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Numerical Estimation of Unsaturated Soil Hydrodynamic Parameters

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ABSTRACT: In this work, a numerical estimation of the unsaturated physical soil hydrodynamic parameters is proposed. The mass transfer of water in the unsaturated porous media is described by the Richards model, which represents a combination of the Darcy law and the mass conservation equation. The control of the solution of the Richards equation is nonlinear and very sensitive to the knowledge of the Van Genuchten parameters {ks, θs, θr, n and α}.The purpose of this study is to estimate these parameters from measurements of water content obtained at different depths of soil in the M'nasra area (north-west of Morocco). The obtained results are compared with those of the literature that refer to the study zone. A good agreement is observed between the estimated and measurement data.

KEYWORDS: unsaturated porous media, Richards model, nonlinear, water content.

1 INTRODUCTION

The soil-moisture θ (h) and hydraulic conductivity K(h) curves are two basic hydraulic properties of soils. Current direct laboratory and in-situ methods for their determination are often costly and time consuming. Parameter optimization is an inverse approach that makes it possible to obtain K(h) and θ (h) simultaneously from transient flow data [Kool et al., 1987]. In this case, a water flow event is modeled with an appropriate governing equation and analytical expressions of K(h) and θ (h). The unknown parameters of K(h) and θ (h) are obtained by minimization of an objective function describing the differences between some measured flow variables and those simulated with a numerical flow Fortran 90 code using Levenberg- Marquardt algorithm.

Thus, for better treatment of the inverse problem requires an understanding of the direct problem, within this context, we based our study on measures of soil moisture conducted in the laboratory LIRNE, driven by A.Hmimo and al(2014). Also we compared the results obtained by the VZM code (Z.Saadi and al.1999) with measurement values. The obtained results are in good agreement with experiment

2 STUDY AREA

The study area is the experimental station located in the Mnasra area, 30 km north of the city of Kenitra, in northwestern Morocco Mnasra Region is a strip of land parallel to the Atlantic coast, of about 50,000 ha, located on the right bank of the Sebou River, 7 to 14 km wide and over about 50 km in length extending from the North of the Kenitra City. It is characterized by several agriculture activity. The experimental site is equipped with a weather station that provided various data, such as rainfall and minimum and maximum temperatures .

3 GOVERNING FLOW EQUATION

The governing flow equation for one-dimensional isothermal Darcian flow in a variably saturated rigid porous medium is given by the following modified form of the Richards equation:

$$C\frac{\partial h}{\partial t}=\frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z}-1\right)\right] \quad , \qquad C=\frac{\partial \theta}{\partial h} \qquad (1)$$

Where θ is the volumetric water content (L^3/L^3), h is the soil-water pressure head (L), K is the hydraulic conductivity (L/T), z is a vertical coordinate (L) positive upward, and t is time (T). For one dimensional vertical flow, subject to the following initial and boundary conditions:

$$h(z,0)=h_0(z)$$

at the top

$$\left[-k(h)\frac{\partial h(0,t)}{\partial z}-k(h)\right]_{z=0}=q_0(t) \qquad (2)$$

and at the lower boundary of the spatial domain:

$$\left[-k(h)\frac{\partial h(L,t)}{\partial z}-k(h)\right]_{z=L}=q_L(t) \qquad (3)$$

Where $q_0(t)$ and $q_L(t)$ are the imposed upper and lower fluid density fluxes, respectively.

4 HYDRODYNAMIC CHARACTERIZATION OF SOIL

The VZM algorithm (Vadose Zone Modelisation) [5] was used to define soil hydraulic properties. This specific algorithm determines characteristic hydraulic curves that take into account the Van Genuchten equation [3] for the water retention curve, θ(h), and hydraulic conductivity curve, K(h), (Van Genuchten, 1980):

$$\theta(h)=\theta_r+(\theta_s-\theta_r)\left[1+(|\alpha h|)^n\right]^{-m} \qquad (4)$$

