International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 27 No. 2 Sep. 2019, pp. 632-637

http://www.ijias.issr-journals.org/

SOME RESULTS ON FUZZY DIVISOR CORDIAL GRAPHS

Mohammed M. Ali Al-Shamiri[1-3], S.I. Nada[2], A.I. Elrokh[2], and Yasser Elmshtaye[1]

[1]Department of Mathematics, Faculty of Sciences and Arts, Mohayel Assir, King Khalid University, Saudi Arabia

[2]Department of Mathematics, Faculty of Sciences, Munofia University, Monofia, Egypt

[3]Department of Mathematics and computer, Faculty of science, Ibb University, Ibb, Yemen

ABSTRACT: A fuzzy divisor cordial labeling of a fuzzy simple graph $\mathrm{G} = (\sigma, \mu)$ be a bijection σ from V to $[0,1]$ such that if each edge $\rightarrow$ is assigned the label d if either $\sigma(u) \mid \sigma(v)$ or $\sigma(v) \mid \sigma(v)$ where $d \in (0,1)$, and the label 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with d differ by at most 1. If a graph has a fuzzy divisor cordial labeling, then it is called fuzzy divisor cordial graph. In this paper, we proved that path, cycle, wheel graph, star graph, some complete bipartite graphs, shell graph S_n,$S(K_{1,n})$ graph, graph $< \mathrm{K}_{1,n}^{(1)}, \mathrm{K}_{1,n}^{(2)} >$ and the Helm $\mathrm{H_n}$ graph are fuzzy divisor cordial graph.

KEYWORDS: Fuzzy Divisor Cordial Labeling, Fuzzy Divisor Cordial Graph, Fuzzy Labeling.

1 INTRODUCTION

The origin of graph labeling problem dates back to the thirteenth century when the Chinese Mathematician Yang Hui and others studied the labeling of geometric figures, which are now classified as plane graphs. The efforts to find solutions to many practical problems in real life situations have also led to the development of several graph labeling methods - graceful, harmonious, prime, divisor, magic, anti-magic, cordial, product cordial, prime cordial etc. Various labeling methods also generate so many new types of graphs. Familiar family graphs have become increasingly useful from mathematical models to a wide range of applications such as design of good radar type codes with automatic correlation properties, identification of ambiguity in X-ray crystallography, formulation of network communication system processing, and optimal circuit layout determination. Diversification of graphic labeling in some references. Marking is a strong connection between number theory and graph structure. By combining the concept of schism in number theory and the concept of a quiet mark in graphing, where the concepts of divisibility play an important role in number theory. Cahit introduced the concept of cordial labeling. And he proved some results related to cordial labeling. Sundaram, Ponraj and Somasundaram in [5] have introduced the notion of prime cordial labeling. And proved that some graphs are prime cordial. Motivated by the concept of prime cordial labeling, R. Varatharajan in [10] introduced a new special type of cordial labeling called divisor cordial labeling. And he proved the standard graphs such as path, cycle, wheel, star and some complete bipartite graphs are divisor cordial. Also show that a complete graph K_n is not divisor cordial for $n \geq 7$. More than that, the special graphs $G * K_{1,n}$, $G * K_{2,n}$ and $G * K_{3,n}$ are divisor cordial.M. Sumathi and A.A gwin in [9] charles introduced a new concept called fuzzy divisor cordial labeling. It is a conversion of crisp graph into fuzzy graph under the new condition namely fuzzy divisor cordial labeling,he proved that a complete n-nary tree is a fuzzy divisor cordial graph, and Every complete graph $\mathrm{k_n}$ is a fuzzy divisor cordial graph. In divisor cordial labeling it is not possible to label all the crisp graphs due to the condition of its definition. Suppose if we consider a graph of size 5, it will be possible to label all the vertices as in the combination of vertex set $\{1,2,3,4,5\}$. So for n vertices, we need to label all the vertices as a combination of all the vertices without repetition, without neglecting any vertex among them. Here discussion about the edge labeling is trivial. So it is clear that all the crisp graphs can't be divisor cordial graphs. However in fuzzy divisor cordial graph for any vertices we can label any fuzzy membership value from $[0,1]$. Since the interval consists of infinite number of terms, there are infinite number of chances for labeling a vertex in fuzzy divisor cordial labeling. Fuzzy graphs have been witnessing a tremendous growth and findingapplications in many branches of engineering and technology so far. In

