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Soft πg-continuous Functions and irresolute Functions

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ABSTRACT: The purpose of this paper is to define a new class of continuous functions called Soft πg-continuous functions and Soft πg-irresolute functions in soft topological spaces. We get several characterizations and some of their properties. Also we investigate its relationships with other soft continuous functions.

KEYWORDS: Soft πg-closed set, Soft πg-open set, Soft πg-continuous functions, Soft πg-irresolute functions, Soft πg-open function, Soft πg-closed function.

1 INTRODUCTION

The concept of soft set theory has been initiated by Molodtsov[6] in 1999 as a general mathematical tool for modeling uncertainties. After the introduction of the notion of soft sets several researchers improved this concept. Topological structure of soft sets was initiated by Shabir and Naz[9] and studied the concepts of soft open set, soft interior point, soft neighborhood of a point, soft separation axioms and subspace of a soft topological space. Many researchers extended the results of generalization of various soft closed sets in many directions. Athar Kharal and B. Ahmad[3]defined the notion of a mapping on soft classes and studied several properties of images and inverse images of soft sets. In this present study, we discuss soft πg-continuous and soft πg-irresolute Functions in soft topological space and some of their properties.

2 PRELIMINARIES

DEFINITION 2.1[6]

Let U be the initial universe and P (U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P (U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For Ɛ ∈ A, F (Ɛ) may be considered as the set Ɛ- approximate elements of the soft set (F, A).

DEFINITION: 2.2[6]

For two soft sets (F, A) and (G, B) over a common universe U, we say that (F , A) is a soft subset of (G, B) if (1) A $\tilde{\subseteq}$ B and (2) for all e ∈ A, F (e) and G(e) are identical approximations. We write (F, A) $\tilde{\subseteq}$ (G, B). (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F , A). We denote it by (F, A)$\tilde{\supseteq}$(G, B).

DEFINITION: 2.3[6]

Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F , A) is a soft subsetof (G, B) and (G, B) is a soft subset of (F , A).

DEFINITION: 2.4[6]

The union of two soft sets of (F , A) and (G, B) over the common universe U is the soft set (H, C), where C = A ∪ B and for all e ∈ C,

$$H(e) = \begin{cases} F(e)\, if e \in A - B \\ G(e)\, if\ e \in B - A \\ F\ (e) \cup G(e)\, if\ e \in A \cap B. \end{cases}$$

We write (F, A) ∪ (G, B) = (H, C).

DEFINITION: 2.5 [6]

The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) ∩ (G, B), is defined as C = A ∩ B, and H(e) = F (e) ∩ G(e) for all e ∈ C.

DEFINITION: 2.6[6]

Then the soft closure of (F, E), denoted by cl(F, E) is the intersection of all soft closed supersets of (F,E).Clearly(F, E) is the smallest soft closed set over X which contains (F, E).

DEFINITION: 2.7 [9]

Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if (1) Φ, X belong to τ , (2) the union of any number of soft sets in τ belongs to τ , (3) the intersection of any two soft sets in τ belongs to τ.The triplet (X,τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

DEFINITION: 2.8 [9]

Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. The soft interior of (F, E), denoted by int (F, E) is the union of all soft open subsets of (F, E). Clearly (F, E) is the largest soft open set over X which is contained in (F, E).

DEFINITION: 2.9[5]

A subset (A,E) of a topological space X is called soft generalized-closed **(**soft g **-**closed**)** if cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

DEFINITION: 2.10[8]

A subset (A, E) of a topological space X is called soft regular closed , if cl(int(A,E))=(A,E). The complement of soft regular closed set is soft regular open set.

DEFINITION: 2.11[8]

The finite union of soft regular open sets is said to be soft π-open. The complement of soft π-open is said to be soft π-closed.

DEFINITION: 2.12[2]

A subset (A, E) of a topological space X is called soft πg-closed in a soft topological space (X, τ, E), if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X.

DEFINITION: 2.13[1]

Let (F, E) be a soft set over X. The soft set (F, E) is called soft point, denoted by (x_e, E), if for element e∈E, F(e) = {x} and F(e') = φ for all e'∈ E−{e}.

DEFINITION: 2.14[10]

Let (X, τ, E) and (Y, τ', E) be two topological spaces. A function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be Soft Semi continuous(Soft pre-continuous, Soft α -continuous, Soft β -continuous), if f^{-1}(G, E) is soft semi open(soft pre-open, soft α -open, soft β -open) in (X, τ, E) for every soft open set (G, E) of (Y, τ', E).

