

## BER Performance of SFBC - OFDM System with Frequency Domain Equalization for the Nakagami MIMO Fading Channel

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**ABSTRACT:** The extension of orthogonal frequency division multiplexing (OFDM) with Multiple input multiple output (MIMO) in Wireless communication is nostrum to fading. This brilliant combination has potential to expedient high data rate requirements and high performance over various impugning channels i.e. Time selective and Frequency selective. This paper presents the Nakagami Multiple input multiple output (MIMO) fading channel simulation and the comparison of BER performance of the frequency domain equalizers i.e. Decision feedback and maximum likelihood detection (ML). The result is provided for two transmitter and two receiver, 16 QAM modulation. The simulation result shows that the high diversity gain stability can be acquired by using the domain equalizers for SFBC-OFDM in Nakagami frequency selective fading channel. The implementation of domain equalizers for SFBC-OFDM system for the arbitrary values of  $m$  minimizes the intersymbol interference (ISI). ML is the best coding for minimizing the error probability.

**KEYWORDS:** Nakagami- $m$  fading channel, SFBC-OFDM system, Frequency domain equalization, QAM.

### 1 INTRODUCTION

In the propagation environment of digital mobile communication, the radio signal experiences reflection, diffraction and scattering between the transmitter and the receiver, collectively referred to as multipath fading. Therefore, design of persuasive equalization and channel estimation algorithms for such channels becomes a basic obstacle. The sequel of an equalization system is to atone for transmission channel impairments such as frequency dependent phase and amplitude distortion. Besides amending for channel frequency response deviations, the equalizer can efface the effects of multipath signal components. Several probability density functions (P.d.f.) are proposed to model the fading random variable [1]. The Rayleigh fading models are basically utilized in simulating high frequency signal propagation. But the Rayleigh fading is not enough for the larger distance with adequate precise results. Nakagami was the first to scrutinize this and he gave a parametric gamma distribution based density function [2]. Nakagami- $m$  distribution can be used for arbitrary value of ' $m$ ' [1]. To meet the plea of high data rates and the transmission reliability, Multiple input multiple output (MIMO) is the solution which can mollify the multipath fading, resulting in the augmentation of the channel performance, increases transmission reliability and high data rates achievement [6]. In the previous work, the competent combination of space time code and the OFDM system is the SFBC-OFDM. Performance analysis of SFBC-OFDM system with frequency domain equalization for Rayleigh fading MIMO channel is evaluated [7]. A way to do space frequency coding is to take the space time codes and implement them in frequency domain rather than time domain[6]-[7].The integration of MIMO and OFDM can greatly improve the system performance over flat fading channel with the reasonable complexity [13]. Use of space-time block codes in the frequency domain is attractive method because of their low complexity decoding and is suitable for fast fading [7]. The Nakagami- $m$  fading channel simulation method which is based on the implementation of the square-root beta process and complex Gaussian noise process[11].This method is restricted to the value of  $0.5 \leq m \leq 1$ . Another method which is based on semi-empirical method, partially simulation based. This simulation method requires determining the values of certain

coefficients from measured data [12]. The efficient method for Nakagami- $m$  fading channel simulation is given in [1] which is applicable for arbitrary values of  $m$ .

In this paper we have evaluated the two domain equalizers for same Nakagami MIMO fading channel. The BER performances of both the domain equalizers for Nakagami fading channel are taken into the consideration. This paper is organized as follows; Section 2 presents the description of Nakagami -  $m$  fading channel model section 3 gives the details of SFBC -OFDM system. Section 4 has the description of domain equalization. Numerical results and discussion are provided in Section 5. Following section 6, this provides the conclusion.

