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A Comparative View between Topological Space, Fuzzy Topological Space and Soft Topological Space

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ABSTRACT: The aim of this search is to study the relation between crisp set, fuzzy set, soft set and study the relation between topological space, fuzzy topological space, soft topological space.

KEYWORDS: Soft set, soft topology, fuzzy set, fuzzy topology, α.level sets, color pixel accuracy.

1 INTRODUCTION

Soft sets was introduced by the Russian Demetry Molodtsove 1999[3] as a general mathematical tool for dealing with uncertain objects, operations on soft set was introduced by P.K. Maji , R. Biswas and A.R. Roy 2003[8], Sabir and Nas 2011 [6] introduce and study the concept of soft topological spaces over soft set and some related concepts, Let X be an initial universe and E be a set of parameters, P(X) denote the power set of X , A pair (F,E) is called a soft set over X , where F is a mapping given by

F : E → P(X) , For two soft sets (F,A) and (G,B) over common universe X , we say that (F,E) is a soft subset (G,E) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$,null soft set, denoted by Φ where if for each $e \in E$, $F(e) = \Phi$, absolute soft set denoted by $\widetilde{X}$, if for each $e \in E$, $F(e) = X$, union of two soft sets of (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$, and $\forall e \in C$, we write $(F,A) \,\widetilde{\cup}\, (G,B) = (H,C)$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

The intersection of two soft sets (F,A) and (G,B) over a common universe X is the soft set (H,C), where $C = A \cap B$, and , $H(e) = F(e) \cap G(e)$, we write $(F,A) \,\widetilde{\cap}\, (G,B) = (H,C)\ \forall e \in C$.

The (α.level) of a fuzzy set $\widehat{A}$ is defined by {α.level $(\widehat{A}) = \{x \in X |\ M_A(x) \geq \alpha\ \}$,

We will denote to fuzzy set $\widehat{A}$, fuzzy topology $\widehat{T}$, soft set $\widetilde{A}$ and soft topology $\widetilde{T}$.

2 STUDY THE RELATION BETWEEN CRISP SETS, FUZZY SETS, SOFT SETS

The beginning of set theory as a branch of mathematics is often marked by the publication of Cantor's 1874 . the first publication in fuzzy set theory by Zadeh 1965 [4] **,** soft sets was introduced by the Russian Demetry Molodtsove 1999[3] , In this part we will study the relation between crisp sets, fuzzy sets , soft sets with examples and counter examples .

Definition 2.1.[7] A classical (orcrisp) set A⊆X is a set characterized by the function $\chi_A : X \longrightarrow \{0,1\}$ called the characteristic function and A is defined by

$$A = \left\{ x \in X \mid \chi_{(x)} = \begin{cases} 0 \text{ if } x \notin A \\ 1 \text{ if } x \in A \end{cases} \right\}$$

Definition 2.2.[7] A fuzzy set $\widehat{A}$ over X is a set characterized by a membership function of $\widehat{A}$, $M_A: X \longrightarrow I$ and $\widehat{A}$ represented by an ordered pairs $\widehat{A} = \{(x, M_{A(x)}) | x \in X, M_A(x) \in I\}$, M_A(x) is called the grade (or degree) of membership of x in set $\widehat{A}$.

Definition2.3.[3] Let X be an initial universe set, E be a set of parameters, P(X) set of all subsets of X, a pair (F,E) is called a soft set over X if and only if F is a mapping of E in to the set of all subsets of the set X (i.e. $F: E \to P(X)$) is called soft mapping .

Example2.4. Let a soft set (F,E) describe the attractive houses assume the four houses in the universe X={h_1, h_2, h_3, h_4} under the consideration E={e_1= cheep, e_2=expensive, e_3=comfortable} is the set of parameters, F(e_1)={h_1,h_2}, F(e_2)={h_3,h_4} F(e_3)={h_1,h_3,h_4} then the soft set (F,E) = {F(e_1), F(e_2), F(e_3)}={{h_1,h_2}, {h_3,h_4}, {h_1,h_3,h_4}} .

Remark2.5. Every fuzzy set is a generalized of crisp set since so crisp set is fuzzy set but the converse is not necessary true.

Examples2.6. Let $X = \{h_1, h_2, h_3, h_4\}$

1. $A = \{h_1, h_2\}$ is crisp set it is also fuzzy set since we can write A as the form

   $A = \{(h_1, 1), (h_2, 1), (h_3, 0), (h_2, 0)\} = \widehat{A}$

2. $\widehat{B} = \{(h_1, 0.1), (h_2, 0.2)\}$ is fuzzy set but not crisp set.