$$K(h)=k_s S_e^l\left[1-(1-S_e^{\frac{1}{m}})^m\right]^2 \qquad (5)$$

Where S_e is the effective water content, k_s is the saturated hydraulic conductivity (L/T), θr and θs denote the residual and saturated water contents (L^3/L^3) respectively, l is a pore connectivity parameter, and α (L^{-1}), n, and m (m=1-1/n) are empirical parameters. The predictive K(h) model is based on the capillary model of Mualem (1976) in conjunction with Eq. (6). The pore-connectivity parameter l in the hydraulic conductivity function was estimated by Mualem (1976) to be 0.5 as an average for many soils. The hydraulic characteristics defined by Eq. (4) and (5) contain five unknown parameters: θ_r, θ_s, α, n, and k_s, they are determined by direct and inverse methods.

5 NUMERICAL METHOD

The finite difference method is used to solve the the Richard's equation. A forward scheme is used for the temporal term and centered scheme is used for the diffusion term [7]. Then the approached form the equation (1) is as follow:

$$C_i^j\frac{h_i^{j+1}-h_i^j}{\Delta t}=\left(\frac{2}{\Delta z_i+\Delta z_{i-1}}\left(K_{i+1/2}^j\left(\frac{h_{i+1}^{j+1}-h_i^j}{\Delta z_i}-1\right)-K_{i-1/2}^j\left(\frac{h_i^{j+1}-h_{i-1}^{j+1}}{\Delta z_{i-1}}-1\right)\right) \qquad (6)$$

Where the subscript i and the superscript j represent respectively grid location and time step [7]. The obtained tridiagonal system, presented below, is solved by using the TDMA algorithm (Remson et al. 1978).

$$A_i^j.h_{i-1}^{j+1} + B_i^j.h_{i-1}^{j+1} + D_i^j.h_{i-1}^{j+1} = E_i^j \quad (7)$$

$$A_i^j = \frac{2K_{i-1/2}^j}{\Delta z_{i-1}(\Delta z_{i-1} + \Delta z_i)} \quad (8)$$

$$B_{i,j} = -A_i^j - D_i^j - C_i^j / \Delta t \quad (9)$$

$$E_i^j = \Delta z_i.D_i^j - \Delta z_{i-1}.A_i^j - C_i^j.h_i^j / \Delta t \quad (10)$$

$$D_{i,j} = -\frac{2K_{i+1/2}^j}{\Delta z_i\left(\Delta z_{i-1} + \Delta z_i\right)} \quad (11)$$

6 PARAMETERS ESTIMATION

In the equations (1) to (5), the set of parameters p={θr, θs, n, α} are very influences on the soil-water pressure head h and a significant changes of the hydraulic conductivity K(h) and the capillary capacity C(h) with pressure head and water content θ(h), give it a very non-linear character. The inverse problem under analysis is formulated as a least square minimization problem involving the following objective functional [1,2] :

$$\Phi(p) = \sum_{i=1}^{N}\int_0^{t_f}\left[\sigma_\theta(\theta(p_i) - \theta_{meas,i})^2\right]dt \quad (12)$$

With the variances of the measured water content

$$\sigma_\theta = \frac{1}{(\theta_{meas,Max} - \theta_{meas,Min})^2} \quad (13)$$

Where:

$\theta_{meas,i}$ is the measured water contents at location z_i , $\theta(p_i)$ is the predicted water contents of the porous medium computed at the sensor location z_i with given set of parameters p={θ_r, θ_s, n, α}, N is the number of observations for water content.

To solve this inverse problem, the Levenberg–Marquardt method is used to minimize the unconstrained optimization problem (12) under the direct problem (1) to (5). The estimated vector of parameters is obtained by the iterative procedure:

$$p_{k+1} = p_k + \Delta p_k \quad (14)$$

The variation ΔPk is computed by solving the following linear system:

$$(J^T(p).J(p) + \lambda.I)\Delta p = -g(p) \quad (15)$$

Where: $J(p) = \frac{\partial\theta(p)}{\partial p}$ is the Jacobian matrix, g(p) is the gradient of the objective function $\Phi(p)$, I is the identity matrix and λ is the damping coefficient (a positive scalar) which controls both the magnitude and direction of Δp_k .