3 SOFT πg-CONTINUOUS FUNCTIONS

DEFINITION: 3.1

Let (X, τ, E) and (Y, τ', E) be two topological spaces. A function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be Soft regular continuous(Soft π-continuous, Soft g-continuous, Soft πg-continuous), if f^{-1}(G, E) is soft regular open(soft π-open, soft g-open, soft πg-open) in (X, τ, E) for every soft open set (G, E) of (Y, τ', E).

THEOREM: 3.2

For a function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ the following hold

1) Every soft regular continuous function is soft π-continuous.
2) Every soft π-continuous function is soft continuous.
3) Every soft π- continuous function is soft πg-continuous.
4) Every soft continuous function is soft g-continuous.
5) Every soft g-continuous function is soft πg-continuous.

Proof: Obvious

From the above result we have the following implication:

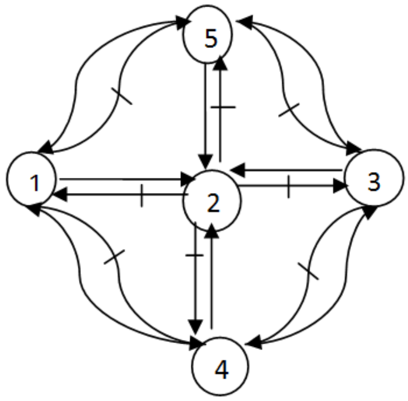
(1) Soft g-continuous

(2) Soft πg-Continuous

(3) Soft π-continuous

(4) Soft continuous

(5) Soft regular continuous



The converse of the above is not true as shown in the following example:

EXAMPLE: 3.3

Let X = Y= {h_1, h_2, h_3}, E= {e_1, e_2}. Then τ = {$\widetilde{\emptyset}$, $\widetilde{X}$, (F_1, E) (F_2, E), (F_3, E), (F_4, E)} is a soft topological space over X and τ' = {$\widetilde{\emptyset}$, $\widetilde{Y}$, (G_1, E), (G_2, E), (G_3, E), (G_4, E)} is a soft topological space over Y. Here (F_1, E), (F_2, E), (F_3, E), (F_4, E) are soft sets over X and (G_1, E), (G_2, E), (G_3, E), (G_4, E) are soft sets over Y defined as follows: $F_1(e_1)$ = {h_2} , $F_1(e_2)$ = {h_1}; $F_2(e_1)$ = {h_2, h_3}, $F_2(e_2)$ = {h_1, h_2}; $F_3(e_1)$ = { h_3}, $F_3(e_2)$ = {h_1, h_3}; $F_4(e_1)$ = {Ø} , $F_4(e_2)$ = {h_1} and $G_1(e_1)$ = {h_2} , $G_1(e_2)$ = {h_1}; $G_2(e_1)$ = {h_2, h_3}, $G_2(e_2)$ = {h_1, h_2}; $G_3(e_1)$= { h_1, h_2}, $G_3(e_2)$ = { X}; $G_4(e_1)$ = { h_2} , $G_4(e_2)$ = {h_1, h_2}. If the function $f : (X, \tau, E) \rightarrow (Y, \tau, E)$ be an identity function then f is soft πg-continuous but not soft π-continuous, since the inverse image of soft open sets in Y are not soft π-open sets in X.

EXAMPLE: 3.4

Let X = Y= {x_1, x_2, x_3}, E= {e_1, e_2}. Then τ = {$\widetilde{\emptyset}$, $\widetilde{X}$, (F_1, E) (F_2, E), (F_3, E)} is a soft topological space over X and τ' = {$\widetilde{\emptyset}$, $\widetilde{Y}$, (G_1, E), (G_2, E)} is a soft topological space over Y. Here (F_1, E), (F_2, E), (F_3, E) are soft sets over X and (G_1, E), (G_2, E) are soft sets over Y defined as follows: $F_1(e_1)$ = {x_1} , $F_1(e_2)$ = {x_1}; $F_2(e_1)$ = {x_2}, $F_2(e_2)$ = {x_2}; $F_3(e_1)$ = { x_1, x_2}, $F_3(e_2)$ = { x_1, x_2} and $G_1(e_1)$ ={x_1}, $G_1(e_2)$ = {x_1}; $G_2(e_1)$ = { x_1, x_2}, $G_2(e_2)$ = { x_1, x_2} . If the function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is defined as $f(x_1) = x_1$, $f(x_2) = x_3$, $f(x_3) = x_2$ then f is soft πg-continuous but not soft g-continuous, since the inverse image of soft open sets in Y are not soft g-open sets in X.

