# Robustness of Feedforward Notch and Sallen-Key Compensators used with Second-Order Process

Galal A. Hassaan

Department of Mechanical Design & Production, Faculty of Engineering, Cairo University, Giza, Egypt

Copyright © 2014 ISSR Journals. This is an open access article distributed under the *Creative Commons Attribution License*, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

**ABSTRACT:** Robustness is one of the requirements used in controllers and compensators design. This paper examines the robustness of a Notch and a Sallen-Key compensator when used to control a highly oscillating second-order process. A variation of  $\pm$  20 % in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned compensators. With a feedforward notch compensator, the variation in process damping ratio has small effect on the settling time, maximum percentage overshoot, and phase margin of the control system, while the change in the process damping ratio has a clear effect on the control system performance. For a negative change in the process parameters, the control system is unstable.

With the Sallen-Key compensator, the control system is stable for the whole range of the process parameters variation ( $\pm$  20 %). The change in the process damping ratio has a minor effect on the control system settling time, maximum percentage overshoot and phase margin. The change in the process natural frequency has a minor effect on the control system settling time and maximum percentage overshoot. The phase margin changes in the range 40-47 degrees corresponding to the  $\pm$  20 % change in process natural frequency.

**Keywords:** Notch and Sallen-Key compensators, Compensators robustness, Variation in process parameters, Control system performance.

## 1 INTRODUCTION

Processes are subject to uncertainty in their parameters during operation. Therefore, it is worth to investigate the effectiveness of the used compensators with such process uncertainty. This will investigated for two types of forward compensators suggested by the author to control highly oscillating second-order processes.

Hu, Chang, Yeh and Kwatny (2000) used the H $\infty$  approximate I/O linearization formulation and  $\mu$ -synthesis to design a nonlinear controller for an aircraft longitudinal flight control problem and address tracking, regulation and robustness issues [1]. Gong and Yao (2001) generalized a neural network adaptive robust control design to synthesize performance oriented control laws for a class of nonlinear systems in semi-strict feedback forms through the incorporation of backstepping design techniques [2]. Lee and Na (2002) designed a robust controller for a nuclear power control system. They used the Kharitonov and edge theorem in the determination of the controller which was simpler than that obtained by the H $\infty$  [3]. Arvanitis, Syrkos, Stellas and Sigrimis (2003) analyzed PDF controllers designed and tuned to control integrator plus dead time processes in terms of robustness. They performed the robustness analysis in terms of structured parametric uncertainty description [4]. Lhommeau, Hardouin, Cottenceau and Laulin (2004) discussed the existence and the computation of a robust controller set for uncertain systems described by parametric models with unknown parameters assumed to vary between known bounds [5]. Dechanupaprittha, Hongesombut, Watanabe, Mitani and Ngammroo (2005) proposed the design of robust superconducting magnetic energy storage controller in a multimachine power system by using hybrid tabu search and

evolutionary programming. The objective function of the optimization problem considered the disturbance attenuation performance and robust stability index [6].

Chin, Lau, Low and Seet (2006) proposed a robust PID controller based on actuated dynamics and an unactuated dynamics shown to be global ....... bounded by the Sordalen lemma giving the necessary sufficient condition to guarantee the global asymptotic stability of the URV system [7]. Vagja and Tzes (2007) designed a robust PID controller coupled into a Feedforward compensator for set point regulation of an electrostatic micromechanical actuator. They tuned the PID controller using the LMI-approach for robustness against the switching nature of the linearized system dynamics [8]. Fiorentini and Bolender (2008) described the design of a nonlinear robust/adaptive controller for an air-breathing hypersonic vehicle model. They adapted a nonlinear sequential loop-closure approach to design a dynamic state-feedback control for stable tracking of velocity and altitude reference trajectories [9]. Labibi, Marquez and Chen (2009) presented a scheme to design decentralized robust PI controllers for uncertain LTI multi-variable systems. They obtained sufficient conditions for closed-loop stability of multi-variable systems and robust performance of the overall system [10]. Matusu, Vanekova, Porkop and Bakosova (2010) presented a possible approach to design simple PI robust controllers and demonstrate their applicability during control of a laboratory model with uncertain parameters through PLC [11].

