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Young American's diet problem: A linear programing application

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ABSTRACT: This study employs linear programming to find a healthy diet at minimum cost to avoid overweight and obesity that affect 9–18-year-old girls in U.S. The data includes the most consumed food in USA based on the USDA ERS data base, and nutrients that are selected from the Dietary Reference Intake Vitamins and Elements list: Protein, Calcium, Iron, Vitamin A, and Vitamin C.

KEYWORDS: Diet-problem, linear programming, USA, cost minimization.

1 INTRODUCTION

The nutrition problem is one of the most recurrent problems in the world ranging from the overweight in the US to underfed mothers in the third world (thesis). By nature, an individual doesn't necessarily know how to balance his diet, so it is essential to coordinate foods and their nutrients for planning nutritionally adequate diets.

From the early 30's and 40's, economists has been fascinated by the Diet Problem since it is a direct application of the constraint cost minimization procedure that is taught to students at all levels, and many mathematical methods are available which could perform the necessary calculations to find the changes to foods. The "Diet Problem" has a long history, whereas most solutions for comparable diet problems were developed in 2000 or later when more performing computers became accessible (Dooren, 2018).

The linear programming formulation of the diet problem demonstrates both the feasibility and the economic meaning of the notion of an adequate diet (Fare and Karagiannis, 2014).

Adolescence is a unique period that represents immense biological and socio-emotional development alongside with increased autonomy, thus, encouraging a healthy diet increase the probability to extend this practice to adulthood (Keats et al., 2018). In the United States, based on the 2013–2014 National Health and Nutrition Examination Survey (NHANES), 17.2% of children and adolescents aged from 2- to 19-year-old are obese and another 16.2% suffer from overweight. Thus, this study uses linear programming (LP) (Dantzig, 1963; Ignizio, 1982; Chvatal,1983) to address the problem of finding a healthy diet at minimum cost to avoid overweight and obesity that affects 9-18 years American girls. The food data used for this study is collected from the most consumed food in USA based on the USDA ERS data base (USDA ERS), and the nutrients considered are calories and 9 nutrients and elements that were selected from the Dietary Reference Intake Vitamins and Elements list: Protein, Calcium, Iron, Vitamin A, and Vitamin C.

2 BACKGROUND

The diet problem is one of the first optimization problems to be studied back in the 30's and 40's. The primal goal is to meet nutritional requirements while minimizing the cost. Many studies emerged in this area, but one of the first ones was George Stigler's work in 1945 that posed the following problem:

"For a moderately active man (economist) weighing 154 pounds, how much of each of 77 foods should be eaten on a daily basis so that the man's intake of nine nutrients (including calories) will be at least equal to the recommended dietary allowances (RDAs) [1] suggested by the National Research Council in 1943, with the cost of the diet being minimal?".

Stigler based this study on calories, protein, calcium, iron, Vitamin A, thiamine, riboflavin, niacin, and ascorbic acid. Using linear programming with trial and error, the result was an optimal yearly diet cost of $39.93, that consists of 370 lbs. of wheat flour, 57 cans of evaporated milk, 111 lbs. of cabbage, 23LB of spinach and 285 lbs. of navy beans which responds to the nutritional requirements.

Almost a decade later, in 1947, his study was used by Jack Laderman from Mathematical Tables Project of the National Bureau of Standards to test George Dantzig's simplex algorithm for linear programming, which gave a result of $39.69, which is only 24 cents apart from Stigler's optimal solution (Dantzig, 1990).

Since then, many studies developed diet problems and treated them from different aspects that extend or duplicate Stigler's basic structure by using updated RDA's values, prices or recalculated content of foods (Garner and Gass,2001).

Such as Smith's diet problem (1963) which consists of a list of 73 foods that were commonly used by 176 families surveyed in the Lansing, Michigan area in 1955, subject to 13 constraints with bounds on fat and carbohydrate intake and upper bound on total calories. The recommended dietary allowances were set for a healthy and active family of two 45 year old adults and 18 year old daughter. His optimal solution was a daily cost of $0.336 per person that refers to an annual cost of $122.76 per person, comprising: milk (fresh, homogenized, plain), oleomargarine, fresh carrots, fresh potatoes, pork (picnic ham, cured butts), and flour (white, enriched).

We can also cite Beckmann's work (1960), that used same Stigler's nutrient amount but with updated RDA values to 1958, different food list and prices[2] and solved for both: a 45-year-old male, one at the 3000-calorie level and the other for 2200 calories. The optimal 3000 calories Beckmann diet included soybean meal, beef liver, lard, and frozen orange juice and had a daily cost of $0.216 per day or an annual cost of $78.99.

The literature is extensive, and we can cite many other diet-problem related studies such as Foytik (1981) who developed a least-cost diet using updated prices and 12 RDAs, Soden and Fletcher (1992), Hitomi (2015), and Darmon (2002).