[8]Rosenfeld has obtained several concepts in fuzzy graph like fuzzy bridges, fuzzy paths, fuzzy cycles, and fuzzy trees and established some of their properties. In this paper we have introduced some results on fuzzy divisor cordial graph , as an application of labeling fuzzy numbers for the graphs under some new conditions. Fuzzy graph is the generalization of the crisp graph. So it is necessary to know some basic definitions and concepts of fuzzy graph and fuzzy divisor cordial graphs.

2 PRELIMINARIES

DEFINITION2.1 [4]

A Graph $G(V,E)$is of vertices and edges. The vertex set $V(G)$ is non-embty set and the edge set $E(G)$ may be embty. A vertex is simply an element of the vertex set and an edge represents a connection between two elements of unordered pair from the vertex set.

DEFINITION 2.2 [4]

A complete graph $\mathrm{K_n}$ of order n is a graph in which every two distinct vertices are adjacent

DEFINITION 2.3 [4]

A graph G is called bipartite if V can be partitioned into two subsets $\mathrm{V_1}$ and $\mathrm{V_2}$ in such a way that every edge of G joins a vertex of $\mathrm{V_1}$ with a vertex of $\mathrm{V_2}$. If every vertex in $\mathrm{V_1}$ is adjacent to every vertex in $\mathrm{V_2}$, then G is said to be complete bipartite graph, denoted by $\mathrm{K_{m,n}}$, where $\mathrm{m} = |\mathrm{V_1}|$ and $\mathrm{n} = |\mathrm{V_2}|$.

DEFINITION 2.4 [4]

A path of length n − 1, denoted by $\mathrm{P_n}$, is a sequence of distinct edges $\mathrm{v_1v_2}$,$\mathrm{v_2v_3}, \ldots, \mathrm{v_{n-1}v_n}$ with $\mathrm{v_iv_{i+1}} \in \mathrm{E(P_n)}$. A closed path, a path with $\mathrm{v_1} = \mathrm{v_n}$, is called a cycle.

DEFINITION 2.5 [4]

A wheel $\mathrm{W_n}$ is defined as $\mathrm{K_1} + \mathrm{C_{n-1}}, \mathrm{n} \geq 4$.

DEFINITION 2.6

The graph $< \mathrm{K_{1,n}^{(1)}}, \mathrm{K_{1,n}^{(2)}} >$is the graph obtained by joining apex (central) vertices of stars to new vertex x.

DEFINITION 2.7

The subdivision of star $S(K_{1,n})$ is the graph obtained from $K_{1,n}$ by attaching a pendent edge to each vertex of $K_{1,n}$ except root vertex.

DEFINITION 2.8 [3]

The helm graph H_n is the graph obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.

DEFINITION 2.9 [2]

The shell S_n is the graph obtained by taking $n-3$ concurrent chords in cycle C_n .

DEFINITION 2.10 [3]

Let $G = (V,E)$ be a graph. A mapping $f : V(G) \to \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f. For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

DEFINITION 2.11 [1]

A binary vertex labeling of a graph G is called a cordial labelingif $|v_f\ (0) - v_f\ (1)| \leq 1$ and $|e_f\ (0) - e_f\ (1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

DEFINITION 2.12 [5]

A prime cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,\ldots,|\ V\ |\}$ such that if each edge uv assigned the label 1 if $gcd(\ f(u), f(v)) = 1$ and 0 if $gcd(f(u), f(v)) > 1$, then the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1.

