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Asymmetric Smooth Transition for Modeling and Simulation of the Emergence of Hopf Bifurcations and Endogenous Fluctuations of Tunisian Industrial Cycle

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ABSTRACT: There are many areas of research related to the empirical analysis of time series which support The Real Business Cycle Theory, but researchers have rarely explored the possibility that business cycle fluctuations have a nonlinear aspect and that is intrinsically a phenomenon without exogenous chocks. According to the theoretical exception of Keynesian endogenous cycle, cyclical movements are not due to the optimal adjustments of erratic displacements compared the equilibrium or deviations of working economy or of the impulses caused by external shocks or unpredictable events, but rather of vulnerable and fatal instability through which economy is oriented and of how endogenous fluctuations' evolution can affect nonlinear dynamics structures of time series. In this study, an econometric examination of the Tunisian industrial production cycle would prove the existence of endogenous fluctuations through a Hopf bifurcation if the control parameter transition undergoes a change. Also, clarify how bifurcation Theory and endogenous instability can explain how chaos is generated endogenously. The aim of our paper is to investigate the potential asymmetric effects of industrial dynamics. As a special case, we expand upon other works on this issue, by assessing the effect of structural changing processes in an asymmetric transition function on smooth transition autoregressive (STAR) specification that exhibits a limited cycle. But, in many given times, a deterministic dynamical system has a chaotic character that can affect predictability. While our study is based on recent fields of dynamical economy and econometrics' nonlinear processes, our results would concern of different empirical simulations about the endogenous business cycle. Specially, we analyze some aspects of nonlinear dynamics time series including chaos by controlling a parameter's transition in each bifurcation diagram.

KEYWORDS: Nonlinearity, Simulation, Endogenous cycle, Hopf bifurcation, Chaos, Asymmetric smooth transition, Industrial production in Tunisia.

1 INTRODUCTION

It is widely recognized that most of the discussions in macroeconomic analysis are based on the assumption of linearity. Recently, nonlinear time series theory could be developed and applied for various purposes. Especially, a large part of literature in econometrics analysis suggests that predicting the future is one of the fundamental objectives of nonlinear time series analysis. In contrast, it has been long recognized that linear prediction's techniques, such as exponential smoothing and the Box-Jenkins methodology can not help to improve contemporary macro-economic analysis. In addition, these procedures are not able to estimate only the breaks and structural change in time series, but also to consider the asymmetry phenomenon from nonlinear conditional mean models dynamics. This in particular, to describe the pattern of changes in various macro-economic aggregates if we desire to explain the irregularity of cyclical fluctuations, to predict the future movement of economic activity and to clarify the evolution of dynamic behavior in endogenous economic cycle. Thereby, the problem of characterizing predictive nonlinear time series model is still an open question. This issue is discussed by [2] since a long time, who uses threshold autoregressive (TAR) model and smooth transition autoregressive (STAR) model to describe the asymmetric effects in endogenous cycle over monetary policy. He makes use of an endogenous business cycle model in which cyclical behavior arises from the intrinsic investment-profit instability. At this way, a good specification of the asymmetric behavior is crucial in empirical work, especially for reported studies in the statistical modeling writings of asymmetric business cycle, we can see [2]. But now, it is largely recognized that business cycle prediction approach mainly for regularities hypothesis induced by simple linear relationship. It describes that propagate mechanisms of predictive information provided by exogenous shocks are not precise. Accordingly, some known dynamic aspects of traditional modeling methods such as the stability and regularity, result from the reason that they are based on linear evidence of econometric specifications which is unreliable when the underlying phenomenon generating the data that is considered to be non-linear. It is therefore essential to understand how to reproduce the nonlinear dependency behind the data. One major advantage of giving more information about nonlinear dynamics is to understand the real data generation process. In this way, many economists as [7] confirmed that even in the absence of exogenous stochastic shocks the time series can fluctuate inherently in an endogenous deterministic dynamics that can rarely exhibit a chaotic behavior in various situations. However, the presence of nonlinear dynamics and the existence of chaotic behavior have many fundamental implications for predictive modeling and for the nonlinear time series analysis. For that, ([5], [6]) have indicated that characterization of chaos for predictive modeling is known to be a difficult diagnostic problem especially in studies in nonlinear dynamics and econometrics. In this context, [19] observed that the ability of nonlinear dynamical tools to detect the possible presence of chaos has centered the research effort on fluctuations instability domain which is intensively a strong debate to identify whether dynamics in nonlinear time series are endogenous or exogenous. In the same context, [3] show that chaotic business cycles can be endogenously generated in a deterministic setting. For this reason, an important development in business cycle analysis and the prediction related to endogenous cycle theory tend to explain how the endogenous generating realizations with asymmetrical cyclical fluctuations can be used in nonlinear time series prediction analysis. Consequently, writings of the contemporary fluctuation analysis in both axes theoretical and empirical researches given in ([40], [38], [35], [34], [39], [18]) show the importance of considering the hypothesis of nonlinearity in the mean and the instability determined from investigation of the model and prediction of the endogenous cycle. So, all combination of search strategies, have proved that forecast’s parameters are extremely sensitive to each stability conditions which include convergences to equilibrium or an endogenous cycle such as a limit cycle or the chaotic dynamics. On the other hand, the dynamic economic science writings are generally more concentrated on the ability of nonlinear dynamical tools to detect, characterize, and predict chaos but less concerned with the influence that endogenous fluctuation have a coefficient forecasts. However, selected writing on the nonlinear prediction model is largely ignored, as a whole, in economic science journals. While many studies derive inferences from previous and more general studies on nonlinear time series, such as those focusing on endogenous cycle (see [14], [17]).

In the fluctuated instable macroeconomic domain, a natural approach to model and forecast economic cycle with nonlinear time series considered that observation process’s is usually assured in the nonlinear dynamic regime in which the system oscillate like a limit cycle. But, many deterministic dynamic systems are now known to exhibit chaotic behavior. As a result, [22] proposed a theoretical foundations and applications in the social sciences where erratic fluctuations are a dominant cause of irregularity of macroeconomic fluctuations. The evidence of chaotic behavior generated from a large nonlinear economic deterministic time series can justify the existence of endogenous propagation mechanism of cyclical fluctuations resulting from a loss of equilibrium and stable proprieties noted by bifurcation phenomenon [9]. On another side, ([42], [30]) have examined that when cyclical movement exposed a strong asymmetry, the complexity of dynamic system is explained by fort dependent combination phase (or state) with a smooth progressed function. This smoothness can change over the time and indicates any evolution in the dynamics’ structures. Few areas of study can avoid reference to combination nonlinear dynamical systems and econometrics for problematic nonlinear time series prediction.

