International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 13 No. 4 Dec. 2015, pp. 929-934

http://www.ijias.issr-journals.org/

On Fuzzy Semiprime Submodules

Mohammed M. Ali Radman Al-Shamiri

Department of Mathematics and Computer, Faculty of Science, Ibb University, Ibb, Yemen

ABSTRACT: In this paper our aim is to extend some notions of ordinary semiprime submodules into fuzzy semiprime. Also we introduce and study new properties of fuzzy semiprime submodules. Several results on fuzzy semiprime submodules are proved.

KEYWORDS: Fuzzy module, fuzzy ring, fuzzy submodule, fuzzy semiprime submodule, fuzzy semiprime ideal.

1 INTRODUCTION

Throughout this paper by R we mean a commutative ring with identity and M is an R −module.A crisp set is defined by dichotomize the individuals into two types - members and non members.Anyone can observes that there exists distinction between the members and non members of the class in the crisp set. In our live the more situations are very often not crisp and deterministic and they cannot be described precisely. And hence, we, do not exhibit this characteristic, where their boundaries seem vague, and the transition from member to nonmember appears gradual not abrupt. For that, some mathematicians introduced notion of fuzzy set which introduces vagueness by remove the sharp boundary dividing members of the class from non members. Such situations are characterized by imprecision cannot be answered about that by yes or no justly.From the among mathematicians, where he is the first, Lotfi A. Zadeh [1] in 1965 introduced the notion of a fuzzy set to treatment the vagueness mathematically by easy abstractness and he gave a certain grade of membership to each member of a given set. This in fact laid the basic of fuzzy set theory. Zadeh has defined a fuzzy set as a characteristic function of a set wherein the degree of membership of an element is more than merely "yes" or "no". It can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree which is compatible with the concept represented by the fuzzy set. The membership grades are represented by real number values in the closed interval between 0 and 1. The notion of fuzzy set was applied in algebra as one of the first branches from among various branches of pure mathematics . The first paper was published by A. Rosenfeld[2] in 1971, in which the concepts of fuzzy subgroupoid and fuzzy subgroups were introduced. In 1979 J.M. Anthony and H. Sherwood[3] are studied in new notion fuzzy subgroup and some results of Rosenfeld.W. J. Liu[4] introduced the notions of fuzzy normal subgroup, fuzzy subring and the product of fuzzy sets. Liu[5] introduced the notion of a fuzzy ideal of a ring. N. Kuroki[6] demonstrated the utility of the notion of the fuzzy set in the more general setting of semigroups. S. Nanda[7] introduced concepts of fuzzy fields and fuzzy linear spaces. In fuzzifying of concepts of abstract algebra many researchers have been coproduced In its notions as J. N. Mordeson, D. S. Malik, M. M. Zahedi, M. Das, M. K Chakraborty, B. B. Makamba, V. Murali, A. K. Katsaras, D. B. Liu, M. Asaad, P.S. Das,N. P. Mukherjee, P. Bhattacharya, F. I. Sidky, M. A. Mishref, and M. Akhul, T. K. Mukherjee, M. K. Sen, V. N. Dixit, N. Ajmal, R. Kumar And contributed a lot of them to the theory of fuzzy algebraic structures. The concept of fuzzy modules and L-modules were introduced by Negoita and Ralescu [8] and Mashinchi and Zahedi [9] respectively. Subsequently they were further studied by Golan[10] ,Muganda [11], Pan [12-13-14-15], Zahedi and Ameri[16-17-18-19].Until now there is many papers are published in the field of fuzzy set ,we in this paper shall we contribute by introduce some notions in fuzzy semi prime and some related concepts, continuously on my paper " prime fuzzy submodule and primary fuzzy submodule" which was published by IJCST ,6-2, 2015.

Definition 1.1[1] Let M be a non-empty set and let I be the closed interval [0, 1]. A fuzzy set μ

inM (a fuzzy subset μ of M) is a function from M to I.

Definition 1.2[1]A fuzzy set μ of a set M is called constant if $\mu(x) = t$ for all $x \in M$, where $t \in [0,1]$.

Definition 1.3[1] Let μ be a fuzzy set in M. μ is called an empty set denoted by ∅ if and only if $\mu(x) = 0$ for all $x \in M$.

