International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 8 No. 1 Sep. 2014, pp. 195-203

http://www.ijias.issr-journals.org/

Five Point Predictor-Corrector Formulae and Their Comparative Analysis

Mahtab Uddin

Institute of Natural Sciences, United International University, Bangladesh

ABSTRACT: This paper is mainly analytical and comparative. Here I have proposed three new forms of predictor-corrector formulae for solving ordinary differential equation of first order and first degree. These predictor-corrector formulae have derived by taking the general forms of predictor-corrector. These formulae approximate the value of dependent variable based on five initial value of independent variable by predictor formula and then improve that initial crude value of dependent variable by corrector formula. A comparative analysis among proposed three predictor-corrector formula with Milne's predictor-corrector formula and Adam-Moulton's predictor-corrector formula by means of comparing with exact value of dependent variable have expressed as relative error. Finally, conclusive discussions have narrated.

KEYWORDS: ODE, predictor-corrector formula, general form, particular form, comparative analysis, relative error, accuracy.

1 INTRODUCTION

Considering the initial value [4] problem

$$y' = \frac{dy}{dx} = f(x,y);\ y(x_0) = y_0 \tag{1}$$

If the function $f(x,y)$ is continuous in the open interval $a < x < b$ containing $x = x_0$, there exists a unique solution [5] of the equation (1) as $y_r = y(x_r)$; $r = 1,2,3,\dots\dots\dots$ The solution is valid for throughout the interval $a < x < b$. It is required to determine the approximate values of y_r for the exact solution $y = y(x)$ in the given interval for the value $x = x_r = x_0 + rh$; $r = 1,2,3,\dots\dots\dots$

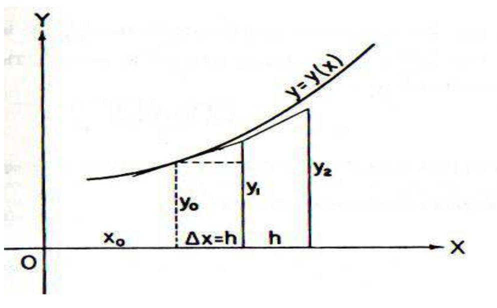

Figure-(1)

Now deriving a tangent line equation for (1), from above figure

$$\frac{\Delta y}{\Delta x} \approx tan\theta$$
$$or, \Delta y \approx \Delta x(tan\theta)$$
$$or, y_1 - y_0 \approx h(\frac{dy}{dx})_0$$
$$or, y_1 \approx y_0 + hf(x_0, y_0) \quad (2)$$

After starting with the initial valuey_0, an approximate value of $y_1 = y_1^{(1)}$ is computed from the relation given by (2) as

$$y_1^{(1)} \approx y_0 + h(\frac{dy}{dx})_0 = y_0 + hf(x_0, y_0)$$

Substituting this approximate value of y_1 in (1) for getting an approximate value of $f(x, y)$ at the end of the first interval as:

$$(\frac{dy}{dx})_1^{(1)} = f(x_1, y_1^{(1)})$$

Now the improved value of Δy is obtained by using the Trapezoidal rule [6,7] as

$$\Delta y \approx \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

Then the second approximation for y_1 is now

$$y_1^{(2)} \approx y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

Substituting this improved value of $y_1^{(1)}$we get the second approximate value of $f(x, y)$ as

$$(\frac{dy}{dx})_1^{(2)} = f(x_1, y_1^{(2)})$$

Then the third approximation for y_1 is now

$$y_1^{(3)} \approx y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$$

Continuing this process, we can find

$$y_1^{(n)} \approx y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(n-1)})] \quad (4)$$

Then the next approximation for $y_1$is obtained as

$$y_1^{(n+1)} \approx y_0 + \frac{h}{2}[f(x_0, y_0) + f\left(x_1, y_1^{(n)}\right)] \quad (5)$$

This process is applied repeatedly until no significant change is produced in two consecutive values of y_1.