DEFINITION 2.13 [10]

Let $G = (V, E)$ be a simple graph and $f: V \rightarrow \{1, 2, \ldots |V|\}$ be a bijection. For each edge uv , assign the label 1 if either $f(u) \mid f(v)$ or $f(v) \mid f(u)$ and the label 0 if $f(u) \nmid f(v)$. f is called a divisor cordial labeling if $|e_f\ (0) - e_f\ (1)| \leq 1$.

If a graph has a divisor cordial labeling, then it is called divisor cordial graph.

DEFINITION 14 [8]

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

DEFINITION 2.15 [7]

A graph $G = (\sigma, \mu)$ is said to be a fuzzy labeling graph if $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

DEFINITION 2.16 [9]

Let $G = (\sigma, \mu)$ be a simple graph and $\sigma : V \rightarrow [0,1]$ be a bijection. For each edge uv, assign the label d if either$\sigma(u) \mid \sigma(v)$ or $\sigma(v) \mid \sigma(v)$ and the label 0 if $\sigma(u) \nmid \sigma(v)$. $\sigma$is called a fuzzy divisor cordial labeling if $\left|e_\mu(0) - e_\mu(d)\right| \leq 1$, where d is a very small positive quantity, which is closure to 0 and $d \in (0,1)$.

3 MAIN RESULTS

Some result in this section is listed in [3] but we introduced in another labeling and prove also some result are new.

THEOREM 3.1.

The path P_n is a fuzzy divisor cordial.

PROOF.

Let $v_1, v_2, \ldots v_n$ be the vertices of the path P_n.

We can label these vertices by divided it to many parties.

Part one consists of v_i vertices, where $m = 1$,and$k = 0,1,2, \ldots.$, and$\mu(v_{i_k}) = \frac{(2m-1)2^{k_m}}{10^k}$; where $(2m - 1)2^{k_m} \leq n$ and $m = 1, k_m \geq 0$.

Part two consists of v_j vertices, where $m = 2$ and $k = 0,1,2, \ldots$,

and $\mu(v_{j_k}) = \frac{(2m-1)2^{k_m}}{10^k}$, where$(2m - 1)2^{k_m} \leq nandm = 2, k_m \geq 0$.

We observe that: $\frac{(2m-1)2^a}{10^k}$ divides $\frac{(2m-1)2^b}{10^k}(a < b)$ and $\frac{(2m-1)2^{k_i}}{10^k}$ does not divide $2m + 1$.

In the above labeling, we see that the consecutive adjacent vertices have the labels even numbers in numerator of fraction and consecutive adjacent vertices have labels odd and even numbers contribute d to each edge. Similarly, the consecutive adjacent vertices have the labels odd numbers in numerator of fraction and consecutive adjacent vertices have labels even and odd numbers contribute 0 to each edge. Thus, $e_\mu(d) = \frac{n}{2}$ and $e_\mu(0) = \frac{n-2}{2}$ if n is even.

And $e_\mu(d) = e_\mu(0) = \frac{n-1}{2}$ if n is odd.

Hence, $|e_\mu(0) - e_\mu(d)| \leq 1$. Thus, P_n is a fuzzy divisor cordial.

THEOREM 3.2.

The cycle C_n is fuzzy divisor cordial.

PROOF.

Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n . We follow the same labeling pattern as in the path, except by interchanging the labels of $v_1$and v_2 . Then it follows C_n is fuzzy divisor cordial.

THEOREM 3.3.

The wheel graph $W_n = K_1 + C_{n-1}$ is fuzzy divisor cordial.

PROOF.

Let v_1 be the central vertex and $v_2, v_3, \ldots, v_n$ be the verticesof C_{n-1}.

Case (i): n is odd.

Label the vertices $v_1, v_2, \ldots, v_n$ as in the labels of cycle $C_n$in the Theorem 2.2 , with the same order.