Kada and Ghazzawi (2011) described the structures and design of a robust PID controller for higher order systems. They presented a design scheme combining deadbeat response, robust control and model reduction techniques to enhance the performance and robustness of the PID controller [12]. Surjan (2012) applied the genetic algorithm for the design of the structure specified optimal robust controllers. The parameters of the chosen controller were obtained by solving the nonlinear constrained optimization problem using IAE, ISE, ITAE and ITSE performance indices. He used constraints on the frequency domain performances with robust stability and disturbance rejection [13]. Jiao, Jin and Wang (2013) analyzed the robustness of a double PID controller for a missile system by changing the aerodynamic coefficients. They viewed the dynamic characteristics as a two-loop system and designed an adaptive PID control strategy for the pitch channel linear model of supersonic missile [14]. Hassaan (2014) published a series of papers aiming at studying the robustness of some controllers and compensators when used with difficult processes [15-17].

## 2 ANALYSIS

#### Process:

The process considered in this analysis has the transfer function,  $G_p(s)$ :

$$G_{p}(s) = \omega_{n}^{2} / (s^{2} + 2\zeta \omega_{n} s + \omega_{n}^{2})$$

Where the process parameters are carefully selected to represent one of the difficult industrial processes having high maximum percentage overshoot and large settling time. The process parameters are:

 $\omega_n$  = process natural frequency = 10 rad/s.  $\zeta$  = process damping ratio = 0.05

#### Feedforward Notch Compensator Tuning:

This compensator was tuned to control this difficult second order process by Hassaan [18]. The compensator has 3 parameters:

- The compensator gain: K.
- The compensator constants: a<sub>1</sub> and b<sub>1</sub>.

The compensator was tuned manually for a satisfactory performance of the closed-loop control system incorporating the compensator and the second order highly oscillating process. The tuning parameters and the control system performace measures are [18]:

$$\begin{split} & \mathsf{K} = 150 \\ & \mathsf{a}_1 = 200 \\ & \mathsf{b}_1 = 100 \\ & \mathsf{OS}_{max} = 0.549 \ \% \\ & \mathsf{T}_{\mathsf{s}} = 0.0287 \ \mathsf{s} \\ & \mathsf{GM} = \infty \ \mathsf{dB} \\ & \mathsf{PM} = 67.5 \ \mathsf{degrees} \end{split}$$

(1)

#### **Process Uncertainty:**

Due to the change in the operating conditions during operation, the process is subjected to parametric changes. It is assumed that this change can be as large as  $\pm$  20 % of the assigned process parameters.

#### Compensator Robustness:

The control system is robust when it has acceptable changes in its performance due to model changes or inaccuracy [19]. On the other hand Lee and Na added the stability requirement to the robustness definition besides the plants having uncertainty [3]. Toscano added that the controller has to be able to stabilize the control system for all the operating conditions [20].

In this work, the robustness of the controller and hence of the whole control system is assessed as follows:

- A nominal process parameters are identified.
- The compensator is tuned for those process parameters.
- A variation of the process parameters is assumed within a certain range.
- Using the same compensator parameters, the step response of the system using the new process parameters is drawn and the control system performance is evaluated through the maximum percentage overshoot and settling time.
- The frequency based relative stability parameters are also evaluated using the open-loop transfer function of the control system.
- The variation in process parameters is increased and the procedure is repeated.

Application of the above procedure results in the fact that with the feedforward Notch compensator almost all the performance parameters change with changing the process natural frequency.

- The control system is unstable for a change in the process natural frequency in the range:
- 20 % ≤ δω<sub>n</sub> ≤ -2.5 %
- The control system is stable for a change in the process natural frequency in the range:  $0\% \le \delta\omega_n \le 20\%$
- The control system is stable for a change in the process damping ratio in the range:
- 20 % ≤ δζ ≤ 20 %
- Fig.1 shows the variation of the settling time against the variation in the process parameters.



Fig.1 Effect of process parameters change on system settling time (Feedforward Notch compensator).