Recently, a nonlinear dynamic association with this functional mode has considered that the industrial production cycle is only a nonlinear and endogenous cycle [25]. Since the Tong’s basic contribution [40], the TAR model has been largely developed and experimented [39]. This shift means sequential linear model class defined by party and assured that nonlinearity can be determined by a threshold effect which exhibits changing regime process. It is absolutely true that every linear sequence in TAR model can contain many points of equilibrium. But the imposed discontinuities in TAR model signify that the passage between different regimes can be assured by brutal adjustment. There are of course many notable exceptions authorizing a slow passage of one regime to the others by using a STAR model, used notably in order to model nonlinear economic cycle; as it is extensively discussed in [37]. In this context, we suggested that a transition of cyclical regime is not brutal but it is a smooth transition. For instance, [16] explains that the business cycles phases are frequently asymmetric and it can pass slowly from expansion situation to recession and vice-versa, using multiple regime smooth transition autoregressive (MRSTAR) models which are introduced by [15]. However, in STAR class model, the means of access between cyclical regimes are usually assured by replacing the indicator function in TAR model by a continuous transitional function which changes smoothly from 0 to 1 and it is examined in [35]. These gradual passages between regimes induced by utilizing the transitional function and that can generate the adjustment structural process. This is proved in [8] and which caused dynamic equilibrium of instability induced by the presence of a phenomena of limit cycle. If the endogenous loss of stability of dynamic equilibrium is attended, then dynamic equilibrium passed from local stability to general instability. In this situation, according to [21] we suppose that a transition in the model structure can be explained by changing in the propriety of stability of a dynamical equilibrium multiplicity. The goal of this paper is to expand upon other works and reexamine how changing parameters in equation of transitional function affect the dynamic equilibrium of instability which caused a transitional structure model. While the study should prove to be useful in all fields in social science, I hope to examine how identification and control of this pure endogenous structure are used for the suppression of instable fluctuation that model and forecast a nonlinear time series cycle. Instability and Nonlinearity affect inferences drawn from nonlinear dynamic time series models of national endogenous industrial production cycle. Due to similarities of national endogenous cycle fluctuation analyses, the results of this study are directly applicable to ten monthly time series analyses of national instable industrial sector. So, we think that there is time for structural reform when industry becomes more stabilized and more organized.

The transitional events can be exhibited by a discrete time evolution and can also refer to the change in observable values of state dynamic process by the passage of structure. In this deterministic formulation, the concept of time evolution may be applicable to fund instability structure principle. Studies such as [23] show that the evolution rule of the dynamical system is a deterministic rule which has been developed to characterize the features of future states which results from the current state. So, the unit to elaborate nonlinear prediction in the domain of instable structure which is usually used by dynamic system for a fundamental task of time series analysis and that mechanist needs to know the law of organized phenomena of transition between broken chain data. But nonlinear prediction with this hypothesis is sometimes affected by a deterministic chaos; this is mentioned in [13]. In many cases, chaotic behavior has been founded to describe only a subset of phase space mechanism. Researchers such as [10] think that it can be more difficult to distinguish between deterministic and stochastic processes because deterministic system always evolves in the same way from a given starting point as suggested by [7]. However, [41] have examined how the organized deterministic chaos can be illustrated for its predictability. After combining the different procedures by using recent empirical methods in both studies of chaos theory and econometric modeling, the same kinds of nonlinear time series prediction problems have been attacked.

This article describes optimal nonlinear time series of prediction models based on theoretic norms to investigate the underlying dynamics of various time series, especially industrial time series. The preceding optimal modelings of combinative techniques have been employed to select a better prediction model. The purpose of this research is to determine whether nonlinear dynamics and econometrics techniques could be used to extend and understand evolution of error propagation mechanism. We focus on how economic fluctuation of endogenous cycle affects statistical inference, specifically in regard to the repression and dissent of nexus. One other question that endogenous theory of cyclical fluctuations poses is, how do errors predictions nest from economics fluctuations and how does errors prediction affect economics fluctuations? That is, how does dynamical economic cycle of fluctuation behavior influence future dynamic behavior? Typically, qualitative bifurcation technique specifies and estimates dynamic recurrent equations in asymmetric transition function and draws inferences from the controlled parameters about the effect of each transition's behavior on one another. A positive coefficient indicates a reciprocal relationship, while a negative coefficient reflects an inverse relationship between state and dissident behavior. Additionally, some researchers perform Granger causality tests to see if one actor's behavior is better predicted by the inclusion of the other actor's past behavior in the model.

The paper begins by defining some terms and introducing the problem. Next, we review both the econometrics and economics science writings on this topic. Then we move to discuss how discrete nonlinear time series may represent an endogenous cycle. In this section, we proposed a chaotic control behavior of state in nonlinear dynamics by simulation using a deterministic part in univariate asymmetric smooth transition autoregressive specification. Following that discussion, we intend to show that the structure of instability affects both dynamical and statistical inference about analysis. Then, we report the findings and evaluate the bifurcation diagram of each parameter's transition control. Finally, we discuss the implications in the conclusion.

2 DATA DESCRIPTION AND TUNISIAN INDUSTRIAL SITUATION

Before submitting the manuscript, author(s) should check the following list. The analysis is carried out on the index of industrial production (IPI); it is used to represent real economic activity and industrial business cycle situation. The instabilities of Tunisian industrial sector can be explained by a branch of industrial recession. The global economic slowdown caused by a slowdown of the production in manufacturing. This framework is applied to ten Tunisian macroeconomic time series typically considered in endogenous business cycle theories. A shift of regimes is also integrated into the structural model. This monthly aggregate data shown in table (1), are data in which a monthly univariate time series of 180 observations associated with ten particular variables of industrial production sector from July 2000 to December 2014, the categories examined in this paper are as follows:

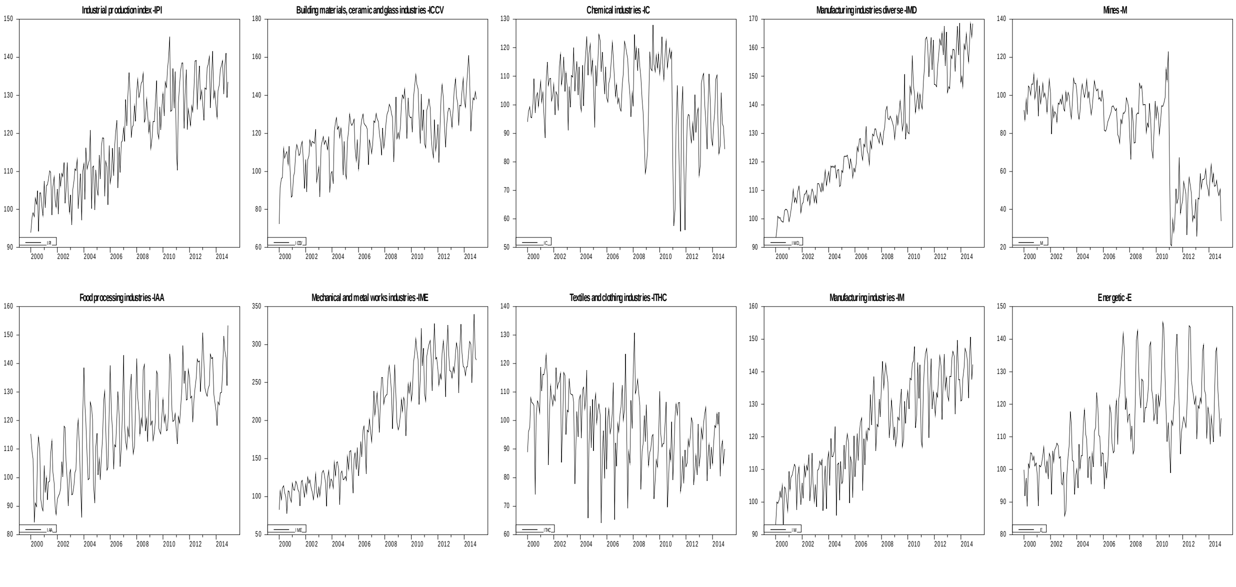
Table 1: Description of monthly data time series*

Series	Seize	Descriptions
IPI	180	Industrial production index
		Manufacturing Data
IAA	180	Food processing industries
ICCV	180	Building materials, ceramic and glass industries
IME	180	Mechanical and metal works industries
IC	180	Chemical industries
ITH	180	Textiles and clothing industries
IMD	180	Diverse Manufacturing industries
IM	180	Manufacturing industries
		Non Manufacturing Data
M	180	Mines
E	180	Energetic

(*) Source: The databases are available at the monthly frequency and accounted for values based 100 at 2000 (started with July 2000 and finished on December 2014) in the Monthly Bulletins of Statistic produced by the National Institution of Statistic. All time series were transformed into growth rates (annual and monthly).

This table gives more economic significances measuring variable accorded data of time series of industrial production that could play any central role in conjectural analysis, especially in the intrinsic dynamics analysis of Tunisian industrial endogenous cycle. The results in Figure (1) illustrate that dynamics of all time series of industrial production during the period January 2000 to December 2014 are with unstable evolution.

Figure 1: Time series from 2000:01 to 2014:12



This fundamental remark can be used for deduction of nonlinear and instability of industrial production cycle. So, it's logical to prefer a STAR methodology specification to analyze and simulate the Tunisian industrial endogenous cycle.

3 RECENT THEORETICAL DEVELOPMENT FOR ASYMMETRIC ENDOGENOUS CYCLE MODELLING AND PREDICTION

As an internal economic conflict, scholars are interested in explaining that the cycle which is essentially nonlinear with endogenous behavior is whether an exogenous or endogenous cycle. To do so, we base the thesis on two fundamental hypotheses such as nonlinearity and inherently the instability of economic structures, which suggests the possible emergence of endogenous cyclical fluctuations but, without damaging the economy. The first one presumes the effect of asymmetric cyclical fluctuations that exhibits a continuous and dynamic change in its states. To expose dynamic propriety, let's imagine the evolution of a dynamic system that can characterize fluctuation movement of endogenous cycle. These evolutions expect changing structures that may suppose instability hypothesis. To join both hypotheses, this has different opinions about how to modulate and forecast endogenous cycle characterized by especially dynamic and progressive behavior. In this paradigm of the domain of instability, to realize a prediction is the aim of recent theoretical development that intensively do examine them in asymmetric endogenous cycle modeling and prediction. When we want to do so, the major difficulty is to defend evolution of dynamic predictive information over time. The problem is what can we use to identify dynamic information behaviors and recognize with precision the slow evolution that can characterize their future positions? Each precise answer needs the choice of a particular trajectory from a larger set of possible forecast trajectories given by a dynamic prediction. So, when we want to select a precise dynamic prediction trajectory what practice technique can we pursue?

Theoretical writings and reports separately study the relationships of deterministic nonlinear dynamics advance and stochastic econometrics proceeding between state and space endogenous cycle forecasting behavior. We begin with writing that focuses on how to join deterministic and stochastic studies to complete discrete nonlinear forecasting time series of Tunisian industrial endogenous cycle. These developments investigated the potential impact of nonlinearity and endogenous structures instability witch can raise the possibility of changes in qualitative proprieties of industrial business cycle, [29] have examined how the effect of these different causes have became generated by endogenous fluctuation. On the one hand, researchers in mobilization in nonlinear dynamics argue that only deterministic treatment imposes costs on statistics characteristics of forecasts information. Thus, only deterministic nonlinear dynamics treatments proved to be unable to capture many interesting probabilistic previous information. On the other hand, similarly, the probabilistic school which is based only on econometric treatment is insufficient because in this case only econometric treatment imposes costs on dynamic propriety of forecasts information's. So, if we desire to realize prediction in unstable structure domain, then we must respect the time and the space as dependent principles that require the joint of different applied methods.

For the purposes of this study, we choose to posit a few general hypotheses that come from this writing and that we can test using data event. In this situation, we suggest a statistical methodology; we begin by hypothesizing that structural time series modeling, which includes endogenous cycles as nested alternatives. Both states respond to one another's dissident actions and reactions (i.e., cause or drive one another's behavior). We should see that states respond to dissident behavior and that dissidents respond to state behavior. In addition, we should see that states respond to their own past behavior and that dissidents respond to their own past behavior.