Proposition 2.7.[7] Every fuzzy set may be considered as a special case of the soft set (F,[0,1]).

Examples 2.8.

1- X={a,b,c}, $\widehat{A} = \{(a, 0.1)\}$ *is a* fuzzy set,

$F(0.1) = \{a \in X \mid M_A(a) \geq 0.1\}$, $\widetilde{A} = (F, E) = \{(0.1, a)\}$ is a soft set

2- X={a,b,c} , $\widehat{A} = \{(a, 0.1), (b, 0.2), (c, 0.3)\}$ is a fuzzy set

$$F(0.1) = \{x \in X, M_A(a) \geq 0.1\} = \{a, b, c\}$$
$$F(0.2) = \{x \in X, M_A(a) \geq 0.2\} = \{b, c\}$$
$$F(0.3) = \{x \in X, M_A(a) \geq 0.3\} = \{c\}$$

Then by Proposition (2.7.)

(F,A)={(0.1,{a,b,c}),(0.2,{b,c}),(0.3,{c})} is a soft set for the fuzzy set $\widehat{A}$

Remark 2.9. Every fuzzy set is soft set but the converse is not necessary true.

Examples 2.10.

1) Let X={p_1,…,p_6} be set of 6. types of a papers ,

E={e_1=best , e_2=good , e_3=fair , e_4=poor }

F(e_1)=F($_{best}$)={(p_1,0.2),(p_2,0.7),(p_5,0.9),(p_6,1.0)}

F(e_4)=F($_{poor}$)={(p_1,0.9),(p_2,0.3),(p_3,1.0),(p_4,1.0),(p_5,0.2)}

F(e_1) and F(e_4) are fuzzy sets Then the α.level for F(e_1), F(e_4) are given by F(e_1)$_{0.2}$={p_1,p_2,p_5,p_6} ,

F(e_1)$_{0.7}$ ={p_2,p_5,p_6} , F(e_1)$_{0.9}$ ={p_5,p_6}, F(e_1)1.0={p_6} , Where A={0.2, 0.7, 0.9, 1.0}$\subset$ [0,1]

$F : A \to P(X), F(e_1)_\alpha \in P(X), \forall \alpha \in A$

Thus the soft set for the fuzzy set F($_{best}$) can be written as (Fe_1,A)

(Fe_1,A)={(0.2,{p_1,p_2,p_5,p_6}),(0.7,{p_2,p_5,p_6}),(0.9,{p_5,p_6}),(1.5,{p_6})}

and the soft set for the fuzzy set F($_{poor}$) can be written as follows where

B={0.2,0.3,0.9,1.0}$\subset$[0,1], $F(e_4): B \to P(X), F_{e4}(\alpha) \in P(X), \forall \alpha \in B$
(Fe_4,B)={(0.2,{p_1,p_2,p_3,p_4,p_5}),(0.3,{p_1,p_3,p_4}),(0.9,{p_1,p_3,p_4},(1.0,{p_3,p_4})}.

This example show that every fuzzy set can be written as a soft set

2) Let $X=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\ldots e_8\}$ let $A=\{e_2, e_3, e_4, e_5, e_7\}$

$F(e_2)= \{h_2, h_3, h_5\}$, $F(e_3)= \{h_2,h_4\}$, $F(e_4)=\{h_1\}$,$F(e_5)= X$, $F(e_7)= \{h_3,h_5\}$

Then $(F,A)= \{F(e_2), F(e_3), F(e_4), F(e_5), F(e_7)\}$

$= \{\{h_2,h_3,h_5\}, \{h_2,h_4\}, \{h_1\}, X, \{h_3,h_5\}\}$ is soft set over X.

But (F,A) is not fuzzy set .

Remark 2.11. Crisp set is soft set but the converse is not necessary true.

Examples 2.12.

1) Let $X = \{h_1, h_2, h_3, h_4\}$, $A = \{h_1, h_2\}$

A is crisp set it can also be considered as soft set if there exist a set of parameters

$E \neq \Phi$ (or $E= \{0,1\}$) , as a special case if $E= \Phi$ then the soft set is crisp set .

2) (F,[0,1]) is soft set but not crisp set.