Definition 1.4[20] Let $x_t : M \longrightarrow [0,1]$ be a fuzzy set in M, where $x \in M$, $t \in [0,1]$, defined by:

$$x_t(y) = \begin{cases} t & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$$

For all $y \in M$, x_t is called a fuzzy singleton or fuzzy point in M. If $x = 0$ and $t = 1$, then:

$$0_1(y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{if } y \neq 0 \end{cases}$$

$x_t \subseteq \mu$ if and only if $x_t(y) \leq \mu(y)$, for all $y \in M$ and if $t > 0$, then $\mu(x) \geq t$. If x_t and y_s fuzzy singletons, then $x_t + y_s = (x+y)_l$ and $x_t \circ y_s = (x.y)_l$,where$l = \min\{t, s\}$

Definition1.5[2] Let $(G,\cdot)$be a group and μ be a fuzzy set in G then μ is called a fuzzy group in G if for each $x, y \in G$:

$\mu(x \cdot y) \geq \min\{\mu(x), \mu(y)\}$.

$\mu(x) = \mu(x^{-1})$.

A fuzzy subgroup of a fuzzy group μ is a fuzzy group λ satisfing $\lambda(x) \leq \mu(x) \quad \forall x \in G$.

Definition1.6[21]Let $(R, +, \cdot)$ be a ring and μ be a fuzzy set in R,then μ is called fuzzy ring in a ring $(R, +, \cdot)$ if for each $x, y \in R$:

$\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$.

$\mu(x) = \mu(x^{-1})$.

$\mu(x \cdot y) \geq \min\{\mu(x), \mu(y)\}$.

A fuzzy subring of a fuzzy ring μ is a fuzzy ring λ satisfing $\lambda(x) \leq \mu(x) \quad \forall x \in R$.

Definition 1.7[20],[22]A fuzzy set μ in a ring R is called a fuzzy left(right) ideal of the ring R if for each $x, y \in R$:

$\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$.

$\mu(xy) \geq \mu(y) \ \ [\mu(xy) \geq \mu(x)]$.

In [23] when R is a commutative, Bhambri.S.K.,Kumar R.and Kumar P introduced the definition:

Definition 1.8[23] A fuzzy set μ in a commutative ring R is called a fuzzy ideal of the ring R if for each $x, y \in R$:

$\mu(x - y) \geq \min\ \{\mu(x), \mu(y)\}$.

$\mu(xy) \geq \max\ \{\mu(x)\ ,\ \mu(y)\}$.

Definition 1.9[24-25]Let R be a ring and let M be a left R −module.A fuzzy set μ in M is called a fuzzy left R −module if for each $x, y \in M$ and $r \in R$:

$\mu(x - y) \geq \min\ \{\mu(x), \mu(y)\}$.

$\mu(rx) \geq \mu(x)$.

$\mu(0) = 1$ (0 the zero element of M).

Definition 1.10[26-27-28] Let μ, λ be two fuzzy subsets of M,and let r be any element of R,define $\mu + \lambda$, $r\mu$ of M as follows: for each $x \in M$.

$(\mu + \lambda)(x) = \sup\{\min\{\mu(a) + \lambda(b)\} : x = a + b\}$for each $a, b \in M$.

$$(r\mu)(x) = \begin{cases} \sup\{\mu(a) : x = ra\ , a \in M\} \\ 0 \qquad \text{otherwise} \end{cases}$$

Definition 1.11[29]Let μ be a fuzzy subsets of a ring R, and λ be a fuzzy subset of an R − module M. For every $x \in M$

define $(\mu\lambda)(x) = \sup\{\inf\{\mu(r_1), \mu(r_2), .. \mu(r_k), \lambda(x_1), \lambda(x_2), .., \lambda(x_k)\}\}$ where $r_i \in R$ and $x_i \in M$.

Proposition 1.1[29] Let μ and λbe fuzzy ideals of a ring R and let ρ and θ be fuzzy submodules of a fuzzy module A of an $\mathrm{R}-$ module M.Then the followings are satisfied.

$\mu+\lambda$ is a fuzzy ideal of R ,

$\rho+\theta$ is a fuzzy submodule of M,

$\mu\lambda$ is a fuzzy ideal of R ,

$\mu\rho$ is a fuzzy submodule of M,

$(\mu+\lambda)\rho=\mu\rho+\lambda\rho$,

$(\mu\lambda)\rho=\mu(\lambda\rho)$.