## 2 NAKAGAMI - $m$ FADING MODEL

The Nakagami fading distributing can model signal fading condition that ranger from severe to moderate, to light or no fading [1].The Probability density function of Nakagami -  $m$  fading model is

$$F_R(r) = \frac{2m^m r^{2m-1}}{\gamma(m)\Omega^m} e^{-(\frac{m}{\Omega})r^2} \tag{1}$$

Where  $\Omega$  is the scale parameter and is given as

$$\Omega = E[R^2] = \bar{R}^2 \tag{2}$$

Where  $E[.]$  is the expectation operator. And

$$m = \frac{(R^2)^2}{(R^2 - \bar{R}^2)^2} \geq \frac{1}{2} \tag{3}$$

Where  $m$  is the shape parameter that controls the severity, or depth of the amplitude fading [1]. The value  $m = 1$  results in Rayleigh fading model and the value of  $m$  less than one given fading more severe than Rayleigh fading and values of ' $m$ ' greater than one gives fading less severe than Rayleigh fading. [1] - [2].The cdf of the Rayleigh fading is given as

$$Y = 1 - e^{-r^2/2\sigma^2} \tag{4}$$

Where  $\sigma_2$  is the second movement of the random variable 'R' that transfer the random variable RV R into uniform RV on [01], [1] [2][4]. Nakagami fading signal can be obtained from the cdf transform that maps the Raleigh fading into Nakagami fading as illustrated in Fig.1 [1].

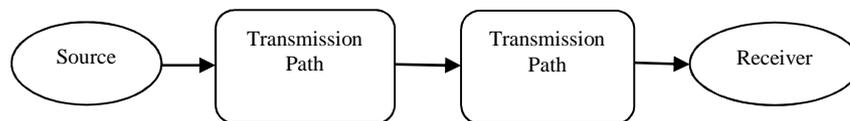


Fig. 1. The cdf corresponds to Nakagami and Rayleigh Fading.

For the integer and half integer values of  $m$  the pdf is that of the amplitude or sum of squared independent Gaussian RVs appropriately normalized as  $R = \sqrt{X_1^2 + X_2^2 + \dots + X_n^2}$  where  $X_i = 1 \dots n$  are the i.i.d Gaussian Random variable with zero-mean and variance  $\sigma_X^2$  and  $n = 2m \times \sigma_X^2$  [1].

## 3 SFBC OFDM SYSTEM MODEL

We consider the SFBC OFDM system with  $N_T$  transmit antenna, and  $N_R$  receive antenna. Let OFDM with  $N_C$  subcarriers.  $N_S$  are the no. of sub bands chosen and  $N_S = N/q$  all the subbands are modulated using BPSK modulation. Thus, a signal vector is given as  $S = \{S[0], S[1] \dots S[R_t - 1]\}$  which is provided at the SFBC encoder.

Where

$$R_t = N_c \times R_c \tag{5}$$

Where  $R_c$  is then code rate .Now, this 'S' signal is transmitted in parallel on  $N_c$  sub carriers by  $N_T$  transmit antenna to SFBC encoder. Now, SFBC encoder provides  $N_T$  blocks, i.e.  $S_1, S_2 \dots S_{N_T}$  each of length ' $N_c$ ' after the IFFT the Blocks generated as follows  $X_1, X_2 \dots X_{N_T}$ .

The fading impulse response of *ith* transmit and *jth* receiving antenna is given as

$$H_{j,i}(t) = \sum_{m=1}^{L-1} \alpha_{m,j,i}(t) \tau(t - \tau_m(t)) \tag{6}$$

Where  $\alpha_{m,j,i}(t)$  is the tap weight,  $\tau_m(t)$  is the time delay of the  $m_{th}$  path and 'L' is the resolvable multipath component.

After the cyclic prefix removal and FFT at the receiver we have demodulated received signals at the *jth* receive antenna which is given as

$$r_j = \sum_{i=1}^{N_t} H_{j,i} * S_i + w_j \tag{7}$$

Where

$$R_j = [r_j(0) \dots r_j(N_c - 1)]^T$$

$$S_i = [S_i(0), S_i(1) \dots S_i(N_c - 1)]^T$$

is the transmitted signal at the *i*-th antenna. And the matrix

$$W_j = [w_j(0) \dots w_j(N_c - 1)]^T$$

Gives the AWGN.  $H_{ij}$  is an  $N \times N$  diagonal matrix with elements corresponding to the DFT of the channel response between the *i<sub>th</sub>* and *j<sub>th</sub>* receive antennas.