Case (ii): n is even

Label the vertices $v_1, v_2, \ldots, v_n$ as in the labels of cycle in the Theorem 2.2,with the same order except by interchanging the labels of the vertices v_2 and v_3 .

In both the cases, we see that $e_\mu(0) = e_\mu(d) = n - 1$.

Hence, $|e_\mu(0) - e_\mu(d)| = 0$. Thus, W_n is fuzzy divisor cordial.

THEOREM 3.4.

The star graph $K_{1,n}$ is fuzzy divisor cordial.

PROOF.

Let v be the central vertex and $v_1, v_2, \ldots, v_n$ be the end ver-tices of the star $K_{1,n}$. Now assign the label $\frac{2}{10}$ to the vertex, the label $\frac{1}{10}$ to the vertex v_1 and the vertices $v_2, \ldots, v_n$label as follows$\mu(v_i) = \frac{i+1}{10^i}$. Then we see that

$$e_\mu(0) - e_\mu(d) = \begin{cases} 0 \text{ if n is even} \\ 1 \text{ if n is odd} \end{cases}$$

Thus, $|e_\mu(0) - e_\mu(d)| \leq 1$ and hence $K_{1,n}$ is divisor cordial.

THEOREM 3.5.

The complete bipartite graph $K_{2,n}$ is fuzzy divisor cordial.

PROOF.

Let $V = V_1 \cup V_2$ be the bipartition of $K_{2,n}$ such that $V_1 = \{x_1, x_2\}$ and $V_2 = \{y_1, y_2, \ldots, y_n\}$. Now assign the label $\frac{1}{10}$ to x_1 and $\frac{p}{10^{\frac{p-1}{2}}}$ where p the largest prime number of numerator of fraction such that $p \leq n + 2$to x_2 , the label $\frac{i+1}{10^i}$ to the vertices $y_i, i = 1,2, \ldots, n$. Then it follows that $e_\mu(0) = e_\mu(d) = n$ and hence, $K_{2,n}$ is fuzzy divisor cordial.

THEOREM3.6.

The complete bipartite graph $K_{3,n}$ is fuzzy divisor cordial.

PROOF.

Let $V = V_1 \cup V_2$ be the bipartition of V such that $V_1 = \{x_1, x_2, x_3\}$ and $V_2 = \{y_1, y_2, \dots, y_n\}$. Now define $\mu(x_1) = \frac{1}{10}, \mu(x_2) = \frac{2}{10}, \mu(x_3) = \frac{p}{10^{\frac{p-1}{2}}}$

Where p is the largest prime number of numerator of fraction such that $p \leq n + 3$ and the label $\frac{i+2}{10^i}$ to the vertices $y_i, i = 1,2, \dots, n$. Then $e_\mu(0) - e_\mu(d) = \begin{cases} 0 \text{ if n is even} \\ 1 \text{ if n is odd} \end{cases}$

Thus $K_{3,n}$ is fuzzy divisor cordial.

THEOREM 3.7.

$S(K_{1,n})$, the subdivision of the star $K_{1,n}$, is fuzzy divisor cordial.

PROOF.

Let $V(S(K_{1,n})) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and let $E(S(K_{1,n})) = \{vv_i, v_iu_i : 1 \leq i \leq n\}$. Label v by $\frac{1}{10}$, and define μ as $\mu(v_i) = \frac{2i}{10^i}(1 \leq i \leq n)$ and $\mu(u_i) = \frac{2i+1}{10^i}(1 \leq i \leq n)$.

Then we have $e_\mu(0) = e_\mu(d) = n$, and therefore $S(K_{1,n})$ is fuzzy divisor cordial.

THEOREM 3.8.

The sell graph S_n is a fuzzy divisor cordial graph

For $n \geq 3$.