Definition 1.12[1] Let μ and λbe fuzzy submodules of A fuzzy module A of an $\mathrm{R}-$ module M. The residual of μ and λ denoted by $[\mu:\lambda]$ is the fuzzy subset of R defined by:

$[\mu:\lambda]=\sup\{t\in[0,1]:r_t\lambda\subseteq\mu\}$for all $r\in R$ that is:

$[\mu:\lambda]=\{r_t:r_t\lambda\subseteq\mu\}$, $r_t$is a fuzzy singleton of R.If $\lambda=(x_k)$, then;

$[\mu:(x_k)]=\{r_t:r_tx_k\subseteq\mu\}$, $r_t$is a fuzzy singleton of R.

Here we mention the equivalent definition and it is very clear to showing the equivalence between the two definitions.

Definition 1.13[30] For $\mu,\upsilon\in I^M$ and $\alpha\in I^R$ define the residual quotient $[\mu:\upsilon]\in I^R$ and $[\mu:\alpha]\in I^M$ as follows :

$[\mu:\upsilon]=\cup\{\eta:\eta\in I^R,\eta.\upsilon\subseteq\mu\}$

$[\mu:\alpha]=\cup\{\sigma:\sigma\in I^M,\alpha.\sigma\subseteq\mu\}$

Theorem 1.1[30] Let $\mu,\upsilon\in I^M$ and $\alpha\in I^R$ .Then

$[\mu:\upsilon].\upsilon\subseteq\mu$.

$\alpha.[\mu:\alpha]\subseteq\mu$.

$\alpha.\upsilon\subseteq\mu\Leftrightarrow\alpha\subseteq[\mu:\upsilon]\Leftrightarrow\upsilon\subseteq[\mu:\alpha]$

Proposition 1.2[1] Let μ and λ be fuzzy submodules of a fuzzy module A of an $\mathrm{R}-$ module M, then $[\mu:\lambda]$ is a fuzzy ideal of R.

Definition 1.14[31]: Let A be a fuzzy module of an $\mathrm{R}-$ module M and μ be a proper fuzzy submodule of A.Then μ is called a prime fuzzy submodule of a fuzzy module A if $r_tx_k\subseteq\mu$ for fuzzy singleton r_t of R and $x_k\subseteq A$, then either $r_t\subseteq[\mu:A]$ or $x_t\subseteq\mu$.

Definition 1.15[9] A fuzzy submodule μ of a fuzzy module A is called a primary fuzzy submodule if for each $r_tx_k\subseteq\mu$, then either $x_k\subseteq\mu$ or $r_t^n\in[\mu:A]$for some $n\in Z^+$ where r_t is a fuzzy singleton of R and $x_k\subseteq A$.

Proposition 1.3[31] If R is an $\mathrm{R}-$ module and A is a fuzzy module of an $\mathrm{R}-$ module R.Then I is a prime fuzzy ideal in R if and only if I is a prime fuzzy submodule of a fuzzy module A of an $\mathrm{R}-$ module R.

Proposition 1.4[31] If μ is a prime fuzzy submodule of a fuzzy module A of an $R-$ module M,then$[\mu:A]$ is a prime.

Definition 1.16 [32] If μ is a non constant fuzzy ideal in R .Then μ is fuzzy semiprime ideal of R if for any fuzzy ideal θ of R , $\theta\circ\theta\subseteq\mu$ implies that $\theta\subseteq\mu$.

Definition 1.17 [33] A fuzzy ideal of a commutative ring R is called fuzzy semiprime if and only of $\mu(x^2)=\mu(x)\ \forall\ x\in R$

Definition 1.18[34] let μ be a proper fuzzy submodule of a fuzzy module A of an $R-$module M, than μ is a fuzzy semiprime of A if and only if $\forall\ r\in R$ and $\forall x_l\subseteq A$ such that $r_t^2x_l\subseteq\mu$ then $r_tx_l\subseteq\mu,\ \ t,l\in[0,1]$.

Definition 1.19[35] An $\mathrm{R}-$module M is called fuzzy multiplication module if and only if for every fuzzy submodule μ of M, there exists a fuzzy ideal σ of R such that $\mu=\sigma 1_M$ where $1_m(a)=1$ for every $a\in M$.