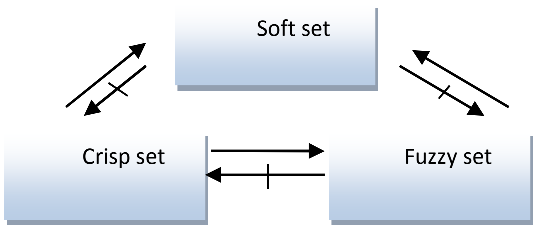

Diagram 1.

Show the relation between crisp set, fuzzy set and soft set

3 A Study of The Relattion Between Topological Space, Fuzzy Topological Space and Soft topological space

In 1968[2] Change used fuzzy set theory to define a fuzzy topological space , The concept of a fuzzy topology on a fuzzy set was introduced by M.K. Chakrabarty, T.M.G. Ahsanullah in1992 [5], Muhammad Sabir and Nas 2011[6] introduce and study the concept of soft topological spaces over soft set and some related concepts, in this section we introduce a comparative view between topological space , fuzzy topological space and soft topological space .

Definition 3.1.[2] A fuzzy topology is a family $\widehat{T}$ of fuzzy sets in X which satisfies the following conditions :

1) $\widehat{\Phi}, \widehat{X} \in \widehat{T}$

2) if $\widehat{A}, \widehat{B} \in \widehat{T}$ then $\widehat{A} \widehat{\cap} \widehat{B} \in \widehat{T}$

3) if $\widehat{A}_i \in \widehat{T}$ for all $i \in j$ then $\widehat{\cup} \widehat{A}_{i\ i\in j} \in \widehat{T}$.

$\widehat{T}$ is called a fuzzy topology for X and the pair (X, $\widehat{T}$) is called a fuzzy topological space denoted by (F.T.S.) members of $\widehat{T}$ are called fuzzy open sets in (X, $\widehat{T}$), complement of the members of $\widehat{T}$ are called fuzzy closed sets of (X, $\widehat{T}$).

Examples 3.2.[2]

1- Let $X \neq \Phi$,$\widehat{T}$ contains only $\widehat{\Phi}, \widehat{X}$ then (X, $\widehat{T}_i$) is fuzzy topological space which is called the indiscrete fuzzy topological space.

2- Let $X \neq \Phi$, $\widehat{T}$ be the collection of all fuzzy subsets of X then (X, $\widehat{T}_d$) is fuzzy topological space which is called the discrete fuzzy topological space**.**

Definition 3.3.[6] Let $\widetilde{T}$ be the collection of soft sets over X then $\widetilde{T}$ is said to be soft topology on X if

1- $\widetilde{\Phi}, \widetilde{X}$ belong to $\widetilde{T}$.

2- The union of any number of soft sets in $\widetilde{T}$ belongs to $\widetilde{T}$.

3- The intersection of any two soft sets in $\widetilde{T}$ belongs to $\widetilde{T}$.

The triple (X,$\widetilde{T}$,E) is called a soft topological space over X denoted by (S.T.S.)

Definition 3.4.[6] Let (X,$\widetilde{T}$,E) be a soft space over X , then the members of $\widetilde{T}$ are said to be soft open sets in X.

Definition 3.5.[6] Let (X,$\widetilde{T}$,E) be a soft space over X, a soft set (F,E) over X is said to be a soft closed set in X, if its relative complement (F,E)′ belongs to $\widetilde{T}$.

Examples 3.6.

1) Let X={L_1,…,L_{10}},E={e_1=very costly, e_2=costly, e_3=cheap, e_4=with multi colors, e_5=contain a free programs, e_6=have insurance , e_7=modern, e_8=expensive in repair, e_9=cheap in repair}

suppose A={e_1, e_2 , e_3 } , let F(e_1)={L_2,L_4,L_7,L_8} , F(e_2)={L_1,L_3,L_5} , F(e_3)={L_6,L_9}.

B={ e_3 , e_4 , e_5} , G(e_3)={L_6,L_9,L_{10}},G(e_4)={L_5,L_6,L_8} , G(e_5)= {L_2,L_3,L_7} .

Then the intersection of (F,A) with (G,B) we get (H,C) , H(e_3)=H(cheap)={L_5,L_9} .