A STBC is represented by a matrix. Each row and column represents time slot and one antenna's transmission over time. The STBC is given as

$$G_{stbc} = \begin{bmatrix} g_{1,1} & g_{1,2} & \dots & -g_{1,NT} \\ g_{2,1} & g_{2,2} & \dots & -g_{2,NT} \\ \dots & \dots & \dots & \dots \\ g_{q,1} & g_{q,2} & \dots & -g_{q,NT} \end{bmatrix} \tag{8}$$

Where  $G_{ij}$  is the modulated symbol to be transmitted in time slot *i* from antenna *j* and *q* is the symbol period.

#### 4 EQUALIZATION

In MIMO system, Inter Symbol Interference (ISI) occurs due to Multipath followed by the signal, thus it is difficult to maintain the orthogonality of the transmitted signal, resulting in bit error at the receiver. Equalization is a technique used to minimize the error between actual output and throughput by continuous updating the filter's coefficients. Equalization can be implemented in both time and frequency domain. In this paper we have used the Decision Feedback equalization and the ML detection equalization for the SFBC OFDM systems.

##### 4.1 DECISION FEEDBACK EQUALIZATION (DFE)

This is the nonlinear adaptive equalizer. The basic principle of DFE is that once the value of current transmitted symbol is determined, the value of ISI caused by that symbol can be removed for received future symbols. The nonlinear effect is due to the decision device which endeavours to determine which symbol of a set of discrete levels was basically transmitted. Once the current symbol has been decided, the filter structure can calculate the ISI effect it would tend to have on subsequent received symbols and reimburse the input to the decision device for the following samples. The DFE functional block diagram is given in Fig. 2.

The Feedback filter is driven by decision on the output of the detector, and its coefficients are adjusted such as to cancel the ISI component on the current symbol from past detected symbols. RLS (recursive least squares) algorithm is used for determining the coefficient of an adaptive filter [16]. RLS (recursive least squares) algorithm is used for determining the coefficient of an adaptive filter [16]. RLS algorithm uses information from all past input samples to estimate the autocorrelation matrix of the input vector. To minimize the influence of transmitted input samples, a tap weight for the influence of each sample is used. First process is the filtering in which RLS computes the output of a linear filter in response to an input signal and generates an estimation error. Secondly, the adjustment of parameters of the filter in congruity with the estimation error.

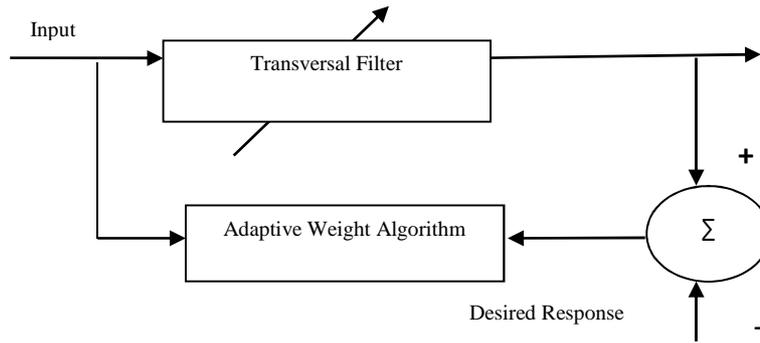


Fig. 2. Functional Block Diagram of DEF

The filter algorithm is given [3] as

$$r(n) = wH(n)C(n) \tag{9}$$

Where  $w$  is the weight vector and  $C(n)$  are the feedback tap coefficients. The error estimation is given as

$$e(n) = D(n) - r(n) \tag{10}$$

Where  $D(n)$  is the desired response and is given as

$$D(n) = w r(n) \tag{11}$$

The tap –weight vector update is given as

$$w(nr + 1) = w(nr) + \mu C(n)e^*(n) \tag{12}$$

Where  $nr$  is the number of iteration and  $\mu$  is the step size.