We introduced the definition of fuzzy multiplication module as general case as follows:

Definition 1.20 A fuzzy module A of an $\mathrm{R}-$module M is called a fuzzy multiplication module if and only if for every fuzzy submodule μ of A, there exists a fuzzy ideal σ of R such that $\mu=\sigma A$.

2 MAIN RESULTS

Theorem 2.1: Every fuzzy prime submodule is a fuzzy semiprime submodule of a fuzzy module A of an R −module M.

Proof: Let μ be a fuzzy prime submodule and let $r \in R, x_l \subset A$ such that $r_t^2 x_t \subseteq \mu$. Sinceμ is fuzzy prime submodule than $r_t x_l \subseteq \mu$ or $r_t^2 \subseteq [\mu : A]$. If$x_l \subseteq \mu$ than $r_t x_l \subseteq \mu$ and if $r_t^2 \subseteq [\mu: A]$ we have got $r_t \subseteq [\mu: A]$ because $[\mu: A]$ is a prime ideal (by proposition 1.5), this means $r_t A \subseteq \mu$ and this implies that $r_t x_l \subseteq \mu$.

The converse is not true in general, for example:

Let $z = R$ and let $A: M \longrightarrow [0,1]$ defined by $A(x) = \begin{cases} 1 & if\ \ x \in 12z \\ \frac{1}{2} & if\ \ x\ \in 6z - 12z \\ \frac{1}{3} & otherwise \end{cases}$ A is a fuzzy module of an z-module z.

And define $\mu: M \longrightarrow [0,1]$ by

$\mu(x) = \begin{cases} 1 & if\ \ x\ \in\ \{0\} \\ \frac{1}{2} & if\ \ x\ \in 6z - \{0\} \\ 0 & otherewise \end{cases}$ μ is a fuzzy submodule of A.

μ is fuzzy semiprime submodule of A,since $\mu_0 = Z$, $\mu_{\frac{1}{2}} = 6Z$ and $\mu_t = \{0\}$ if $t > \frac{1}{2}$.

But μ is not fuzzy prime submodule Since $5_{\frac{1}{3}}.6_{\frac{2}{3}} = 30_{\frac{1}{3}} \subseteq \mu$, $6_{\frac{2}{3}} \nsubseteq \mu$, $5_{\frac{1}{3}} \nsubseteq [\mu: A]$ this mean $5_{\frac{1}{3}}A \nsubseteq \mu$ since $3_{\frac{1}{4}} \subseteq A$ but $5_{\frac{1}{3}} \cdot 3_{\frac{1}{4}} = 15_{\frac{1}{4}} \nsubseteq \mu$.

Theorem2.2: Let μ be a proper fuzzy submodule of a fuzzy module A of an R −module M.Then the fallowing statements are equivalents :

a- μ is a fuzzy semi prime submodule of a fuzzy module A of an R −module M.

b- The fuzzy ideal $[\mu: \lambda]$ is fuzzy semiprime for all fuzzy submodule λ contains μ properly.

c- The fuzzy ideal $[\mu: \langle x_t \rangle]$ is fuzzy simeprime for all $x_t \subseteq A$; $x_t \nsubseteq \mu$.

Proof: (a ⇒ b)

It is clear that the fuzzy ideal $[\mu: \lambda]$ is properly in R for all fuzzy submodule λ of a fuzzy module A of an R −module M contains μ proporly. Now let $r_t \subseteq \sqrt{[\mu: \lambda]}$,and hence there exists positive integer n such that $r_t^n \subseteq [\mu: \lambda]$ this means $r_t^n \lambda \subseteq \mu$ and this means also $r_t^n x_l \subseteq \mu$ where $x_l \subseteq \lambda$. But μ is a fuzzy semiprime submodule ,therefore $r_t x_l \subseteq \mu$, and hence $r_t \subseteq [\mu: \lambda]$ from this we observe that $\sqrt{[\mu: \lambda]} \subseteq [\mu: \lambda]$,and we known that $[\mu: \lambda] \subseteq \sqrt{[\mu: \lambda]}$ this implies $[\mu: \lambda] = \sqrt{[\mu: \lambda]}$,therefore $[\mu: \lambda]$ is a fuzzy semiprime ideal in R for all fuzzy submodule λ of a fuzzy module A contains μ proporly.