Next, we put forth two opposed hypotheses that, again, applicable to both actors. First, we suggest that when one actor increases its levels of cooperation, the other actor increases its levels of cooperation (escalation hypothesis). That is, there is a positive linear relationship that exists between the two actors' behavior. Finally, the opposing deterrent hypothesis, hypothesizes that an increasing in one actor's cooperative behavior will lead to the other actor's decreasing cooperative behavior (i.e., increasing conflict).

In order to model and predict business cycle behavior, [36] remarked that cyclical fluctuations are usually asymmetric. Researchers such as [38] presented discusses the topic of the nonlinearity of business cycle as early as Mitchell's studies since 1927, and suggested that evidence is both in favor and against the asymmetry of business cycles. So, [16] noted that STAR models were originally introduced by [38] are needed to describe the data generating mechanism of inherently asymmetrical realization which can endogenously characterize not only cyclical fluctuation, but also changes in their synchronization.

4 RESEARCH DESIGN

4.1 THE MODEL

The presence of asymmetries in macroeconomic time series can have two fundamental implications. First, linear and Gaussian models are incapable of generating asymmetrical fluctuations cycles. Second, if a nonlinear prediction problem is treated as a symmetric linear one, then the estimate of the unpredictable components of time series would contain too much information. Infact, if we desire to evaluate predictability of complex behavior produced by asymmetric endogenous cycle, we need a large theoretical and empirical complimentary in both nonlinear dynamics and econometrics studies. The important questions about endogenously generated cycle remains to be answered using the econometric techniques envisaged by calibrated nonlinear models of business cycle and endogenous cycle theories. The purpose of this study is to model and analyze by simulation the dynamics of endogenously created oscillations. So, we are paying attention to the connection between dynamic propriety and econometric characteristics STAR models which can predict endogenous cycle. Let's start by taking a look at [35], mathematic representation model. If $Y_t \sim STAR(p=1),$ then the associate STAR model is given by:

$$Y_t = f\left(Y_{t-1}\right) + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right) \tag{1}$$

With,

$$f\left(Y_{t-1}\right) = (\psi_0^{(1)} + \sum_{i=1}^{P} \psi_i^{(1)} Y_{t-i})[1 - G\left(s_t;\gamma,c\right)] + \left(\psi_0^{(2)} + \sum_{i=1}^{p} \psi_i^{(2)} Y_{t-i}\right) G\left(s_t;\gamma,c\right),$$
$$\gamma > 0, 1 \le d \le p,\ t = 1,2,...,T.$$

Where the continuous transition functions $G(.)$ with parameters γ and $c,$ which changes smoothly from $G(s_t;\gamma,c)=0$ to $G(s_t;\gamma,c)=1,$ as $s_t = Y_{t-1}$ increases. A stationary transition variable s_t is endogenous variable see this in [36]. The parameter γ determines the smoothness of the change in the value of the transitional function, and thus the transition from one regime to the other is gradual, see [14]. As γ is large as well as the transition between tow regimes is brutal. So, [24] have affirmed that STAR model is linear if the transition variable has a constant value. In this way, [27] considered the transition determined by the generalized logistic function ALSTAR which can generate asymmetrical behavior. But, [5] extends the exponential transitional function to be asymmetric around the threshold value, hereafter referred as the AESTAR model. Recently, the exponential smooth transition autoregressive (AESTAR) model is proposed by ([31], [32]) to examine asymmetric nonlinear mean of reversion. A popular asymmetric logistic and exponential transition function are represented at table (2),

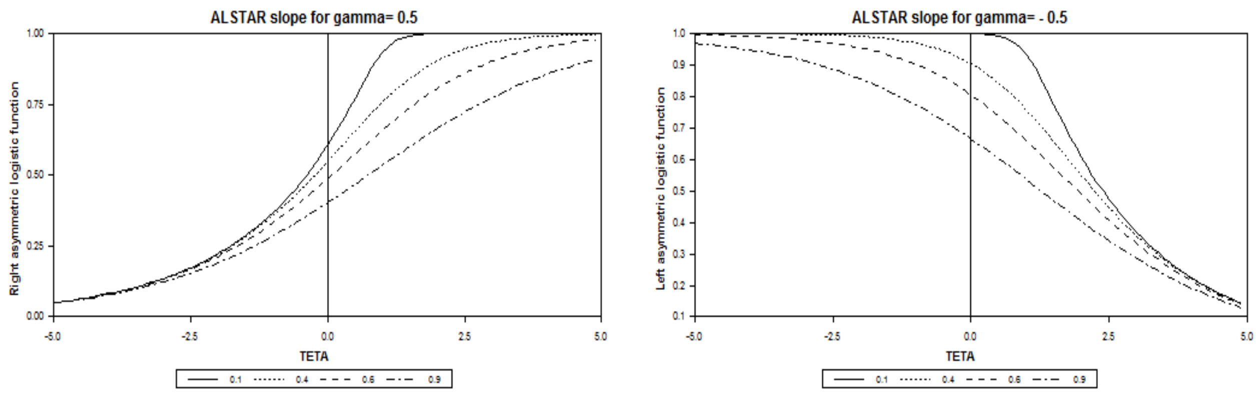
Table 2: Some Asymmetric Transition functions

Logistic and Exponential Asymmetric Transition Function G(Y_{t-1}) and Parameters	
ALSTAR (*)	$G(Y_{t-1};\gamma,c,\theta) = [1+\exp\{-\gamma\,(Y_{t-1}-c)/\theta\}]^{-\theta}, \quad 0<\theta\le 1.$
AESTAR (**)	$G(Y_{t-1};\gamma,c,\lambda) = 1-\exp\{-\gamma(Y_{t-1}-c)^2\left[1/2+(1+\exp\{-\lambda(Y_{t-1}-c)\})^{-1}\right]\},$ $\gamma>0,\ \lambda\ne 0.$

(*) ALSTAR: Asymmetric Logistic Smooth Transition Autoregressive;
(**) AESTAR: Asymmetric Exponential Smooth Transition Autoregressive.