#### 4.2 MAXIMUM LIKELIHOOD (ML) DETECTION

ML equalization technique is used for measuring the Euclidean distance between all the received signal vector and finding the lowest value from the entire transmitted signal vector in channel  $H$ . Maximum posteriority detection is used for maximum optimal detection. It requires knowledge of channel characteristics in order to compute the metrics for making decisions. It also requires the knowledge of statistical distribution of the noise, which determines the form of metric for optimum demodulation of the received signal. Assuming that perfect CSI is available, the receiver chooses  $S = (s_1, s_2, \dots, s_N)$  from the transmission constellation  $C$  that minimize the following decision metric is given as [3]

$$\|r - Hs\|^2 = \sum_{m=1}^{N_r} \sum_{t=1}^L d^2(r_{t,m} \sum_{n=1}^{N_t} h_{n,m} S_{n,t}) \tag{13}$$

Finally, under the channel state information (CSI) at the receiver, Maximum likelihood detection can be used for the SEFBC decoding of the received signal given as

$$S[k] = \sum_{j=1}^{N_r} (h_{j,i}^*[2n]r_j[2n] + [2n]r_j^*[2n + 1]) \tag{14}$$

$$S[k + 1] = \sum_{j=1}^{N_r} (h_{j,i}^*[2n + 1]r_j[2n] - h_{j,1}[2n + 1]r_j^*[2n + 1]) \tag{15}$$

The minimization of equation (12), (13) results in ML decoding which can be written as [3]

$$\hat{s} = \arg \min \|r - Hs\|^2 \tag{16}$$

Where  $\|A\|$  denotes the Euclidean norm of matrix  $A$  defined by  $\|A\|^2 = tr(A^H A)$ ,  $tr(A)$  and  $A^H$  denotes the trace and Hermitian transpose of matrix  $A$ , and  $d^2(a, b)$  is the squared Euclidean distance between the signals and the  $b$  is calculated as

$$d^2(a, b) = |a - b|^2 = (a - b)(a^* - b^*) \tag{17}$$

This is the best coding for minimizing error probability. The complexity involved is also large and increases exponentially with increase in the number of antennas. The job of the equalizer is to implement an inverse filter to counter its ill effect.

5 NUMERICAL RESULTS AND DISCUSSION

We have evaluated the BER performances of SFBC OFDM for frequency selective Nakagami MIMO fading channel for  $m = 2, m = 4$  by varying SNR. In the performance results, for SFBC we have used 312.5 KHz the subcarrier spacing, FFT sampling frequency is 20 MHz,  $N_c = 52$  subcarriers and a cyclic prefix longer than delay spread. The number of multipath followed is 10. Here we have used  $N_t = 2$  and  $N_r = 2$ . The simulation is carried for  $10^6$  number of bits using 16 QAM modulation. Assuming that the channel information is perfectly known to receiver. The behaviour of diversity is studied for the value of  $m=2$  and  $m=4$  in Fig.3 and Fig.4.

In Fig 1, we have plotted the BER performance of the SFBC-OFDM system for two transmitters and two receiver for  $m=2$ . The BER is 0.0004 for ML detection and 0.0015 for DFE for SNR= 10dB. The ISI caused due to Nakagami frequency selective fading channel is less as the BER is 0.0134 when no diversity is applied. As the Nakagami distribution is the one that gives accurate results for high frequencies, also for less and more severe than Rayleigh fading provided in [1]-[2]. The Nakagami MIMO fading channel is complex to simulate than Rayleigh MIMO fading channel, but gives better BER results.