$(b \Rightarrow c)$

Let $x_l \subseteq A$; $x_l \not\subset \mu$ it is observe that the fuzzy ideal $[\mu : \langle x_l \rangle]$ is properly in R . suppose $r_t \subseteq \sqrt{[\mu: \langle x_l \rangle}$ therefore there exists a positive integer n such that $r_t^n \subseteq [\mu: \langle x_l \rangle]$. But $\mu \subsetneq \mu + \langle x_l \rangle$. It is clear that $r_t^n \subseteq [\mu: \mu + \langle x_l \rangle]$ by b we get $r_t \subseteq [\mu: \mu + \langle x_l \rangle]$ and here $r_t \subseteq [\mu: \langle x_l \rangle]$ and this implies that $\sqrt{[\mu : \langle x_l \rangle]} \subseteq [\mu: \langle x_l \rangle]$, since $[\mu: \langle x_l \rangle] \subseteq \sqrt{[\mu: \langle x_l \rangle]}$.

Thus $[\mu: \langle x_l \rangle] = \sqrt{[\mu: \langle x_l \rangle]}$ and the ideal $[\mu: \langle x_l \rangle]$ fuzzy semiprime ideal in R for all $x_l \subseteq A, x_l \nsubseteq \mu$.

$(c \Rightarrow a)$

Let $r \in R, x_l \subseteq A$ such that $r_t^2 x_l \subseteq \mu$. If $x_l \subseteq \mu$ then $r_t x_l \subseteq \mu$ suppose $x_l \nsubseteq \mu$. Since $r_t^2 \subseteq [\mu: \langle x_l \rangle]$ and by c we get $r_t \subseteq [\mu: \langle x_l \rangle]$ this means $r_t x_l \subseteq \mu$ and therefore the proper fuzzy submodule μ is semiprime fuzzy module A of an R −module M.

From the theorem above we get

Corollary2.1 : If μ is a fuzzy semiprime submodule of a fuzzy module A of an R-module M, then $[\mu: A]$ is a fuzzy semiprime ideal in R.

Remark2.1: The converse of this corollary is not true in general for example:

Let $M = Z \oplus Z$ as a Z-module, let $A: M \longrightarrow [0,1]$, $\mu: M \longrightarrow [0,1]$ defined by:

$A(a,b) = 1 \ \ \forall (a,b) \in Z \oplus Z$

$\mu(a,b) = \begin{cases} 1 & if\ (a,b) \in 4Z \oplus \langle 0 \rangle \\ 0 & \text{otherwise} \end{cases}$ it is clear that A is a fuzzy module of M and μ is a fuzzy submodule of A. Since $[\mu_t : A_t] = [4Z \oplus \langle 0 \rangle : Z \oplus Z] = \langle 0 \rangle$ for all $t > 0$. Hence $[\mu: A]_t = \langle 0 \rangle$ and $[\mu: A](r) = \begin{cases} 1 & if\ r \in \langle 0 \rangle \\ 0 & \text{otherwise} \end{cases} = 0_1$, which can be easily a fuzzy semiprime ideal in Z. But μ is not fuzzy semiprime submodule of A. Since $2^2_{\frac{1}{2}}(1,0)_{\frac{1}{2}} = (4,0)_{\frac{1}{2}} \subseteq \mu$ but $2_{\frac{1}{2}}(1,0)_{\frac{1}{2}} = (2,0)_{\frac{1}{2}} \nsubseteq \mu$.

Proposition 2.1 : If μ is a fuzzy primary submodule of a fuzzy module A, then the fuzzy ideal $[\mu: A]$ is fuzzy semiprime ideal in R if and only if μ is a fuzzy prime submodule in A.

Proof: Let μ be a fuzzy primary submodule of A and $[\mu: A]$ be a fuzzy semiprime ideal in R. and let $r \in R$,$x_l \subseteq A$ such that $r_t x_l \subseteq \mu$ .Suppose that $x_l \subseteq \mu$,since μ is a fuzzy primary submodule then $r_t \subseteq \sqrt{[\mu: A]}$ but $[\mu: A]$ is a fuzzy semiprime ideal in R ,then $r_t \subseteq [\mu: A]$ this means μ is a fuzzy prime submodule of A.

The second direction immediately from corollary 2.1.