THEOREM: 3.5

Let f : $(\mathrm{X}, \tau, \mathrm{E}) \rightarrow (\mathrm{Y}, \tau', \mathrm{E})$ be a function. Then the following are Equivalent:

1) f is soft πg-continuous.

2) The inverse image of each soft open set in $(\mathrm{Y}, \tau', \mathrm{E})$ is soft πg-open in $(\mathrm{X}, \tau, \mathrm{E})$.

Proof:

Suppose f is soft πg-continuous. Then it follows from the definition that inverse image of every soft closed set (G,E) of Y is soft closed. Hence Y−(G,E) is soft open in Y. Therefore inverse image of every soft open set in Y is soft πg-open in X.

THEOREM: 3.6

If a function f : $(\mathrm{X}, \tau, \mathrm{E}) \rightarrow (\mathrm{Y}, \tau', \mathrm{E})$ is soft πg-continuous if and only if f($\tilde{S}\pi$g-cl(F,E)) $\tilde{\subseteq}$ cl(f(F, E)) for every soft open set (F, E) of X.

Proof:

Let f : $(\mathrm{X}, \tau, \mathrm{E}) \rightarrow (\mathrm{Y}, \tau', \mathrm{E})$ be soft πg-continuous and (F, E) $\tilde{\subseteq}$ X. Then cl(f(F, E)) is soft closed in Y. Since f is soft πg-continuous, f^{-1}(cl(f(F,E))) is soft πg-closed in X and (F, E) $\tilde{\subseteq}$ f^{-1} (f(F, E)) $\tilde{\subseteq}$ f^{-1}(cl(f(F,E))). As πg-cl(F,E)) is the smallest soft πg-closed set containing (F, E), πg-cl(F,E)) $\tilde{\subseteq}$ f^{-1}(cl(f(F,E))). Hence f($\tilde{S}\pi$g-cl(F,E)) $\tilde{\subseteq}$ cl(f(F, E)).

Conversely,

Let (G, E) be any soft closed set of Y. Then f^{-1}(G, E) ∈ X and so f($\tilde{S}\pi$g-cl(f^{-1}(G,E))) $\tilde{\subseteq}$ cl(f(f^{-1}(G,E))). Therefore f($\tilde{S}\pi$g-cl(f^{-1}(G,E))) $\tilde{\subseteq}$ cl (G,E) which implies that $\tilde{S}\pi$g-cl(f^{-1}(G,E)) $\tilde{\subseteq}$ f^{-1} (G, E). In general f^{-1} (G, E) $\tilde{\subseteq}$ $\tilde{S}\pi$g-cl(f^{-1}(G,E)). Thus f^{-1} (G,E) = $\tilde{S}\pi$g-cl(f^{-1}(G,E)). Hence f^{-1} (G, E) is soft πg-closed . Therefore f is soft πg-continuous.

THEOREM: 3.7

If a function f : $(\mathrm{X}, \tau, \mathrm{E}) \rightarrow (\mathrm{Y}, \tau', \mathrm{E})$ is soft πg-continuous if and only if f^{-1} (int(G,E)) $\tilde{\subseteq}$ ($\tilde{S}\pi$g-int(f^{-1} (G, E))) for every soft open set (G, E) of X.

Proof:

Let f : $(\mathrm{X}, \tau, \mathrm{E}) \rightarrow (\mathrm{Y}, \tau', \mathrm{E})$ be soft πg-continuous. Now int(f(G, E)) is a soft open set of Y.

Then by the soft πg-continuity of f , f^{-1} (int(f(G,E))) is soft πg-open and f^{-1} (int(f(G,E))) $\tilde{\subseteq}$ (G, E). As $\tilde{S}\pi$g-int (G, E) is the largest soft πg-open set contained in (G, E), f^{-1}(int(f(G,E))) $\tilde{\subseteq}$ $\tilde{S}\pi$g-int(G, E).

Conversely,

Assume that f^{-1} (int(G,E)) $\tilde{\subseteq}$ ($\tilde{S}\pi$g-int(f^{-1} (G, E))) for every soft open set (G, E) of X.

Then f^{-1}(G, E) $\tilde{\subseteq}$ $\tilde{S}\pi$g-int(f^{-1} (G, E)). In general $\tilde{S}\pi$g-int(f^{-1} (G, E)) $\tilde{\subseteq}$ f^{-1}(G, E). Therefore

f^{-1}(G, E) = $\tilde{S}\pi$g-int(f^{-1} (G, E)). Hence f^{-1} (G, E) is soft πg-open. This proves that f is soft πg-continuous.