The above process is applied for the first interval and same manner can be hold for the next intervals also. Then the general formula takes the following form

$$y_{m+1} \approx y_m + hf(x_m, y_m) \tag{6}$$

$$y_{m+1}^{(n+1)} \approx y_m + \frac{h}{2}[f(x_m, y_m) + f(x_{m+1}, y_{m+1}^{(n)})] \tag{7}$$

The value of y_{m+1} will be determined using (6) and is to be substituted on right hand side of (7) for giving a better approximation of y_{m+1}. This method of refining an initial crude estimate of y_{m+1} by means of a more accurate formula is known as predictor-corrector [1] method. In the above (6) is called the predictor formula and (7) is called the corrector formula for obtain y_{m+1}.

The predictor-corrector method is a multi-step, iterative and self-correcting [8] process to the value of dependent variable by means of some initial value of independent variable.

In the above process the predictor-corrector method is used for a single initial value problem but that method is applicable for several initial value [3] problems.

2 DERIVATION

2.1 DERIVATION OF PREDICTOR FORMULAE

The general linear predictor formula [2] which involves the information about the function and its derivative at the past four points together with the value of the at the given point being computed as

$$\begin{aligned} y_{n+1} &= A_0 y_n + A_1 y_{n-1} + A_2 y_{n-2} + A_3 y_{n-3} + A_4 y_{n-4} \\ &\quad + h[B_0 y_n' + B_1 y_{n-1}' + B_2 y_{n-2}' + B_3 y_{n-3}' + B_4 y_{n-4}'] \end{aligned} \tag{8}$$

Above equation contains ten unknowns. Suppose it holds for polynomials up-to degree five. Hence take $y(x) = x^n\,; n = 0,1,2,3,4,5$. Let the space between the consecutive values of x be unity. i.e. taking $h = 1$.

Now putting $h = 1$ and $y(x) = 1, x, x^2, x^3, x^4, x^5$ successively in (8), obtain as

$$\begin{aligned} 1 &= A_0 + A_1 + A_2 + A_3 + A_4 \\ 1 &= -A_1 - 2A_2 - 3A_3 - 4A_4 + B_0 + B_1 + B_2 + B_3 + B_4 \\ 1 &= A_1 + 4A_2 + 9A_3 + 16A_4 - 2B_1 - 4B_2 - 6B_3 - 8B_4 \\ 1 &= -A_1 - 8A_2 - 27A_3 - 64A_4 + 3B_1 + 12B_2 + 27B_3 + 48B_4 \\ 1 &= A_1 + 16A_2 + 81A_3 + 256A_4 - 4B_1 - 32B_2 - 108B_3 - 256B_4 \\ 1 &= -A_1 - 32A_2 - 243A_3 - 1024A_4 + 5B_1 + 80B_2 + 405B_3 + 1280B_4 \end{aligned} \tag{9}$$

Here six equations with ten unknowns. Introducing three assumptions as followings

For first assumption taking A_0, A_1, A_2 & A_3 as parameter, then (9) gives

$$\begin{aligned} A_4 &= 1 - A_0 - A_1 - A_2 - A_3 \\ B_0 &= \frac{1}{720}[2125 - 224A_0 + 27A_1 + 8A_2 + 19A_3] \\ B_1 &= \frac{1}{360}[-875 - 512A_0 - 189A_1 - 16A_2 - 53A_3] \\ B_2 &= \frac{1}{30}[125 - 16A_0 - 27A_1 - 8A_2 + 11A_3] \\ B_3 &= \frac{1}{360}[-125 - 512A_0 - 459A_1 - 496A_2 - 323A_3] \\ B_4 &= \frac{1}{720}[475 - 224A_0 - 243A_1 - 232A_2 - 251A_3] \end{aligned} \tag{10}$$

Since A_0, A_1, A_2 & A_3 are arbitrary, choosing $A_0 = A_1 = A_2 = A_3 = 0$. Then from (10) get the followings

$$A_4 = 1, B_0 = \frac{425}{144}, B_1 = -\frac{175}{72}, B_2 = \frac{25}{6}, B_3 = -\frac{25}{72}, B_4 = \frac{95}{144}$$