Crollary2.2: If μ is a fuzzy primary submodule of a fuzzy module A of an R-module M,then the fuzzy ideal $[\mu: A]$ is fuzzy semiprime ideal in R if and only if μ is a fuzzy semiprime of a fuzzy module A.

Proof: immediately from corollary2.1 , Proposition2.1 and theorem2.1.

Theorem2.3: Let μ be a fuzzy submodule of a fuzzy multiplication Module A,then If μ is a fuzzy semiprime submodule of A if and only if the fuzzy ideal $[\mu: A]$ is a fuzzy semiprime ideal in R.

Proof:Suppose the fuzzy ideal $[\mu: A]$ is a fuzzy semiprime ideal in R, then it is proper fuzzy ideal in R ,and hence μ is a proper fuzzy submodule of A. Now let $r \in R, x_l \subseteq A, t, l \in [0,1]$ such that $r_t x_t \subseteq \mu$. Since A a fuzzy multiplication module then $[\langle x_l \rangle : A]A = \langle x_l \rangle$ where $\langle x_l \rangle$ is the fuzzy submodule of A generated by x_l and hence $x_l = r_{t_1} m_{s_1} + r_{t_2} m_{s_2} + \cdots + r_{t_n} m_{s_n}$,$r_{t_i}[\langle x_l \rangle : A]$, $m_{s_i} \subseteq A$,$i = 1,2, \ldots, n$, but $r_t^2 r_{t_i}^2 \subseteq [\langle x_l \rangle : A] \subseteq [\mu: A], \forall\ i = 1,2,3, \ldots n$ and hence $r_t x_l = r_t r_{t_1} m_{s_1} + r_t r_{t_2} m_{s_2} + \cdots + r_t r_{t_n} m_{s_n} \subseteq \mu$, and this means μ is a fuzzy semiprime submodule of A.

The second direction immediately from corollary 2.1.

Now we introduce the condition to become the converse of theorem 2.1 is true before that we give the definition of fuzzy irreducible submodule.

Definition2.1: let μ be a fuzzy submodule of a fuzzy module A of R —module M . thus μ is called fuzzy irreducible submodule of for all two fuzzy submodule $\lambda_1$and $\lambda_2$such that $\lambda_1 \cap \lambda_2 = \mu$ then $\lambda_1 = \mu$ or $\lambda_2 = \mu$ otherwise μ is called reducible .

Theorem2.4: let μ be a fuzzy irreducible submodule of fuzzy module A an R-module$M$.μ is a fuzzy semiprime submodule if and only if μ is a fuzzy prime

Proof:- To sufficient to prove that if μ is fuzzy irreducible submodule and semiprime of A then μ

Is prime. Suppose μ is not fuzzy prime this mean there exists $r \epsilon R$ and $x_l \subseteq A$ such that $r_t x_l \subseteq \mu$ and $x_t \not\subset \mu$ and $r_t \nsubseteq [\mu: A]$, since $r_t \nsubseteq [\mu: A]$,then there exists $w_s \subseteq A$ such that $r_t w_s \nsubseteq \mu$. We claim that $\lambda_1 \cap \lambda_2 = \mu$ where $\lambda_1 = \mu + \langle r_t w_s \rangle \lambda_2 = \mu + \langle r_t w_s \rangle$ for that let $y_p \subseteq \lambda_1 \cap \lambda_2$, this means $y_p \subseteq \mu + \langle x_l \rangle$ and $y_p \subseteq \mu + \langle x_l \rangle$, therefore, there exists $q_f, z_U \subseteq A$, $a, b \in R$ such that $y_p = q_f + a_d r_t w_s = z_u + b_c x_l$ this implies $z_u - q_f + b_c x_l = a_d r_t^2 w_s$ and hence $r_t z_u - r_t q_f + r_t b_c x_l = a_d r_t^2 w_s$ and here $a_d r_t^2 w_s \subseteq \mu$ but μ is fuzzy semiprime submodule this implies $q_f + a_d r_t w_s \subseteq \mu$ and hence we get $q_f + a_d r_t w_s \subseteq \mu$ this means $y_p \subseteq \mu$ and thus $\lambda_1 \cap \lambda_2 \subseteq \mu$,it is observe that, $\mu \subseteq \lambda_1 \cap \lambda_2$, this means $\mu = \lambda_1 \cap \lambda_2$,but this contradiction with μ is fuzzy irreducible submodule, this proved μ is fuzzy prime .