The union of (F,A) and (G,B) is (N,P)

N(e_1)={L_2,L_4,L_7,L_8} , N(e_2)=N(costly)={L_1,L_3,L_5}, N(e_3)={L_6,L_9,L_{10}} ,

N(e_4)={L_5,L_6,L_8}, N(e_5)={L_2,L_3,L_7},

(F,A)= $\{F(e_i), i = 1,2,3\}$={{L_2,L_4,L_7,L_8},{L_1,L_3,L_5},{L_6,L_9},Φ, Φ}

(G,B)= $\{G(e_i), i = 3,4,5\}$ ={Φ, Φ,{L_6,L_9,L_{10}} ,{L_5,L_6,L_8} , {L_2,L_3,L_7}}

(H,C)= $\{H(e_i), i = 3\}$ ={Φ, Φ,{ L_5,L_9},Φ, Φ} ,(N,D)= $\{N(e_i), i = 1,2,3,4,5\}$

={{ L_2,L_4,L_7,L_8},{L_1,L_3,L_5},{L_6,L_9,L_{10}},{L_5,L_6,L_8},{L_2,L_3,L_7}}

The family $\widetilde{T}$={$\widetilde{\Phi}$,$\widetilde{X}$,(F,A),(G,B),(H,C),(N,D)} is soft topology on X,

$(F,A) \widetilde{\cap} (G,B) = (H,C)$ and $(F,A) \widetilde{\cup} (G,B) = (N,D)$ hence (X,$\widetilde{T}$,E) is S.T.S. over X. (F,A), (G,B), (H,C), (N,D) are soft open sets.

2) Let X={u_1,u_2,u_3}, E={x_1,x_2,x_3}, A= {x_1,x_2}$\widetilde{\subseteq}$ E then all soft subsets of the soft set (F,A)= {(x_1,{u_1,u_2}), (x_2, {u_2,u_3})} given by the following

$(F,A)_1$= {(x_1,{u_1})}, $(F,A)_2$= {(x_1,{u_2})}, $(F,A)_3$= {(x_1,{u_2, u_2})},

$(F,A)_4$= {(x_2,{u_2})}, $(F,A)_5$= {(x_2,{u_3})},$(F,A)_6$= {(x_2,{u_2,u_3})},

$(F,A)_7$= {(x_1,{u_1}), (x_2, {u_2})}, $(F,A)_8$= {(x_1,{u_1}), (x_2, {u_3})},

$(F,A)_9$= {(x_1,{u_1}), (x_2, {u_2,u_3})},

$(F,A)_{10}$= {(x_1,{u_2}), (x_2, {u_2})},$(F,A)_{11}$= {(x_1,{u_2}), (x_2,{u_3})},

$(F,A)_{12}$= {(x_1,{u_2}), (x_2,{u_2,u_3})} , $(F,A)_{13}$={(x_1,{u_1,u_2}), (x_2,{u_2})},

$(F,A)_{14}$= {(x_1,{u_1, u_2}), (x_2, {u_3}) , $(F,A)_{15}$= (F,A) ,

$(F,A)_{16}$= $\widetilde{\Phi}_A$, Where P(F,A)= 2^4=16 , Then $\widetilde{T}_1$= {$\widetilde{\Phi}_A$, (F,A)} , $\widetilde{T}_2$= P((F,A)) ,

$\widetilde{T}_3$= {$\widetilde{\Phi}_A$, (F,A), $(F,A)_2$, $(F,A)_{11}$, $(F,A)_{13}$},$\widetilde{T}_1$, $\widetilde{T}_2$, $\widetilde{T}_3$ are soft topologies on (F,A) .

Propositions 3.7.

1- Every topological space is fuzzy topological space.

2- Every topological space is soft topological space.

3- Every fuzzy topological space is soft topological space.

Proof the proofs are obvious .

Remark 3.8. The converse of Proposition (3.7.1) is not necessary true.

Example 3.9. Let X = {a, b, c} , $\hat{T}$ **=** {$\hat{\Phi}$, $\hat{X}$, (a, 0.9)} then (X,$\hat{T}$) is fuzzy topological space but not topological space.

Remark 3.10 The converse of propositions (3.7.2) ,(3.7.3) are not necessary true see example (3.6(1)).

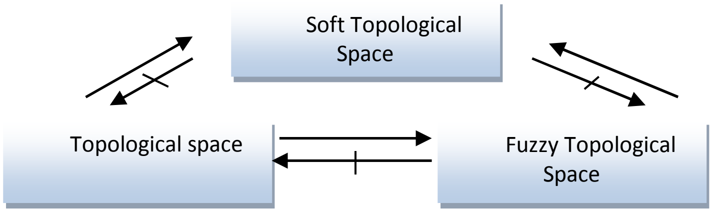

Diagram 2.

show the relation between Topological Space ,Fuzzy Topological Space and Soft Topological Space.