To get a predictor formula substituting these values in (8), as follows

$$y_{n+1}^{p} = (1)y_{n-4} + h[\left(\frac{425}{144}\right)y_n' + \left(-\frac{175}{72}\right)y_{n-1}' + \left(\frac{25}{6}\right)y_{n-2}' + \left(-\frac{25}{72}\right)y_{n-3}' + (\frac{95}{144})y_{n-4}']$$

$$or, y_{n+1}^{p} = y_{n-4} + \frac{h}{144}[425y_n' - 350y_{n-1}' + 600y_{n-2}' - 50y_{n-3}' + 95y_{n-4}'] \quad (11)$$

For second assumption taking A_0, A_1, A_3 & A_4 as parameter, then (9) gives

$$A_2 = 1 - A_0 - A_1 - A_3 - A_4$$
$$B_0 = \frac{1}{720}[2133 - 232A_0 + 19A_1 + 11A_3 - 8A_4]$$
$$B_1 = \frac{1}{360}[-891 - 496A_0 - 173A_1 - 37A_3 + 16A_4]$$
$$B_2 = \frac{1}{30}[117 - 8A_0 - 19A_1 + 19A_3 + 8A_4]$$
$$B_3 = \frac{1}{360}[-621 - 16A_0 + 37A_1 + 173A_3 + 496A_4]$$
$$B_4 = \frac{1}{720}[243 + 8A_0 - 11A_1 - 19A_3 + 232A_4] \quad (12)$$

Since A_0, A_1, A_3 & A_4 are arbitrary, choosing $A_0 = A_1 = A_3 = A_4 = 0$. Then from (12) get the followings

$$A_2 = 1, B_0 = \frac{237}{80}, B_1 = -\frac{99}{40}, B_2 = \frac{39}{10}, B_3 = -\frac{69}{40}, B_4 = \frac{27}{80}$$

To get a predictor formula substituting these values in (8), as follows

$$y_{n+1}^{p} = (1)y_{n-2} + h[\left(\frac{237}{80}\right)y_n' + \left(-\frac{99}{40}\right)y_{n-1}' + \left(\frac{39}{10}\right)y_{n-2}' + \left(-\frac{69}{40}\right)y_{n-3}' + (\frac{27}{80})y_{n-4}']$$

$$or, y_{n+1}^{p} = y_{n-2} + \frac{h}{80}[237y_n' - 198y_{n-1}' + 312y_{n-2}' - 138y_{n-3}' + 27y_{n-4}'] \quad (13)$$

For third assumption taking A_1, A_2, A_3 & A_4 as parameter, then (9) gives

$$A_0 = 1 - A_1 - A_2 - A_3 - A_4$$
$$B_0 = \frac{1}{720}[1901 + 251A_1 + 232A_2 + 243A_3 + 224A_4]$$
$$B_1 = \frac{1}{360}[-1387 + 323A_1 + 496A_2 + 496A_3 + 512A_4]$$
$$B_2 = \frac{1}{30}[109 - 11A_1 + 8A_2 + 27A_3 + 16A_4]$$
$$B_3 = \frac{1}{360}[-637 + 53A_1 + 16A_2 + 189A_3 + 512A_4]$$
$$B_4 = \frac{1}{720}[251 - 19A_1 - 8A_2 - 27A_3 + 224A_4] \quad (14)$$

Since A_1, A_2, A_3 & A_4 are arbitrary, choosing $A_1 = A_2 = A_3 = A_4 = 0$. Then from (14) get the followings

$$A_0 = 1, B_0 = \frac{1901}{720}, B_1 = -\frac{1387}{360}, B_2 = \frac{109}{30}, B_3 = -\frac{637}{360}, B_4 = \frac{251}{720}$$