Theorem2.5: If μ is a fuzzy semiprime of a fuzzy module A of an R —module M and λ fuzzy submodule of A such that $\lambda \nsubseteq \mu$. Then the fuzzy submodule λ ∩ μ is a fuzzy semiprime of a fuzzy module A of an R − module M and λ fuzzy submodule of A such that λ ⊈ μ , then the fuzzy submodule in $\lambda \cap \mu$ is a fuzzy semiprime submodule in λ.

Proof: Since $\lambda \nsubseteq \mu$ then the fuzzy submodule $\lambda \cap \mu$ properly in λ, now let $r \epsilon R$, $x_l \subseteq \lambda$ such that $r_t^2 x_l \subseteq \lambda \cap \mu$ and hence $r_t^2 x_l \subseteq \mu$ and $r_t^2 x_l \subseteq \lambda$ but $x_l \subseteq \lambda$ then $r_t x_l \subseteq \lambda$ and since μ is a fuzzy semiprime in A then $r_t x_l \subseteq \mu$ by this we have get $r_t x_l \subseteq \lambda \cap \mu$ this means the $\lambda \cap \mu$is a fuzzy semiprime submodule in λ.

Theorem2.6: let μ be a fuzzy submodule of a fuzzy module A of an R −module M . If μ is an intersection of fuzzy prime submodules then the fuzzy submodule μ of a fuzzy module A is a fuzzy semiprime submodule of A.

Proof: Let $\mu = \bigcap_{i\in I} \lambda_i$ such that λ_i fuzzy prime submodule of $A\ \forall i \in I$. Now let $r \in R$, $x_l \subseteq A$ such that $r_t^2 x_l \subseteq \mu$ therefor $r_t^2 x_l \subseteq \bigcap_{i\in I} \lambda_i$ and $\forall_i$, $r_t^2 x_l \subseteq \lambda_i$, but λ_i is fuzzy prime submodule of A , then either $x_l \subseteq \lambda_i$ or $r_t^2 \subseteq [\lambda_1 : A]$. In the case $x_l \subseteq \lambda_1$ then $r_t x_l \subseteq \lambda_i$ and in the case $r_t^2 \subseteq [\lambda_i : A]$ then $r_t \subseteq [\lambda_i : A]$ because $[\lambda_i : A]$ is a fuzzy prime submodule. Hence in the two cases we get $\forall_{i\epsilon I}, r_t x_l \subseteq \lambda_i$ and therefore $r_t x_l \subseteq \bigcap_{i\in I} \lambda_i = \mu$, this means μ is a fuzzy semeprime submodule.

REFERENCES

[1] Zadeh, L.A., “Fuzzy sets”. Inf. Control,Vol. 8, 1965 pp. 338-353.

[2] Rosenfeld, A., “Fuzzy groups”. J. Math. Anal. Appl.Vol.35, 1971, pp: 512-517.

[3] Anthony J.M. and H. Sherwood. Fuzzy group redefined. J1. Math. Anal. AppI. Vol.69, 1979, pp: l24-130.

[4] Liu, W. J, Fuzzy Invariant Subgroups and Fuzzy Ideals, Fuzzy Set and Systems. Vol. 8, pp. 133-139, 1982.

[5] Liu W. J. Operations on fuzzy ideals. Fuzzy Sets and Systems, Vol. 11, pp. 31-41, 1983.

[6] Kuroki N. On fuzzy ideals and fuzzy bi-ideals in semigroups. Fuzzy Sets and Systems , Vol. 5,pp.203-215, 1981.

[7] Nanda S. Fuzzy fields and fuzzy linear spaces. Fuzzy Sets and Systems, Vol.19, pp. 89 -94,1986.

[8] Negoita C.V. and D.A. Ralescu. Applications of Fuzzy Sets and System Analysis. Birkhauser, Basel, 1975.

[9] Mashinchi M and Zahedi M.M., On L-Fuzzy Primary submodules, Fuzzy sets and system, Vol. 49, pp. 231-236, 1992.

[10] Golan J .S.. Making modules fuzzy. Fuzzy Sets and Systems, Vol.32, pp.91-94 ,1989.

[11] Muganda G. C. Free fuzzy modules and their bases. Inform. Sci., Vol.72, pp.65-82, I993.

