa, Lars Halldahl, Bruno Hay, Hubert Lobo, Angela Dawson, and Crispin Allen, "Intercomparison of thermal conductivity and thermal diffusivity methods for plastics," *Polymer Testing*, vol. 28, pp. 480-489, 2009.
- [2] Jian-Jun Zheng, Xin-Zhu Zhou, Yu-Fei Wub, and Xian-Yu Jin, "A numerical method for the chloride diffusivity in concrete with aggregate shape effect," *Construction and Building Materials*, vol. 31, pp. 151-156, 2012.
- [3] F. Cernuschi, P. Bison, S. Marinetti, and E. Campagnoli, "Thermal diffusivity measurement by thermographic technique for the non-destructive integrity assessment of TBCs coupons," *Surface & Coatings Technology*, vol. 205, pp. 498-505, 2010.
- [4] I.O. Mohamed, "Development of a simple and robust inverse method for determination of thermal diffusivity of solid foods," *Journal of Food Engineering*, vol. 101, pp. 1-7, 2010.
- [5] Masanori Monde, Masataka Kosaka, and Yuichi Mitsutake, "Simple measurement of thermal diffusivity and thermal conductivity using inverse solution for one-dimensional heat conduction," *International Journal of Heat and Mass Transfer*, vol. 53, pp. 5343-5349, 2010.
- [6] C. Boué, and S. Holé, "Infrared thermography protocol for simple measurements of thermal diffusivity and conductivity," *Infrared Physics & Technology*, vol. 55, no. 4, pp. 376-379, 2012.
- [7] M. F. Couturier, K. George, and M. H. Schneider, "Thermophysical properties of wood-polymer composites," *Wood Science and Technology*, vol. 30, pp. 179-196, 1996.
- [8] Xiangtao Yin, Scott A. Morris, and William D. O'Brien Jr, "Ultrasonic Pulse-Echo Subwavelength Defect Detection Mechanism: Experiment and Simulation," *Journal of Nondestructive Evaluation*, Vol. 22, No. 3, 2003.