To get a predictor formula substituting these values in (8), as follows

$$y_{n+1}^{p} = (1)y_n + h[\left(\frac{1901}{720}\right) y_n^{'} + \left(-\frac{1387}{360}\right) y_{n-1}^{'} + \left(\frac{109}{30}\right) y_{n-2}^{'} + \left(-\frac{637}{360}\right) y_{n-3}^{'} + (\frac{251}{720}) y_{n-4}^{'}]$$

$$or, y_{n+1}^{p} = y_n + \frac{h}{720}[1901y_n^{'} - 2774y_{n-1}^{'} + 2616y_{n-2}^{'} - 1274y_{n-3}^{'} + 251y_{n-4}^{'}] \quad (15)$$

2.2 DERIVATION OF CORRECTOR FORMULAE

The general linear corrector formula [2] which involves the information about the function and its derivative at the past four points together with the value of the at the given point being computed as

$$y_{n+1} = a_0 y_n + a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4}$$
$$+h[b_{-1} y'_{n+1} + b_0 y'_n + b_1 y'_{n-1} + b_2 y'_{n-2} + b_3 y'_{n-3}] \quad (16)$$

Above equation contains ten unknowns. Suppose it holds for polynomials up-to degree five. Hence take $y(x) = x^n\,; n = 0,1,2,3,4,5$. Let the space between the consecutive values of x be unity. i.e. taking $h = 1$.

Now putting $h = 1$ and $y(x) = 1, x, x^2, x^3, x^4, x^5$ successively in (16), obtain as

$$1 = a_0 + a_1 + a_2 + a_3 + a_4$$
$$1 = -a_1 - 2a_2 - 3a_3 - 4a_4 + b_{-1} + b_0 + b_1 + b_2 + b_3$$
$$1 = a_1 + 4a_2 + 9a_3 + 16a_4 + 2b_{-1} - 2b_1 - 4b_2 - 6b_3$$
$$1 = -a_1 - 8a_2 - 27a_3 - 64a_4 + 3b_{-1} + 3b_1 + 12b_2 + 27b_3$$
$$1 = a_1 + 16a_2 + 81a_3 + 256a_4 + 4b_{-1} - 4b_1 - 32b_2 - 108b_3$$
$$1 = -a_1 - 32a_2 - 243a_3 - 1024a_4 + 5b_{-1} + 5b_1 + 80b_2 + 405b_3 \quad (17)$$

Here six equations with ten unknowns. Introducing three assumptions as followings

For first assumption taking a_0, a_1, a_2 & a_3 as parameter, then (17) gives

$$a_4 = 1 - a_0 - a_1 - a_2 - a_3$$
$$b_{-1} = \frac{1}{720}[475 - 224a_0 - 243a_1 - 232a_2 - 251a_3]$$
$$b_0 = \frac{1}{360}[-125 + 448a_0 + 621a_1 + 584a_2 + 637a_3]$$
$$b_1 = \frac{1}{30}[125 - 136a_0 - 117a_1 - 98a_2 - 109a_3]$$
$$b_2 = \frac{1}{360}[875 + 928a_0 + 981a_1 + 1064a_2 + 1387a_3]$$
$$b_3 = \frac{1}{720}[2125 - 2144a_0 - 2133\, a_1 - 2152a_2 - 1901a_3] \quad (18)$$

Since a_0, a_1, a_2 & a_3 are arbitrary, choosing $a_0 = a_1 = a_2 = a_3 = 0$. Then from (18) get the followings

$$a_4 = 1, b_{-1} = \frac{95}{144}, b_0 = -\frac{25}{72}, b_1 = \frac{25}{6}, b_2 = -\frac{175}{72}, b_3 = \frac{425}{144}$$

To get a corrector formula substituting these values in (16), as follows

$$y_{n+1}^{c} = (1)y_{n-4} + h[(\frac{95}{144}) y_{n+1}^{'} + \left(-\frac{25}{72}\right) y_n^{'} + \left(\frac{25}{6}\right) y_{n-1}^{'} + \left(-\frac{175}{72}\right) y_{n-2}^{'} + \left(\frac{425}{144}\right) y_{n-3}^{'}]$$

$$or, y_{n+1}^{c} = y_{n-4} + \frac{h}{144}[95y_{n+1}^{'} - 50y_n^{'} + 600y_{n-1}^{'} - 350y_{n-2}^{'} + 425y_{n-3}^{'}] \quad (19)$$