- 4.5 4 30 8 3.5 3 Unstable control 2 2.5 system OSmax 1.5 0.5 20 -15 -10 -5 Ū 5 10 15 20 δσ<sub>n</sub> : δζ (%)
- Fig.2 shows the variation of the maximum percentage overshoot against the variation in the process parameters.

Fig.2 Effect of process parameters change on system maximum overshoot (Feedforward Notch compensator).

Fig.3 shows the variation of the phase margin against the variation in the process parameters.



Fig.3 Effect of process parameters change on system phase margin (Feedforward Notch compensator).

## Feedforward Sallen-Key Compensator Controlling a Highly Oscillating Second Order Process:

Hassaan used a manual tuning approach to tune a feedforward Sallen-Key compensator when used with a highly oscillating second-order process [21].

The compensator parameters and the system performace measures are:

$$\begin{split} & K_c = 99 \\ & \omega_{nf} = 0.04 \\ & \zeta_f = 10 \\ & \text{Maximum percentage overshoot: 0 \%} \\ & \text{Settling time: 11.819 s} \\ & \text{Gain margin: 43.8 dB} \\ & \text{Phase margin: 76.9 degrees} \end{split}$$

The robustrness investigation procedure is applied on the resulting control system for process variation in the range  $\pm$  20 % from the nominal values. The results are as follows:

- The change in settling time , maximum percentage overshoot and phase margin with natural frequency variation is negligible.
- The minimum and maximum change in the gain margin with natural frequency change is -8.4 % and 6.8 % respectively.
- The change in damping ratio almost has no effect on all the performance parameters of the control system.
- Fig.4 shows the effect of the natural frequency change on the system settling time.



Fig.4 Effect of process parameters change of system settling time. (Sallen-Key compensator).

- Fig.5 shows the variation of the maximum percentage overshoot against the variation in the process parameters.



Fig.5 Effect of process parameters change on system maximum overshoot. (Sallen-Key compensator).

- Fig.6 shows the variation of the phase margin against the variation in the process parameters.



Fig.6 Effect of process parameters change on system phase margin (Sallen-Key compensator).

## 3 CONCLUSION

- Variation in second-order process parameters within ± 20 % was considered.
- The judgment on the robustness condition of a controller is based on an accepted range of both gain margin and phase margin of the closed-loop control system.
- According to Ogata [22], a recommended range is:  $GM \ge 6 dB$  and  $30 \le PM \le 60 degrees$ .
- According to Lei and Man [23], the phase margin range can be widened to be:  $30 \le PM \le 90$  degrees.
- The notch compensator sufferred from instability condition associated with negative changes in process natural frequency and damping ratio.
- The Sallen-Key compensator has a robust design since it generated a stable control system for the variation range of ± 20 % of process parameters.
- With Sallen-Key compensator, the variation in process parameters almost did not change the settling time, maximum overshoot and phase margin.
- The maximum change in gain margin with Sallen-Key compensator was only 8.4 %.
- The notch compensator is not robust when used with the highly oscillating second-order process.
- The Sallen-Key compensator is robust when used with the highly oscillating second-order process.