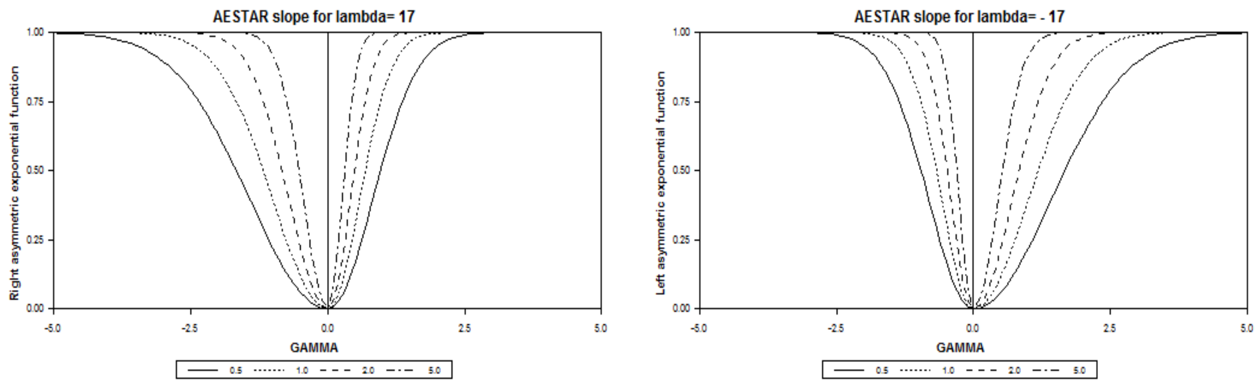
The parameter γ in ALSTAR function governs the speed of transition. Other than the degree of asymmetric is controlled by the parameter $\theta.$ If $\theta=1,$ then a logistic function is earlier symmetric. But, the alternately case may have possible asymmetry. On the other hand, as θ approaches zero, extreme asymmetry is detected. The nature of asymmetry is determined by the sign of parameter γ. for $\gamma > 0$ and $0 < \theta < 1$, a transition starts more slowly than it finishes, while the opposite is the case for $\gamma < 0$. Figure (2) depicts that asymmetric logistic transition function for the ALSTAR model can be left or right introduced through parameter θ. The parameter γ in ALSTAR function governs the speed of transition.

Figure 2: Teta effect in asymmetric logistic function slope with c=1



The Figure (2) show that asymmetric exponential transition function for the AESTAR model can be left or right introduced through parameter γ. The parameter γ in AESTAR function governs the speed of transition. Other than the degree of asymmetric is controlled by the parameter λ.

Figure 3: Gamma effect in asymmetric exponential function slope with c = 0



Although, the need for a practical forecasting framework of endogenous cycle incorporates a combined nonlinear dynamics and econometrics studies under realistic asymmetric loss, thus, we have three related problematic in this work.

With a variety of definitions being offered, the first fundamental question of what constitutes complex endogenous fluctuations is versatile and very complicated theme. For purposes of this discussion, we focus only on the economic dynamic definition of complexity provided and discussed in connection with alternatives by [28]. This definition posits that economic systems are dynamically complex if they fail to converge to a point, a limit cycle, or an exponential expansion or contraction due to endogenous causes. So, to focus on endogenously created oscillations from the dynamics of smooth models, ([2], [42]) have showed that skeleton of smooth transition models can exhibit many types of irregular dynamic patterns of some sort, either sudden discontinuities, a periodic chaotic dynamics subject to sensitive dependence on initial conditions, multi-stability of basins of attraction, or other such irregular patterns. The skeleton of smooth transition maps typically arises as discrete-time models of dynamical systems when the continuous evolution in time is punctuated by impacts or discrete switching events that alter the form of the constitutive structures. Examples of such economic systems include endogenous cycle. As a control parameter is varied in the asymmetric transition function, the fixed point for the Poincaré map of such a system may move in phase space and collide with the border between two smooth regions. When a skeleton of smooth transition map crosses a boundary in a state space many bifurcations phenomena have been much studied it. But if nonlinear dynamic generated by skeleton of smooth transition map exhibits a sensitive dependence on initial conditions, then model structures could have been changed at the situation dramatically, model predictability is affected as the exponential expansion of predictive error. At this situation, uncombined standard techniques forecasts are generally unable to capture the loss structures.

The second fundamental question for forecasting endogenous cycle has been a long practice. While only a few economical scientists such as ([7], [1]) examine the effects of temporal accumulation induced by dynamical propriety on statistic inferences, many other scholars have explored the impacts of dynamic propriety in other forecasting behavior.

Finally, we show how optimal control transition under asymmetric loss may be combined with related econometric techniques for estimation and forecast accuracy comparison under asymmetric loss to produce a flexible framework for forecast model selection. [11] have proposed a new technique for solving forecasting and model selection problems under asymmetric loss using piecewise-linear approximations to the loss function and they have established existence and uniqueness of the optimal predictor. Below, I review particularly the nonlinear dynamics and econometrics, complimentary writings that contribute to the exploration of this issue.

4.2 RESULTS

If all noise terms are set to zero in the nonlinear econometric model (1), then we will refer to a special case of deterministic nonlinear dynamic (skeleton) denoted by:

$$Y_t = f\left(Y_{t-1}\right)$$

In order to appreciate the dynamic proprieties of the nonlinear stochastic econometric model (1) it is important to comprehend the functional law of deterministic nonlinear dynamic skeleton mechanism. Let's start by recognizing the dynamic proprieties of asymmetric functions reported on the table (1).

Dynamically the studies in nonlinear dynamics fluctuation based on deterministic time series models expose the problem that dynamic trajectory of unstable endogenous cyclical fluctuation are usually not stationery and can include a dynamic behavior like limit cycle. But, for many given time a deterministic dynamical system has a dramatically dynamic like chaotic behavior that can affect predictability. In Figures (4-5), we simulate for 130 observations a limit cycle behavior by using ALSTAR function (resp.) AESTAR function.