For second assumption taking a_0, a_1, a_3 & a_4 as parameter, then (17) gives

$$
\begin{aligned}
a_2 &= 1 - a_0 - a_1 - a_3 - a_4 \\
b_{-1} &= \frac{1}{720}[243 + 8a_0 - 11a_1 - 19a_3 + 232a_4] \\
b_0 &= \frac{1}{360}[459 - 136a_0 + 37a_1 + 53a_3 - 584a_4] \\
b_1 &= \frac{1}{30}[27 - 38a_0 - 19a_1 - 11a_3 + 98a_4] \\
b_2 &= \frac{1}{360}[189 - 136a_0 - 173a_1 + 323a_3 - 1064a_4] \\
b_3 &= \frac{1}{720}[-27 + 8a_0 + 19a_1 + 251a_3 + 2152a_4]
\end{aligned} \tag{20}
$$

Since a_0, a_1, a_3 & a_4 are arbitrary, choosing $a_0 = a_1 = a_3 = a_4 = 0$. Then from (20) get the followings

$$a_2 = 1, b_{-1} = \frac{27}{80}, b_0 = \frac{51}{40}, b_1 = \frac{9}{10}, b_2 = \frac{21}{40}, b_3 = -\frac{3}{80}$$

To get a corrector formula substituting these values in (16), as follows

$$y_{n+1}^c = (1)y_{n-2} + h[(\tfrac{27}{80})y'_{n+1} + \left(\tfrac{51}{40}\right)y'_n + \left(\tfrac{9}{10}\right)y'_{n-1} + \left(\tfrac{21}{40}\right)y'_{n-2} + \left(-\tfrac{3}{80}\right)y'_{n-3}]$$

$$or, y_{n+1}^c = y_{n-2} + \frac{h}{80}[27y'_{n+1} + 102y'_n + 72y'_{n-1} + 42y'_{n-2} - 3y'_{n-3}] \tag{21}$$

For third assumption taking a_1, a_2, a_3 & a_4 as parameter, then (17) gives

$$
\begin{aligned}
a_0 &= 1 - a_1 - a_2 - a_3 - a_4 \\
b_{-1} &= \frac{1}{720}[251 - 19a_1 - 8a_2 - 27a_3 + 224a_4] \\
b_0 &= \frac{1}{360}[323 + 173a_1 + 136a_2 + 189a_3 - 448a_4] \\
b_1 &= \frac{1}{30}[-11 + 19a_1 + 38a_2 + 27a_3 + 136a_4] \\
b_2 &= \frac{1}{360}[53 - 37a_1 + 136a_2 + 459a_3 - 928a_4] \\
b_3 &= \frac{1}{720}[-19 + 11a_1 - 8a_2 + 243a_3 + 2144a_4]
\end{aligned} \tag{22}
$$

Since a_1, a_2, a_3 & a_4 are arbitrary, choosing $a_1 = a_2 = a_3 = a_4 = 0$. Then from (20) get the followings

$$a_0 = 1, b_{-1} = \frac{251}{720}, b_0 = \frac{323}{360}, b_1 = -\frac{11}{30}, b_2 = \frac{53}{360}, b_3 = -\frac{19}{720}$$

To get a corrector formula substituting these values in (16), as follows

$$y_{n+1}^c = (1)y_n + h[(\tfrac{251}{720})y'_{n+1} + \left(\tfrac{323}{360}\right)y'_n + \left(-\tfrac{11}{30}\right)y'_{n-1} + \left(\tfrac{53}{360}\right)y'_{n-2} + \left(-\tfrac{19}{720}\right)y'_{n-3}]$$

$$or, y_{n+1}^c = y_n + \frac{h}{720}[251y'_{n+1} + 646y'_n - 264y'_{n-1} + 106y'_{n-2} - 19y'_{n-3}] \tag{23}$$