REMARK:3.7

The composition of two soft πg-continuous function need not be soft πg-continuous in general .

4 SOFT πg-IRRESOLUTE FUNCTIONS

DEFINITION: 4.1

A function f : $(\mathrm{X}, \tau, \mathrm{E}) \rightarrow (\mathrm{Y}, \tau', \mathrm{E})$ is soft πg-irresolute, if f^{-1}(G, E) is soft πg-open in $(\mathrm{X}, \tau, \mathrm{E})$ for every soft πg-open set (G, E) of $(\mathrm{Y}, \tau', \mathrm{E})$.

THEOREM: 4.2

Every soft πg-irresolute function is soft πg-continuous.

Proof: Obvious.

The converse of the above theorem need not be true as shown in the following example:

EXAMPLE: 4.3

Let X = Y= $\{x_1, x_2, x_3\}$, E= $\{e_1, e_2\}$. Then $\tau = \{\widetilde{\emptyset}, \widetilde{X}$, (F_1, E) (F_2, E), $(F_3, E)\}$ is a soft topological space over X and $\tau' = \{\widetilde{\emptyset}, \widetilde{Y}$, (G_1, E), $(G_2, E)\}$ is a soft topological space over Y. Here (F_1, E), (F_2, E), (F_3, E) are soft sets over X and (G_1, E), (G_2, E) are soft sets over Y defined as follows: $F_1(e_1) = \{x_2\}$, $F_1(e_2) = \{x_1\}$; $F_2(e_1) = \{x_1, x_3\}$, $F_2(e_2) = \{x_2, x_3\}$; $F_3(e_1) = \{x_2\}$, $F_3(e_2) = \{X\}$; $F_4(e_1) = \{\emptyset\}$, $F_4(e_2) = \{x_1\}$; $F_5(e_1) = \{x_1, x_3\}$, $F_5(e_2) = \{X\}$; $F_6(e_1) = \{\emptyset\}$, $F_6(e_2) = \{x_2, x_3\}$; $F_7(e_1) = \{\emptyset\}$, $F_7(e_2) = \{X\}$ and $G_1(e_1) =\{x_2\}$, $G_1(e_2) = \{x_1\}$; $G_2(e_1) = \{x_1, x_3\}$, $G_2(e_2) = \{x_2, x_3\}$; $G_3(e_1) =\{x_2\}$, $G_3(e_2) =\{\emptyset\}$. If the function $f : (X, \tau, E) \to (Y, \tau', E)$ is defined as identity function then f is soft πg-continuous but not soft πg-irresolute.

THEOREM: 4.4

Let $f : (X, \tau, E) \to (Y, \tau', E)$ and $g : (Y, \tau', E) \to (Z, \tau'', E)$ be any two functions. Then

1. $g \circ f$ is soft πg-ccontinuous if f is soft πg-irresolute and g is soft πg-continuous.
2. $g \circ f$ is soft πg-irresolute if both f and g are soft πg-irresolute.
3. $g \circ f$ is soft πg-continuous if g is soft continuous and f is soft πg-continuous.

Proof: Straight forward

DEFINITION: 4.5

A function $f : (X, \tau, E) \to (Y, \tau', E)$ is soft πg-open function, if the image of every soft open set in X is soft πg-open in Y.

DEFINITION: 4.6

A function $f : (X, \tau, E) \to (Y, \tau', E)$ is soft πg-closed function, if the image of every soft closed set in X is soft πg-closed in Y.

THEOREM: 4.7

A function $f : (X, \tau, E) \to (Y, \tau', E)$ is soft πg-open if and only if f(int(F, E)) $\widetilde{\subseteq}$ $\tilde{S}\pi$g-int(f(F, E)) for every soft set (F, E) of E.

Proof:

Let $f : (X, \tau, E) \to (Y, \tau', E)$ be soft πg-open. Then f(int(F, E)) = $\tilde{S}\pi$g-int(f(int(F, E))) $\widetilde{\subseteq}$ $\tilde{S}\pi$g-int(f(F, E)).

On the other hand,Let (F, E) be soft open set of X. Then f(F, E) = f(int(F, E)) $\widetilde{\subseteq}$ $\tilde{S}\pi$g-int(f(F, E)). Hence f(F, E) is soft πg-open in Y.