Figure 4: Limit cycle generated by ALSTAR function
for N=130, gamma=-2.3, teta=0.4 and c=0.2

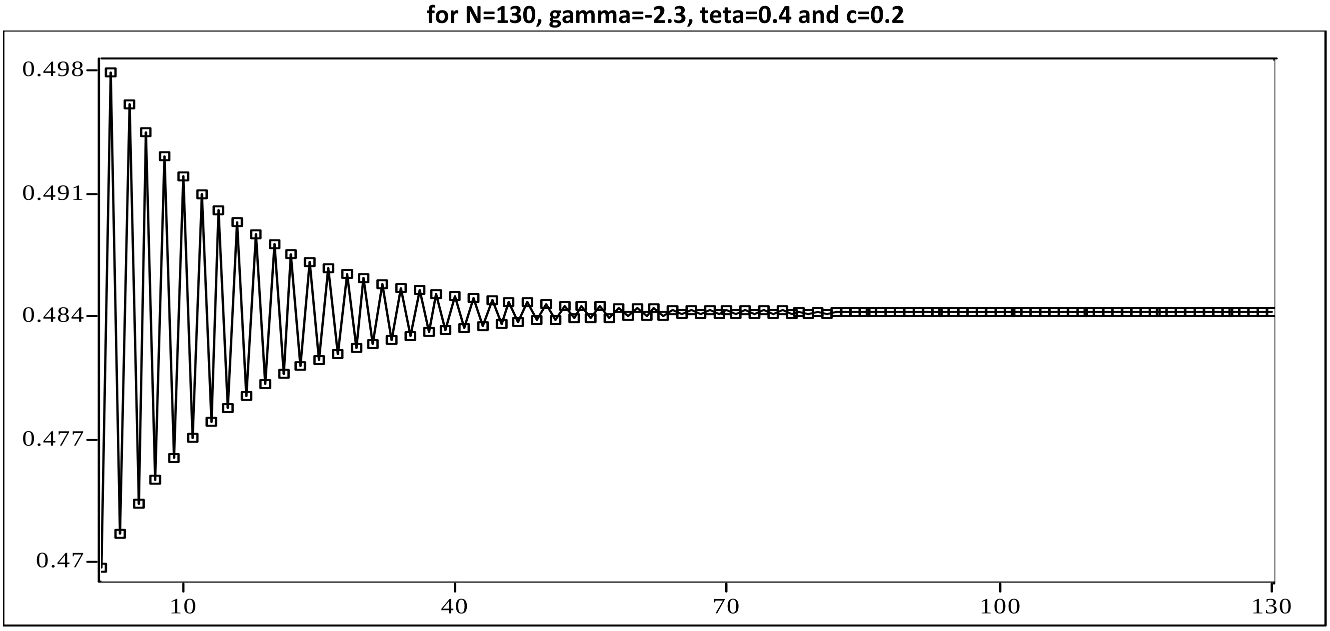
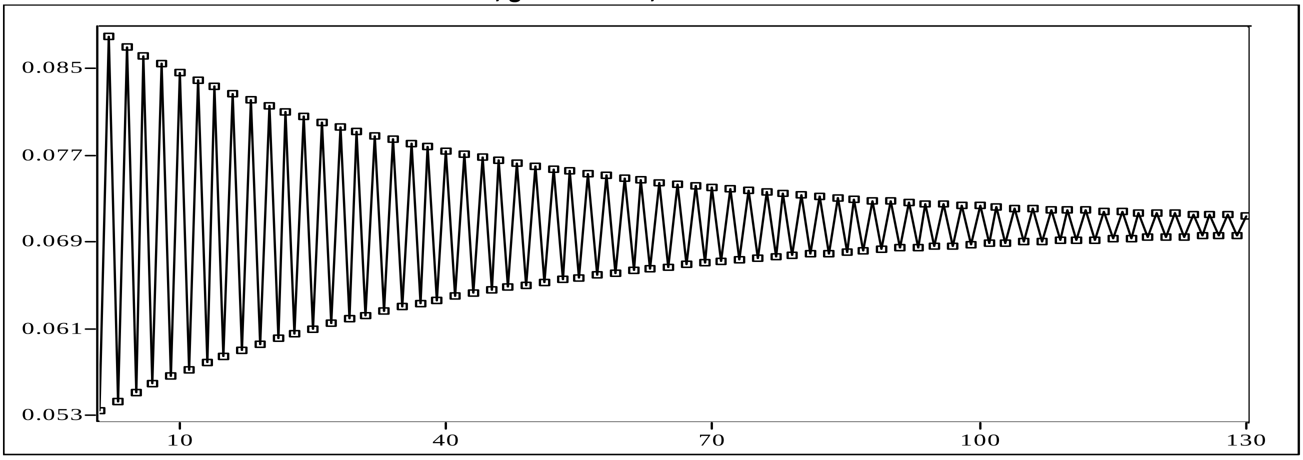


Figure 5: Limit cycle generated by AESTAR function
for N=130, gamma = 5.0, lambda = 4.0 and c=0.2



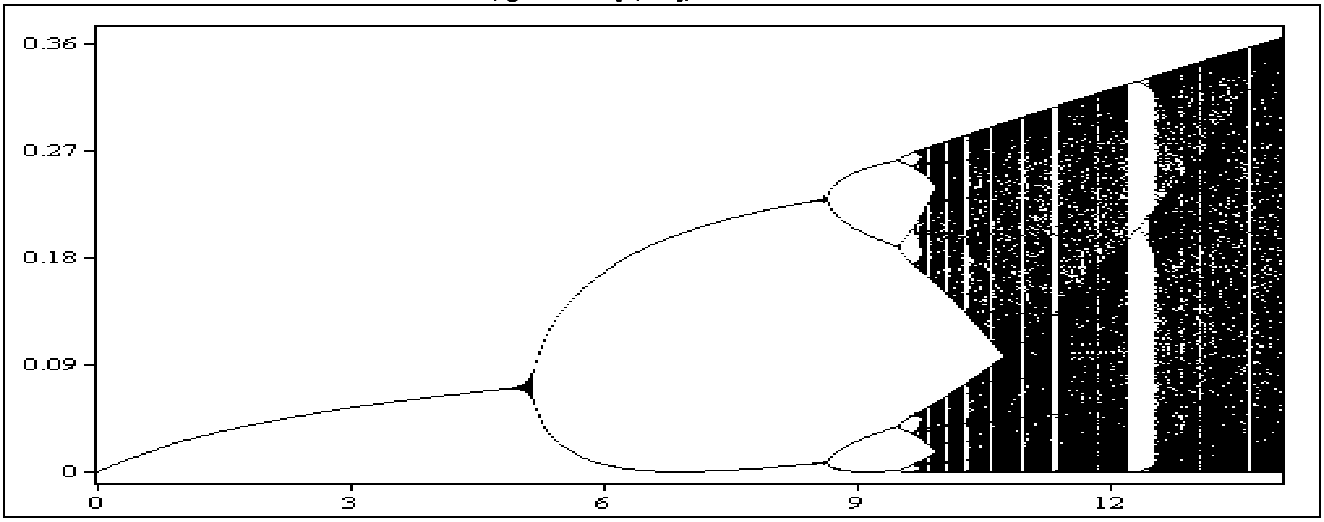
Generally, the fluctuations in limit cycles can characterize second-order bifurcation such as the most importantly transitions from a stable to an unstable state ; those usually are detected near the bifurcation point $\gamma=\gamma_c$, where the structure can change its stability, split into new structures with γ being an external control parameter bifurcation and γ_c is a critical value of the control parameter. The bifurcation theory considers a structure of transformation in phase space and indicates qualitative changing near equilibrium state of the stability properties of dynamical systems that can characterize dynamical endogenous fluctuation. At this point the assimilation of the phase space changes qualitatively in the dynamical system which is more important if we desire to understand and control the transition phenomena. Some bifurcations can lead to very complicated structures in phase space. Consequently, bifurcations can affect with dynamic chaos the predictability of the system. It was in 1972 that Lorenz exposes some conditions of unpredictable behavior: if dynamics exhibit to be highly sensitive to initial conditions denoted *butterfly effect*. At this point, there is explosive growth in error prediction. Figures (6) show that dynamic behavior is dependent on control parameter transition γ, a bifurcation diagrams generated by ALSTAR function can summarize this. By varying the control parameter transition γ, the following behavior is observed:

Figure 6: Bifurcation diagram generated by ALSTAR function
for N=1000, gamma $\in$ [0, 0.08], theta = 0.5 and c=20.0

First, concerning Figure (6), with $0<\gamma<0.0415521$, the processes will quickly stabilize on the approximately critical value $\gamma_c=0.0415521$, for $\gamma=\gamma_c$, there is a bifurcation point when it is dramatically slow, less than linear. With $\gamma>\gamma_c$, the processes may oscillate like limit cycle between two values forever. These two values are dependent on the control of parameter transition γ. In figure (7), similarly by varying the control parameter transition γ in AESTAR transition function, we can summarize the impact of γ on its transition behavior:

Figure 7: Bifurcation diagram generated by AESTAR function

for N=1000, gamma ∈ [0, 14], lambda = 4.0 and c=0.2



With 0<γ<5.0993377, the processes will quickly stabilize on the approximately critical value γc=5.0993377, independent of the initial values. The rate of convergence is linear, except for a critical value γc when it is dramatically slow, less than linear. Withγc<γ<8.6534267 the population may oscillate like limit cycle between two values forever. These two values are dependent on control parameter transition γ. For 8.6534267<γ<9.5496689 (approximately), the processes may oscillate between four values forever. With 9.5496689<γ<9.7041943 (approximately), the processes will probably oscillate between 8 values, then 16, 32, etc. The lengths of the parameter intervals; which yield the same number of oscillations decrease rapidly. This behavior is an example of a period doubling cascade. At γ approximately 9.7041943 is the onset of chaos, at the end of the period-doubling cascade. We can no longer see any oscillations. Slight variations in the initial value yield dramatically to different results over time, a prime characteristic of chaos. Most values beyond 9.7041943 exhibit chaotic behavior, but there are still certain isolated values of γ that appear to show non-chaotic behavior ; these are sometimes called islands of stability. For instance, there is a range of parameters γ which show oscillation between three values, and for slightly higher values of γ oscillation between 6 values, then 12 etc. There are other ranges which yield oscillation between 5 values etc.; all oscillation periods do occur.

After proving the effects of transitions, which can be caused by the endogenous instability and make, evidence a possible bifurcation on the dynamic behaviors that can show dramatically chaotic situations, if transitions are not controlled. To impose control on parameter transition γ, in both ALSTAR and AESTAR transition function, is the first task of selection parameter transition based on chaotic reducing, that needed a developing empirical methods for modelling Tunisian Industrial Endogenous Cycle. The developed empirical methodology for simulation of the emergence of Hopf bifurcations and endogenous fluctuations of nonlinear dynamics time series, based on deterministic chaotic reduction consists to establish some econometric algorithm witch providing empirical evidence that endogenously determined cycles using both ALSTAR and AESTAR process see Figures (8-9).