The next example show a method of transform a topological space in to a fuzzy topological space then in to a soft topological space it is also an application to determine color accuracy of a pixel.

Example 3.11. Let X={R,G,B} be a set of three colors then T is a topology on X, consider a single parameter "the degree of color", L(the degree of color)={A,B,D} each degree associated with its own fuzzy set three of them might be defined as follows

$\hat{F}_A$ = {(R,0.1), (G,0.2)} , $\hat{F}_B$ ={(R,0.3), (G,0.2)} , $\hat{F}_D$ ={(R,0.1), (G,0.2), (B,0.3)}

consider the fuzzy sets $\hat{F}_A$, $\hat{F}_B$, $\hat{F}_D$ and their α.level sets are

$\hat{F}_A$ (0.1)= {R, G} , $\hat{F}_A$ (0.2)= {G} , $\hat{F}_B$ (0.2)= {R, G} , $\hat{F}_B$ (0.3)= {R}

$\hat{F}_D$ (0.1)={R,G,B} , $\hat{F}_D$ (0.2)={G,B} , $\hat{F}_D$ (0.3)={B}

The value H={0.1, 0.2}$\subset$ [0,1] can be treated as a set of parameters s. t. the mapping $\hat{F}_A$: $H \to P(X)$ give an approximate value sets $\hat{F}_A(\alpha)$ for $\alpha \in H$,we can write the equivalent soft set as follows : ($\hat{F}_A$,I) = $\{\ (0.1, \{R, G\})\ , (0.2, \{G\})\ \}$

Similarly for K ={0.2, 0.3}$\subset$[0,1] can be treated as a set of parameters s. t.

$\hat{F}_B$: $K \to P(X)$, we have ($\hat{F}_B$,I)= $\{\ (0.2, \{R, G\})\ , (0.3, \{R\})\ \}$

And for L ={0.1 , 0.2 , 0.3}$\subset$[0,1] can be treated as a set of parameters s. t.

$\hat{F}_D$: $L \to P(X)$, we have ($\hat{F}_D$,I)= $\{\ (0.1, \{R, G, B\})\ , (0.2, \{G, B\})\ , (0.3, \{R\})\ \}$

So we can define a (discrete topology)

T= { Φ , X , R , G , B , R $\cap$ G , R $\cap$ B, G $\cap$ B , R $\cup$ G , R $\cup$ B, G $\cup$ B }

And define the (fuzzy discrete topology)

$\hat{T}$={ $\hat{\Phi}$, $\hat{X}$, $\hat{F}_A$, $\hat{F}_B$, $\hat{F}_D$, $\hat{F}_A \hat{\cap} \hat{F}_B$, $\hat{F}_A \hat{\cap} \hat{F}_D$, $\hat{F}_B \hat{\cap} \hat{F}_D$, $\hat{F}_A \hat{\cup} \hat{F}_B$, $\hat{F}_A \hat{\cup} \hat{F}_D$, $\hat{F}_B \hat{\cup} \hat{F}_D$}

More over we can define the equivalent (soft discrete topology)

$\tilde{T}$={$\tilde{\Phi}$,$\tilde{X}$, ($\hat{F}_A$,I), ($\hat{F}_B$, I) , ($\hat{F}_D$, I) , ($\hat{F}_A$, I) $\tilde{\cap}$($\hat{F}_B$, I) , ($\hat{F}_A$, I) $\tilde{\cap}$ ($\hat{F}_D$, I),

($\hat{F}_B$, I) $\tilde{\cap}$($\hat{F}_D$, I) , ($\hat{F}_A$, I) $\tilde{\cup}$ ($\hat{F}_B$, I) , ($\hat{F}_A$, I) $\tilde{\cup}$ ($\hat{F}_D$, I), ($\hat{F}_B$, I) $\tilde{\cup}$ ($\hat{F}_D$, I) }

And in general $\tilde{\Phi}$ = {{(αi,Φ)}, $\forall \alpha i \in I$ } , $\tilde{X}$ = {{(αi, X)}, $\forall \alpha i \in I$ } .

4 CONCLUSION

In this search we introduce a study of the relation between crisp set, fuzzy set and soft set use it to study the relation between topological space, fuzzy topological space , soft topological space we conclude that (each topological space is fuzzy topological space and each fuzzy topological space is soft topological space but the converse is not necessary true) , finally we give a practical example on this study .

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