PROOF:

Let G be shell graph S_n. Let $v_1, v_2, \dots, v_n$be vertices of G with v_1 an apex vertex of G.Then $|V(G)| = n$ and $|E(G)| = 2n - 3$, label U_i by $\mu(v_i) = \frac{2i-1}{10^i}$ $\forall\, 1 \leq i \leq n$. Then we have $e_\mu(0) = n - 2$ and $e_\mu(d) = n - 1$.

Therefore $|e_\mu(0) - e_\mu(d)| \leq 1$.

Hence, S_n is fuzzy divisor cordial graph.

THEOREM 3.9.

The graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$ is a fuzzy divisor cordial graph.

PROOF:

Let G be a graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$. Let $v_1, v_2, \dots, v_n$ be pedant vertices of $K_{1,n}^{(1)}$ and $u_1, u_2, \dots, u_n$be pedant

vertices of $K_{1,n}^{(1)}$, and $K_{1,n}^{(2)}$ respectively, w the a common vertex adjacent to them. Then $|V(G)| = 2n + 3$, and $|E(G)| = 2n + 2$, lable the vertices by $\mu(u) = \frac{1}{10}$, $\mu(v) = \frac{p}{10^{\frac{p-1}{2}}}$ where p the largest prime number from 3 to $4n + 5$, and $\mu(v_i) = \frac{2i+1}{10^i}$, $\mu(w) = \frac{2n+3}{10^{\frac{p-1}{2}}}$ and $\mu(u_i) = \frac{2n+2i+3}{10^i}$. Here we get $e_\mu(d) = n + 1$ and $e_\mu(0) = n + 1$

And therefore $|e_\mu(0) - e_\mu(d)| = 0 < 1$.

Hence $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$ is a fuzzy divisor cordial graph.

THEOREM 3.10.

The Helm $H_n$is a Fuzzy divisor Cordial graph.

PROOF:

Let G be a HelmH_n. Let V be the apex vertices of degree 4 and the $u_1, u_2, \dots, u_n$ be the pendant vertices of H_n .

Then $|V(G)| = 2n + 1$, $|E(G)| = 3n$ Vertex labeling $\mu: V(G) \longrightarrow [0,1]$ define as follows:

$\mu(V) = \frac{1}{10}$.

Case (i). $n = 3$

$f(v_i) = \frac{2i+1}{10^i}$, $1 \leq i \leq n$ and $f(u_i) = \frac{2i+7}{10^i}$, $1 \leq i \leq n$.

Then $e_\mu(d) = 4$ and $e_\mu(0) = 5$. Therefore, $\left|e_\mu(0) - e_\mu(d)\right| \leq 1$

Case (ii). $n \geq 4$

Consider $\left\lfloor\frac{n}{4}\right\rfloor = k$ and $\left\lfloor\frac{n}{2}\right\rfloor = m$

$\mu(v_i) = \frac{2k+2i+1}{10^i}$, $\mu(u_i) = 3\mu(v_i)$, $1 \leq i \leq m$

Labels to the remaining vertices $v_j, v_{j+1}, \dots, v_n$ where $j = m + 1, m + 2, \dots, n$ and $u_j, u_{j+1}, \dots, u_n$

Where $j = m + 1, m + 2, \dots, n$ by $\mu\left(v_j\right) = \frac{2j+1}{10^j}$ such that $\mu(v_i) \nmid \mu(v_{i+1})$

Where $m \leq i \leq n - 1$ and $\mu(v_n) \nmid \mu(v_1)$, and $\mu(v_i) \nmid \mu(u_i) m + 1 \leq i \leq n$.

Hence we have $e_\mu(d) = \left\lfloor\frac{3n}{2}\right\rfloor$, and $e_\mu(0) = \left\lceil\frac{3n}{2}\right\rceil$.

Therefor$\left|e_\mu(0) - e_\mu(d)\right| \leq 1$.

Hence H_n is fuzzy divisor cordial.

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