Figure 8: Limite cycles generated with ALSTAR over time from 2000:01 to 2014:12

alstar_dly(t) = (1+exp(-gamma(dly(t-1)-c)/teta))^-teta*

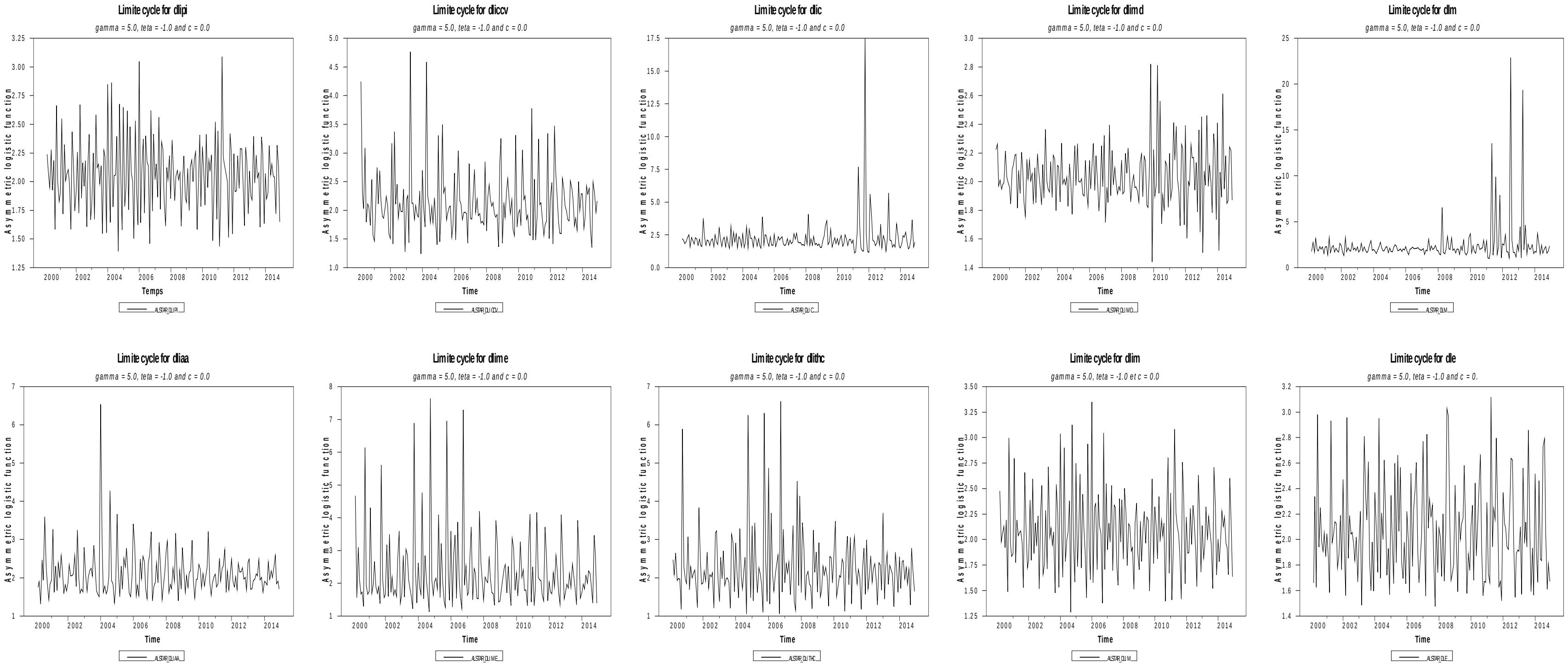


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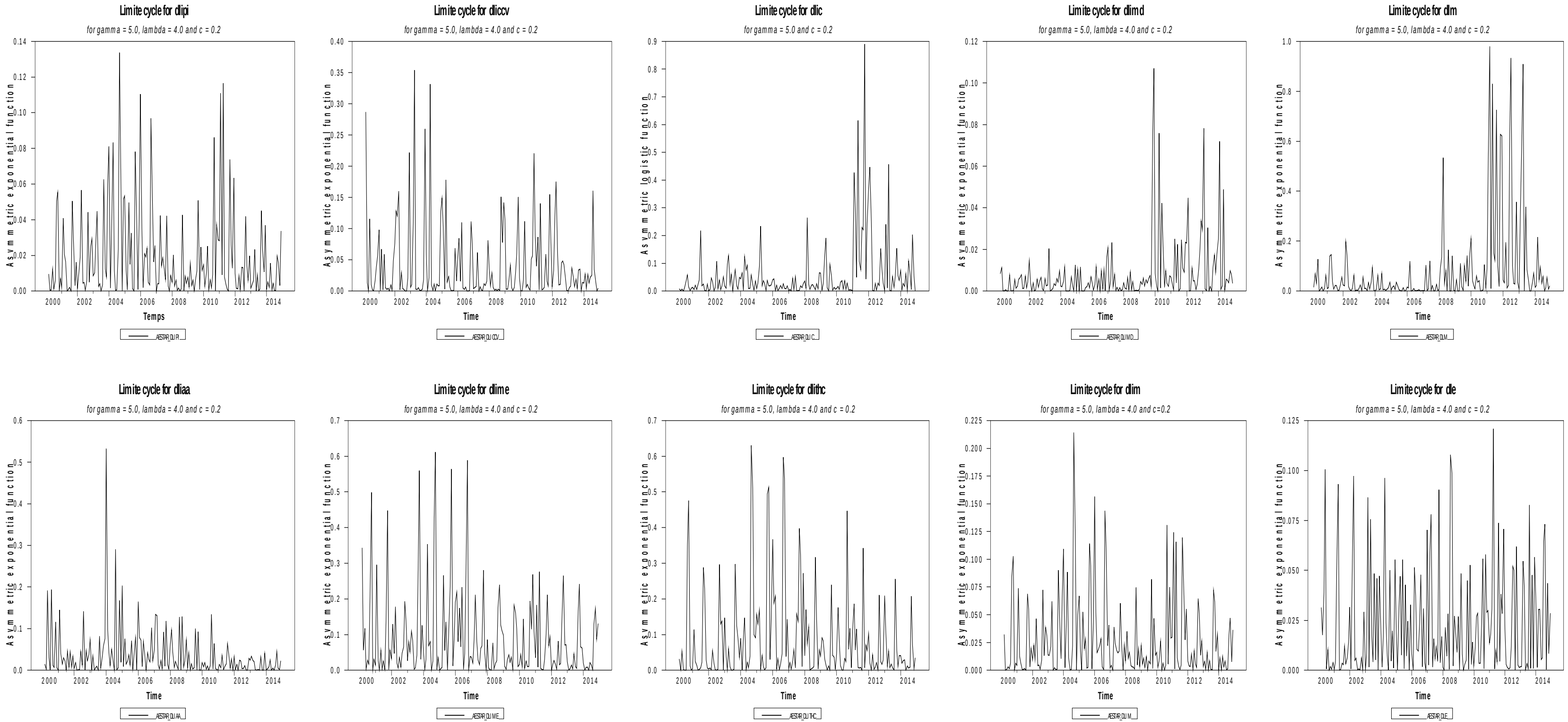
Figure 9: Limite cycles generated with AESTAR over time from 2000:01 to 2014:12

aestar_dly(t) = 1-exp((-gamma(dly(t-1)-c)^2)*(0.5+(1+exp(-lambda*(dly(t-1)-c)))^-1))*



5 CONCLUDING REMARKS

Our original contribution in this paper is to examine how the modeling and simulation of emergence fluctuation of the endogenous structure dynamics in asymmetric smooth transition models can provide a strong evidence of endogenous cycles. For that reason, we have discussed how good practice of new piecewise-linear model where endogenous business cycle dynamics may be envisaged intrinsically by following a period-doubling process and when the dynamics equilibrium exchanges its qualitative properties. This paper tends to focus on nonlinear models of business cycle making and exactly to discuss how the cycle does endogenously generated. Our arguments may be summarized in two parts:

First, it is usually argued that endogenous business cycle dynamics may be related to the appearance and disappearance of endogenous fluctuations, to qualitative changes in their amplitude and to complex structure behavior of stable and unstable sets of the same saddle cycle. But, to focus on nonlinear models of business cycle fluctuations the dynamics in asymmetric smooth models contribute to our understanding of endogenous business cycle dynamics. Our research findings are essentially focus on the period doubling bifurcations within the feasible parameter range which are a dominant cause of irregularity of inherently macroeconomic fluctuations that can be transformed into erratic fluctuations.

Secondly, thus contributes to our understanding to business cycle dynamics and how can be triggered via a center bifurcation where endogenous business cycle dynamics may be generated. The purpose of this study is to model and analyze by simulating the dynamics of endogenously created cycle by bifurcation.

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