The estimated parameters p={θ_r, θ_s, n, α} are used to estimate the fifth parameter k_s by using linear regression method of functions K(h) and K(h)/ks .The relative hydraulic conductivity can also be expressed in terms of the pressure head by substituting Eq. (4) into Eq. (5):

$$K(h)/k_s = \frac{\left[(1-(\alpha h)^{n-1})(1+(\alpha h)^{-m}\right]^2}{(1+(\alpha h)^n)^{m/2}}, \quad m = 1 - \frac{1}{n} \quad (16)$$

From the Darcy flow, the hydraulic conductivity is defined as:

$$q = -k(h)(\frac{\partial h}{\partial z} - 1) \quad (17)$$

$$k(h) = -q / (\frac{\partial h}{\partial z} - 1) \quad (18)$$

As a result of curves of equations (16) and (18), k_s is obtained at each space point. And k_s final is calculated as follow:

$$k_S = \frac{1}{N}\sum_{i=1}^{N} k_{S,i} \quad (19)$$

7 Results and discussion

The data for the optimization problem were taken from the in-situ experiment conducted at the LIRNE laboratory, Université Ibn Tofail, Kénitra, Morroco, [4,5,6]. More than three month [8], the temporal and spatial distributions of soil water content and soil water pressure head were monitored during the course of the experiment. The results of the optimization by direct method are depicted in Figs. 2, 3 and 4 in which the optimized values of the parameters produce acceptable fit for the soil water content data.

The water content measured (in-sittu) and originally fitted parameters are compared in Figures 2 to 4. These figures show a high correlation between calculated and fitted parameters, with R^2 between 0.83 and 0.931. Hence, one can consider that Van Genuchten parameters can be estimated adequately using the direct model VZM code.

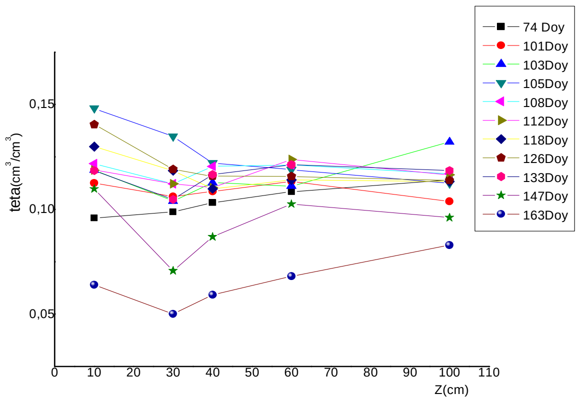

Figure (1): Spatial distributions of soil water content during the course of experiment.

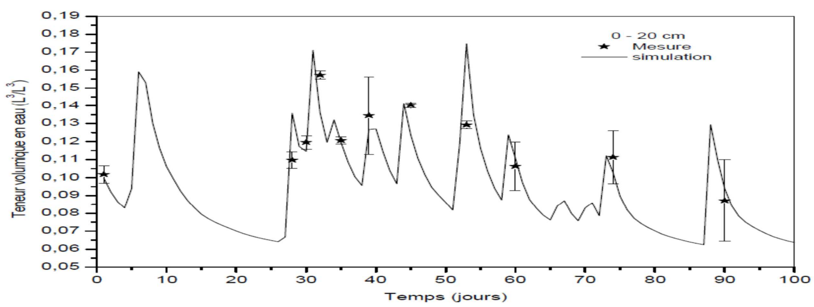

Figure (2): Time evolution of the water Content, depth (0-20cm)

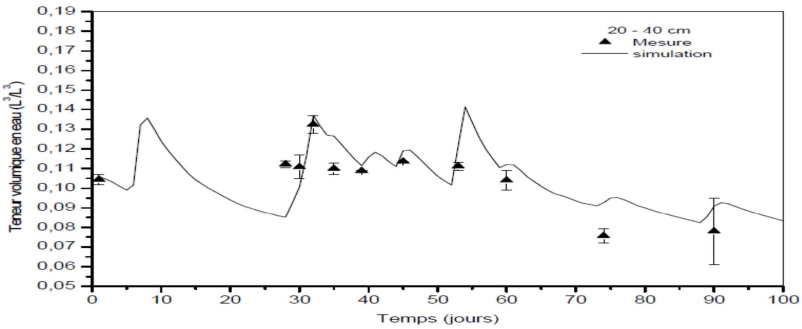

Figure (3): Time evolution of the water Content, depth (20-40cm)