THEOREM: 4.8

A function f $(X, \tau, E) \to (Y, \tau', E)$ is soft πg-closed if and only if $\tilde{S}\pi$g-cl(f(F, E)) $\widetilde{\subseteq}$ f(cl(F, E))

for every soft set (F, E) of E.

Proof:

Let $f : (X, \tau, E) \to (Y, \tau', E)$ be soft πg-closed. Then $\tilde{S}\pi$g-cl(f(cl(F, E))) = $\tilde{S}\pi$g-cl(f(F, E)) $\widetilde{\subseteq}$ f(cl(F,E)).

On the other hand,Let (F, E) be soft closed set of X. Then $\tilde{S}\pi$g-cl(f(F, E)) $\widetilde{\subseteq}$ f(cl(F, E)) = f(F, E) . Hence f(F, E) is soft πg-closed in Y.

THEOREM: 4.9

A function f : $(X, \tau, E) \to (Y, \tau', E)$ be a bijection. Then the following are equivalent:

(1) f is soft πg-open .

(2) f is soft πg-closed .

(3) f^{-1} is soft πg-continuous .

Proof:

(1) ⇒ (2)

Let (F, E) be soft closed set in X and f be a soft πg-open .Then X−(F, E) is soft open in X. Since f is soft πg-open, f(X−(F, E)) is soft πg-open set in Y. Then Y− f(X−(F, E)) = f(F, E) is soft πg-closed in Y. Hence f is soft πg-closed.

(2) ⇒ (3)

Let (F, E) be soft closed set in X and f be a soft πg-closed. Then f(F, E) is soft πg-closed set in Y. If f(F, E) = $(f^{-1})^{-1}$(F, E), then f^{-1} is soft πg-continuous.

(3) ⇒ (1)

Suppose f^{-1} is soft πg-continuous. Let (F, E) be soft open in X. Since f^{-1} is soft πg-continuous, $(f^{-1})^{-1}$(F, E)= f(F, E) is soft πg-open in Y. Therefore f is soft πg-open.

THEOREM: 4.10

Let f : $(X, \tau, E) \to (Y, \tau', E)$ be soft closed and g : $(Y, \tau', E) \to (Z, \tau'', E)$ be soft πg-closed then $g \circ f$ is soft πg-closed .

Proof:

Let (F, E) be soft closed in X. Then f(F, E) is closed in Y. Since g is soft πg-closed, g(f(F, E)) is soft πg-closed in Z. Then $g \circ f$ is soft πg-closed.

THEOREM: 4.11

Let f : $(X, \tau, E) \to (Y, \tau', E)$ be soft open and g : $(Y, \tau', E) \to (Z, \tau'', E)$ be soft πg-open then $g \circ f$ is soft πg-open.

Proof:

Let (F, E) be soft open in X. Then f(F, E) is soft πg-open in Y. Since g is soft πg-open,

g(f(F, E)) is soft πg-open in Z. Then $g \circ f$ is soft πg-open.

REFERENCES

[1] C.G. Aras and A. Sonmez, On soft mappings, arXiv:1305.4545,(2013).

[2] I.Arockiarani and A.Selvi, Soft πg-operators in soft topological spaces, International Journal of Mathematical archive, 5(4) (2014), 37-43.

[3] Athar Kharal and B. Ahmad, Mappings on soft classes, arXiv:1006.4940(2010).

[4] E.Ekici, C.W. Baker, On πg-closed sets and continuity, Kochi J. Math., 2007(2)35-42.

[5] K. Kannan, “Soft Generalized closed sets in soft Topological Spaces”, Journal of Theoretical and Applied Information Technology,37 (2012), 17-20.

[6] D.Molodtsov, “Soft set Theory- First Results” Computers and Mathematics with Applications (1999), 19-31.

[7] Saziye Yuksel, Naime Tozlu, Zehra Guzel regul, On soft generalized closed sets in Soft topological Spaces”, Journal of Theoretical and Applied Information Technology, 37 (2012), 17-20.

[8] A.Selvi and I. Arockiarani, Soft πg-closed sets in soft topological spaces, 79th Annual Conference of Indian Mathematical Society, December 28-31, 2013.

[9] M.Shabir and M. Naz , On Soft topological spaces, Computers and Mathematics with Applications(2011), vol.61, Issue 7, 1786-1799.
[10] Y.Yumak, A.K. Kayamakci, Soft β-open sets and their application, arXiv:1312.6964(2013).
[11] I. Zortlutuna, M. Akday, W.K.Min, S.Atmaca, Remarks on soft topological spaces, Annals of fuzzy mathematical and Informatics, 3(2012) 171-185.