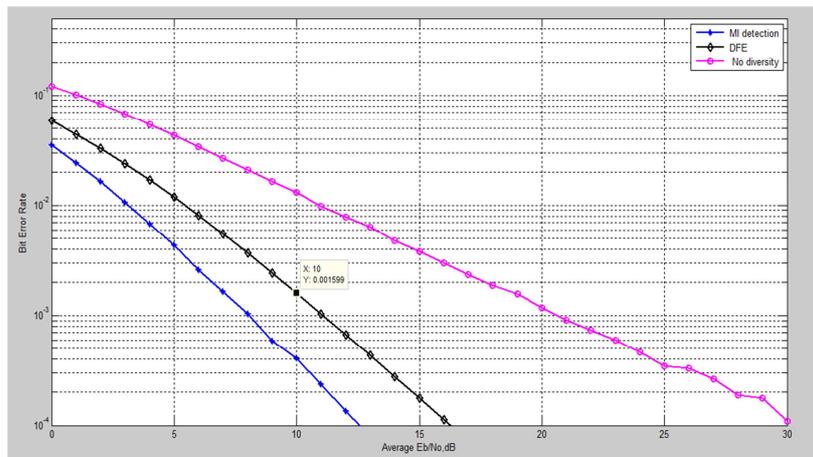


Fig. 3. BER of SFBC-OFDM system using 2Tx and 2Rx for Nakagami MIMO fading channel for  $m=2$ , 16QAM with frequency domain equalization

The plot for  $m=4$  for two transmitter and two receiver is given in fig. 4. As it is shown from the graph that BER is 0.00023 for ML detection and 0.00029 for DFE for SNR= 10dB. And BER when no diversity is applied is 0.009. thus we get refined results when the equalization is applied for SFBC- OFDM system. Thus ISI is greatly reduced for the Nakagami Fading channel for  $m=4$ .

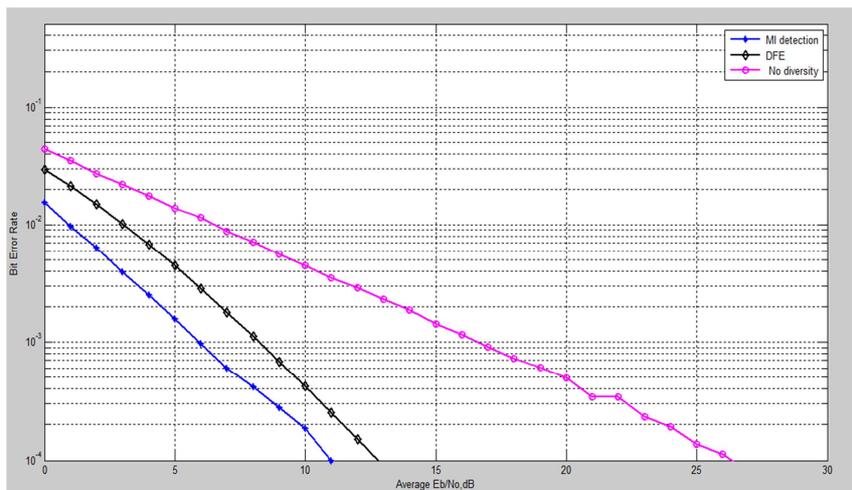


Fig. 4. BER of SFBC-OFDM system using 2Tx and 2Rx for Nakagami MIMO fading channel for  $m=4$ , 16QAM with frequency domain equalization

## 6 CONCLUSION

The SFBC – OFDM diversity scheme is examined for simulated Nakagami MIMO fading channel for the value  $m=2$  and  $m=4$ . Further, the BER performance of SFBC OFDM with the DFE and ML detection equalization is considered. It is evaluated from the result that ML detection performs better but the complexity increases with increase in the numbers of antennas for transmitter and receiver. The results obtained are best for increasing value of  $m$ , as it can be seen that BER decreases with the increasing value of  $m$ . System performance can further be increased by using full diversity and extending the code in three dimensions, i.e. time, space and frequency. Results can be found for the M-ary Modulation.

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