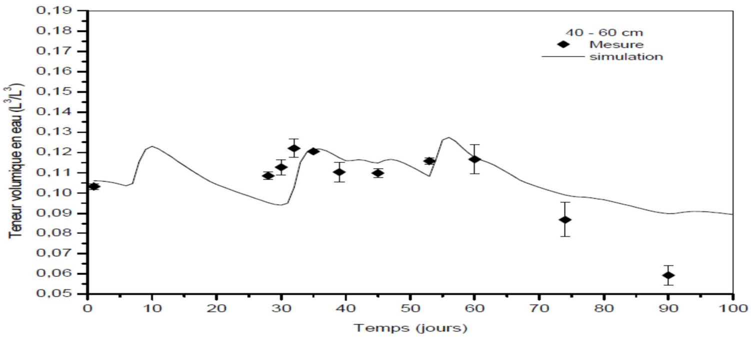

Figure 4. Time evolution of the water Content, depth (40-60cm)

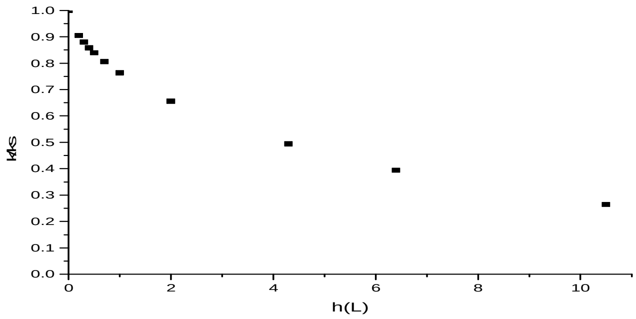

Figure 5. Calculated relative hydraulic conductivity (k/k_s)

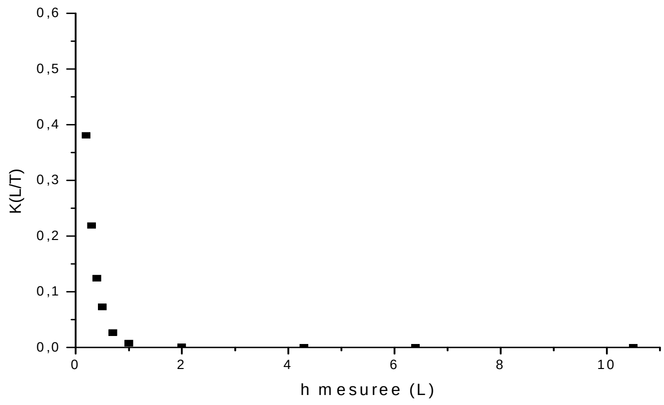

Figure 6. Measured hydraulic conductivity

Tab 1.Comparison between measured and estimated parameters.

	θ_r(cm³/ cm³)	θ_s(cm³/ cm³)	k_s(cm/s)	n	α (cm⁻¹)
Initial guesses	0	0.39	0.0065	2	0.1
measured values	0	0.430	0.0089	2.368	0.366
estimated values	0.013	0.428	0.0081	2.091	0.330

Table 1 presents fives of optimized parameter values obtained by the minimization of objective function using initial guesses for the hydraulic parameters. The optimized values θ_r, θ_s, α and k_s are almost stable with values of 0.013 cm^3/cm^3, 0.428 cm^3/cm^3 and 2.091 respectively. However, the optimized value of the parameter k_s is near to the measured value.

8 CONCLUSION

We have developed an efficient adaptive algorithm for estimating hydraulic parameters in one dimensional unsaturated soil water flow, by using direct method and inverse approach. The obtained results are compared with those measured in-situ and a good agreement is observed.

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