[12] Pan F. Exact sequences of fuzzy linear maps. Fuzzy Sets and Systems, Vol.27, pp. 317-325,1988.

[13] Pan F. Fuzzy finitely generated modules. Fuzzy Sets and Systems, Vol.2l, pp. l05-l13, 1987.

[14] Pan F. Fuzzy quotient modules. Fuzzy Sets and Systems, Vol.28, pp. 85-90,1988.

[15] Pan F. The various structures of fuzzy quotient modules. Fuzzy Sets and Systems, Vol.50, pp. 178-192, 1992.

[16] Zahedi M.M. and A. Ameri. On fuzzy projective and injective modules. J1. Fuzzy Math., pp.l8l-193, 1995.

[17] Zahedi M.M. On L-fuzzy residual quotient modules and π-primary submodules. Fuzzy Sets and Systems, Vol.51, pp. 333-344, 1992.

[18] Zahedi. M.M. Some results on L-fuzzy modules. Fuzzy Sets and Systems, Vol.55, pp., 355 -361, 1993.

[19] Zahedj M.M. and A. Amen. Fuzzy exact sequences in category of filzzy modules. Jl. Fuzzy Math., Vol.2 No.2, pp. 409-424, 1994.

[20] Zahedi M. M., On L-fuzzy Residual Quotient modules and P-Primary submodules, Fuzzy sets and system, Vol. 51, pp. 333-344, 1992.

[21] Nanda, S., Fuzzy Modules over fuzzy rings, Bull. Col. Math. Soc. Vol.81, pp. 197-200 1989.

[22] Martinez , L., Fuzzy subgroups of fuzzy groups and fuzzy ideal of rings , J. Fuzzy Math. Vol.3, pp. 833-849, 1995.

[23] Bhambri. S. K., Kumar R. and Kumar P., On fuzzy Primary submodules. , Bull. Col. Math. Soc. Vo.86, pp. 445-452, 1994.

[24] Zheng Pan, Fu. , Fuzzy Finitely generated Modules, Fuzzy sets and systems, Vol.21,pp. 105-113, 1987.

[25] Zheng Pan, Fu., Fuzzy Finitely Value Distribution of Fuzzy Vector spaces and fuzzy Modules, Fuzzy sets and systems, Vol.55,pp. 319-322 ,1993.

[26] Jianli. Z, Kaiquan S, Mingshan Y.”Fuzzy Modules Over Fuzzy Rings” J. Fuzzy. Math. Vol., pp.1, 531-539, 1993.

[27] Kumar, R. Bhambri S. K and Kumer P.” Fuzzy Submodules :Some analogues and Deviations” Fuzzy Sets and Systems. Vol. 70, pp. 125-130, 1995.

[28] Martinez ,L., Fuzzy modules over fuzzy rings in connection with fuzzy ideal of fuzzy ring, J. Fuzzy Math., Vol.4, pp. 843-857, 1996.

[29] Dong SooLee, Chul Hwan Park., On Fuzzy Prime Submodules of Fuzzy Multiplication Modules, East Asian Mathematical Journal, Vol.27, pp.75-82, 2011.

[30] Modeson J.N. and Malik D.S., Fuzzy commutative Algebra, Word Scientific Publishing Co., 1998.

[31] Abou- Draeb A. T., Al-Zreegawi Badr Al-Duja Yaseen Khudhair, Prime Fuzzy Submodules and Strongly Prime Fuzzy Submodules Over Near-ring, Journal of Babylon Univ. Pure and Applied Science, Vol.22, No. 3, 2014, pp. 984-996.

[32] Zahedi M.M., A note on L-fuzzy primary and semiprime ideals, Fuzzy Sets and System Vol.51, 1992, pp:243-247.

[33] Kumbhajkar, M.S. Bapat, On semiprime fuzzy ideals, Fuzzy Sets and System Vol. 60, 1993, 219-223.

[34] Hadi,I.M.A,Semiprime Fuzzy Submodules of Fuzzy Modules,Ibn Al-Haitham j. of Pure and Appl. Sci., Vol. 17, No. 3, 2004, pp: 112-123.

[35] Dong Soo Lee And Chul Hwan Park,On fuzzy prime submodules of fuzzy multiplication modules,” Fast Asian Mathematical journal” Vol.27, 2011, No.1, pp:75-82.