#### REFERENCES

- [1] S. Hu, B. Chang, H. Yeh and H. Kwatny, "Robust nonlinear controller design for a longitudinal flight control problem", *Asian Journal of Control*, vol.2, no.2, June, pp.111-121, 2000.
- [2] J. Gong and B. Yao, "Neural network adaptive robust control of nonlinear systems in semi-strict feedback form", *Automatica*, vol.37, pp.1149-1160, 2001.
- [3] Y. Lee and M. Na, "Robust controller design of nuclear power reactor by parametric method", *Journal of the Korean Nuclear Society*, vol.34, no.5, October , pp.436-444, 2002.
- [4] K. Arvanitis, G. Dyrkos, I. Stellas and N. Sigrimis, "Controller tuning for integrating processes with time delay Part II: Robustness analysis under structured parametric uncertainty", 11<sup>th</sup> IEEE Mediterranean Conference on Control and Automation, Rodos, Greece, June 28-30, Paper T07-41, 2003.
- [5] M. Lhommeau, L. Hardouin, B. Cottenceau and L. Laulin, "Interval analysis and dioid: application to robust controller design for timed event graphs", *Automatica*, vol.40, pp.1923-1930, 2004.
- [6] S. Dechanupaprittha, K. Hongesombut, M. Watanabe, Y. Mitani and I. Ngamroo, "Design of robust SMES controller in a multimachine power system by using hybrid TS/EP", 15<sup>th</sup> PSCC, Liege, 22-26 August, Session 28, Paper 3, 2005.
- [7] C. Chin, M. Lau, E. Low and G. Seet, "A robust controller design method and stability analysis of an underactuated underwater vehicle", *Inernational Journal of Applied Mathematics and Computer Science*, vol.16, no.3, pp.345-356, 2006.
- [8] M. Vagja and A. Tzes, "Robust PID control design for an electrostatic micromechanical actuator with structured uncertainty", *Proceedings of the 15<sup>th</sup> Mediterranean Conference on Control and Automation*, Athens, Greece, July 27-29, 2007.
- [9] L. Fiorentini and M. Bolender, "Robust nonlinear sequential loop closure control design for an air-breathing hypersonic vehicle model", American Control Conference, Western Seattle Hotel, Seattle, Washington, USA, June 11-13, pp.3458-3463, 2008.
- [10] B. Labibi, H. Marquez and T. Chen, "Decentralized robust PI controller design for an industrial boiler", *Journal of Process Control*, vol.19, pp.216-230, 2009.
- [11] R. Matusu, K. Vanekova, R. Prolop and M. Bakosova, "Design of robust PI controllers and thei application to nonlinear electronic systems", *Journal of Electrical Engineering*, vol.61, no.1, pp.44-51, 2010.
- [12] B. Kada and Y. Ghazzawi, "Robust PID controller design for an UAV flight control system", *Proceedings of the World Congress on Engineering and Computer Science*, San Francisco, USA, October 19-21, Vol.II, 2011.
- [13] R. Surjan, "Design of fixed structure optimal robust controller using genetic algorithm and particle swarm optimization", International Journal of Engineering and Advanced Technology, vol.2, no.1, October, pp.187-190, 2012.
- [14] G. Jiao, Y. Jin and S. Wang, "Research on the robustness of an adaptive PID control of a kind of supersonic missile", *Journal of Applied Sciences, Engineering and Technology*, vol.5, no.1, pp.37-41, 2013.
- [15] G. A. Hassaan, "Robustness of the feedback PD compensator used with second-order and third-order processes", *International Journal of Advanced Research in Computer Science and Technology*, vol.2, no.2, Version 5, 2014 (Accepted for publication).

- [16] G. A. Hassaan, "Robustness of PD and PID controllers used with second-order processes", *International Journal of Scientific and Technology Research*, vol.3, no.9, 2014 (Accepted for publication).
- [17] G. A. Hassaan, "Robustness of I-PD, PD-PI and PI-PD controllers used with second-order processes", *International Journal of Scientific and Technology Research*, vol.3, no.10, 2014 (Accepted for publication).
- [18] G. A. Hassaan, "A novel notch compensator used with a highly oscillating second-order process", *International Journal of Advanced Research in Computer Science and Technology*, vol.2, no.3, July-September, 2014 (Accepted for publication).
  [19] R. Dorf and R. Bishop, "Modern control systems", *Pearson Education Int.*, 2008.
- [20] R. Toscano, "A simple robust PI/PID controller design via numerical optimization approach", Journal of Process Control, vol.15, 2005.
- [21] G. A. Hassaan, "A novel Sallen-Key compensator used with a highly oscillating second-order process", *International Journal of Engineering Sciences and Research Technology*, vol.3, no.8, July-September, 2014 (Accepted for publication).
- [22] K. Ogata, "Modern control engineering", Prentice Hall Int., 2009.
- [23] W. Lei and T. Man, "Advanced approach for optimizing dynamic response for buck converter", *Semiconductor Components Industries*, January 2011.

## BIOGRAPHY



## <u>Galal Ali Hassaan:</u>

- Emeritus professor of System Dynamics and Automatic Control, Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations and Mechanism Synthesis.
- Published 10's of research papers.
- Author of a book on Experimental Systems Control.