2.3 PARTICULAR FORMS

The system of above predictor-corrector schemes can be expressed in particular form by putting $n = 4$ and then these reduce to the following system of predictor-corrector formulae

$$y_5^p = y_0 + \frac{h}{144}[425y_4' - 350y_3' + 600y_2' - 50y_1' + 95y_0']$$
$$y_5^c = y_0 + \frac{h}{144}[95y_5' - 50y_4' + 600y_3' - 350y_2' + 425y_1']$$

$$y_5^p = y_2 + \frac{h}{80}[237y_4' - 198y_3' + 312y_2' - 138y_1' + 27y_0']$$
$$y_5^c = y_2 + \frac{h}{80}[27y_5' + 102y_4' + 72y_3' + 42y_2' - 3y_1']$$

$$y_5^p = y_4 + \frac{h}{720}[1901y_4' - 2774y_3' + 2616y_2' - 1274y_1' + 251y_0']$$
$$y_5^c = y_4 + \frac{h}{720}[251y_5' + 646y_4' - 264y_3' + 106y_2' - 19y_1']$$

3 NUMERICAL EXAMPLES

Problem-1: $Solve\ \ y^/ = \frac{dy}{dx} = x + y - 1\ \ at\ \ x = 1.00$

With initial values are $y(0.00) = 1.00000000$, $y(0.20) = 1.02140276$, $y(0.40) = 1.09182470$, $y(0.60) = 1.22211880$ & $y(0.80) = 1.42554093$.

The analytical solution is $y = e^x - x$

Problem-2: $Solve\ \ y^/ = \frac{dy}{dx} = e^x + y\ \ at\ \ x = 1.25$

With initial values are $y(0.00) = 1.00000000$, $y(0.25) = 1.60503177$, $y(0.50) = 2.47308191$, $y(0.75) = 3.70475003$ & $y(1.00) = 5.43656366$.

The analytical solution is $y = (1 + x)e^x$

Problem-3: $Solve\ y^/ = \frac{dy}{dx} = xy\ \ at\ \ x = 1.25$

With initial values are $y(0.00) = 2.00000000$, $y(0.25) = 2.06348682$, $y(0.50) = 2.26629691$, $y(0.75) = 2.64956952$ & $y(1.00) = 3.29744254$.

The analytical solution is $y = 2e^{\frac{x^2}{2}}$

Problem-04: $Solve\ \ y^/ = \frac{dy}{dx} = \frac{x+y}{2}\ \ at\ \ x = 2.50$

With initial values are $y(0.00) = 2.00000000$, $y(0.50) = 2.6361016$, $y(1.00) = 3.59488508$, $y(1.50) = 4.96800007$ & $y(2.00) = 6.87312731$.

The analytical solution is $y = 4e^{\frac{x}{2}} - x - 2$

Problem-5: $Solve\ \ y^{/} = \frac{dy}{dx} = \frac{y}{1-x}\ \ at\ \ x = 0.50$

With initial values are $y(0.00) = 1.00000000$, $y(0.10) = 1.11111111$, $y(0.20) = 1.25000000$, $y(0.30) = 1.42857143$ & $y(0.40) = 1.66666667$.

The analytical solution is $y = \frac{1}{1-x}$

4 Comparative Analysis Of Numerical Results

Problem No.	Exact value	New method I	New method II	New method III	Milne method	Adam-Moulton method
01	1.71828183	1.71832282 5^{th} iteration $\|E_R\|$ = 0.002386%	1.71828428 4^{th} iteration $\|E_R\|$ = 0.000143%	1.71828432 4^{th} iteration $\|E_R\|$ = 0.000145%	1.71829032 5^{th} iteration $\|E_R\|$ = 0.000494%	1.71830108 5^{th} iteration $\|E_R\|$ = 0.0011203%
02	7.85327165	7.85469550 8^{th} iteration $\|E_R\|$ = 0.018137%	7.85335930 5^{th} iteration $\|E_R\|$ = 0.001116%	7.85336122 6^{th} iteration $\|E_R\|$ = 0.001141%	7.85349775 6^{th} iteration $\|E_R\|$ = 0.002879%	7.85377374 7^{th} iteration $\|E_R\|$ = 0.006393%
03	4.36840162	4.37851870 10^{th} iteration $\|E_R\|$ = 0.231586%	4.36917214 7^{th} iteration $\|E_R\|$ = 0.017639%	4.36920791 7^{th} iteration $\|E_R\|$ = 0.018457%	4.36946805 7^{th} iteration $\|E_R\|$ = 0.024412%	4.37053870 7^{th} iteration $\|E_R\|$ = 0.048921%
04	9.46137183	9.46211459 8^{th} iteration $\|E_R\|$ = 0.007851%	9.46141668 5^{th} iteration $\|E_R\|$ = 0.000474%	9.46141756 5^{th} iteration $\|E_R\|$ = 0.000483%	9.46150093 6^{th} iteration $\|E_R\|$ = 0.001365%	9.46166121 6^{th} iteration $\|E_R\|$ = 0.003059%
05	2.00000000	2.00273381 7^{th} iteration $\|E_R\|$ = 0.136691%	2.00025926 5^{th} iteration $\|E_R\|$ = 0.012963%	2.00027429 5^{th} iteration $\|E_R\|$ = 0.013715%	2.00032394 6^{th} iteration $\|E_R\|$ = 0.016197%	2.00061350 6^{th} iteration $\|E_R\|$ = 0.030675%

5 Conclusions

From above comparison table it is clear that among three new predictor-corrector formulae, second & third formulae give better accuracy and also can minimize the calculating time as it takes less number of iterations but the first one gives very poor result. It is yet to implement proposed formulae to the real world problems. Though new predictor-corrector formulae seem to be lengthy process of solving ordinary differential equations of first order and first degree, it has following advantages over previous methods. Such as; (i) the previous methods estimates the value of y respecting a given value of x by means of four initial conditions whereas the proposed predictor-corrector formulae estimate the value of y respecting a given value of x by means of five initial conditions, which is more logical, (ii) taking more decimal places in these formulae give better accuracy.

Thus it can be said that the second & third proposed formulae can be used to solve ordinary differential equations of first order and first degree with more accuracy and in shorter time than existing predictor-corrector formulae. Also, for less accuracy and much time consume the first one to be rejected obviously. Finally, by considering each of the formula for compare with exact value, it can conclude that the second new predictor-corrector formula is the best.

References

[1] RALSTON and P. RABINOWITZ,“A first course in numerical analysis”, McGraw-Hill Book Company, Auckland, 1988, PP. 193.
[2] R. VASISTHA, V. VASISTHA, “Numerical analysis”, Kedar Nath & Ram Nath, Meerut, 1999, PP.269, 271.
[3] S. GOEL and S. K. MITTAL, “Numerical analysis”, Pragati-Prakashan, India, 1995, PP.518
[4] V. N. VEDAMURTHY and N. Ch. S. N. IYENGAR, “Numerical methods”, Vikas Publishing house Private Ltd., New Delhi, 2002, PP.11.22
[5] JAMES B. SCARBOROUGH, “Numerical mathematical analysis” Oxford and IBM Publishing Co. Pvt. Ltd., 1966, PP.310.
[6] S. BALACHANDRA RAO and C. K. SHANTHA, “Numerical methods”, Universities Press India Ltd., Hyderabad, 2000, PP.386.

[7] S. S. SASTRY, 2002. "Introductory methods of numerical analysis", Third edition, Prentic-Hall of India Private Ltd., New Delhi, 2002, PP.303.

[8] WARD CHENEY, DAVID KINCAID, 1980. Numerical mathematics and computing (Books/Cole Publishing Company Monterey, California.). P.362