3 DATA AND METHODOLOGY

3.1 DATA

Our study gathers items from the five food groups as described by "The 2015-2020 Dietary Guidelines for Americans":

(1) Fresh Vegies
(2) Fresh Fruits
(3) Dairy
(4) Grains
(5) Proteins

Each group includes a variety of foods that are similar in nutritional makeup, and each group plays a key role in an overall healthy eating pattern. This classification simplifies dietary recommendations by focusing on foods instead of nutrients so, when individuals eat the recommended amounts, they can meet their nutritional needs without having to track dozens of individual nutrients.

That is, to make our first data selection, we used the "Packer 2017 Fresh Trends" that is based on the percentage of primary shoppers buying in the last 12 months in the U.S., to choose the most popular fruits and vegetables purchased, and we used the 2010 USDA ERS Food Availability (Per Capita) Data System and the Agricultural Research Service National Agricultural Library to find the most consumed food from dairy, grains and proteins food groups, to which we assigned its retail price based

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[1] *Where RDA's are the levels of intake of essential nutrients that, on the basis of scientific knowledge, are judged by the Food and Nutrition board to be adequate to meet the known nutrient needs of practically all healthy persons" (National Research Council 1989).*
[2] Food prices as found in Providence, Rhode Island in the fall of 1959

on the USDA ERS 2013 Data[3], then, to simplify the calculation, we computed the average price of the items of each of the five food groups[4].

For the use of our second data and following Stigler's work, we selected from the Dietary Reference Intake (DRIs) list: Calories, Iron, Protein, Vitamin C, Vitamin A, and Calcium, to which we allocated the lower bounds that correspond to the 9-18 years old girls range (table 1).

We should remind that it is not necessary for the individual to meet these allowances on average over a 5-8 day-period (Food and Nutrition Board, 1980, p 36).

Table 1. Dietary Reference Intakes (DRIs): Recommended Dietary Allowances and Adequate Intakes, Elements for 9-18 years girls

Calcium (mg/d)	Calories (µg/d)	Vitamin A	Vitamin C (mg/d)	Iron (µg/d)	Protein
1,300	2350	700	65	15	40

3.2 METHODOLOGY

We used linear programming to find the minimum cost of the healthiest diet. The theory of linear programming is used to describe the interrelations of the components of a system and is concerned with scientific procedures for arriving at the best design, given the technology, the required specifications, and the stated objective, with respect to a certain assumptions of proportionality, nonnegativity, and additivity (Dantzig, 1968).

Before being used to solve Diet problems, this method has served industries and businesses by providing a novel view of operations, research in the mathematical analysis of the structure of industrial systems and being an important management tool to improve the efficiency of their operations (Dantzig,1968).

Practically, linear programming minimizes a linear function (objective function) given a set of constraints (Snjadha et al., 2011), and for diet problem purpose, the objective function is the food expenditure subject to the constraints which are the required daily allowance of nutrients.

Our study follows Stigler's Diet problem formulation process with the aim to find the minimal cost of a healthy diet using linear programming. The objective is to minimize per capita per day food expenditure subject to the constraints of daily intake of calories, proteins, iron, calcium, vitamin A, and vitamin C.

The basic linear programming diet problem model is given by *Min Cx,* subject to *AX*≥*b*, *X*≥0, where *c* is a vector of prices for the foods *X*, each column of the matrix *A* contains the nutrient content of the corresponding food, and the vector *b* is the set of lower bounds for the RDAs. Let's specify that we set these constraints with lower limits because these nutrients are well tolerated in amounts that exceed the allowances by as much as two to three times, and a substantial proportion of the population commonly consume an excess over the RDAs for several nutrients without evidence of adverse effects (Food and Nutrition Board, 1980, p30).

The formulation of the final linear programming for our model is given by:

$$Min\ Total\ Cost\ \{Z\} = \ p_i x_i \ (1)$$

subject to seven constraints:

1) Total Daily Calories requirements= $ax_i \ \geq g \ (where\ g = 2350\ kcal)$
2) Total Daily Proteins requirements= $bx_i \ \geq h \ (where\ h = 40\ mg)$
3) Total Daily Calcium requirements= $cx_i \ \geq i \ (where\ i = 1300\ mg)$
4) Total Daily Iron requirements= $dx_i \ \geq j \ (where\ j = 15\ mg)$
5) Total Daily Vitamin A requirements= $ex_i \ \geq k \ (where\ k = 700\ UI)$

[3] *http://supplementsos.com/nutrition-stats/most-consumed-foods/most-eaten-dairy-products-usa/*
Calculated by ERS, USDA from 2013 IRI Infoscan data; the USDA National Nutrient Database for Standard Reference, Release 26 (SR); and the 2009-2010 Food Patterns Equivalents Database (FPED) as well as the FPED's accompanying Methodology and User Guide.
[4] *The group classification of foods follows USDA food classification*

6) Total Daily Vitamin C requirements= $fx_i \geq l \ (where\ l = 65\ mg)$
7) And, $X_j \geq 0 \ (Non - negativity\ restriction)$,

where:

Z = Per capita per day food expenditure,
X_j = Decision variable, namely average per capita per day consumption of food group (j=1,…,5), such as: x_1 corresponds to vegetables, x_2 corresponds to fruits, x_3 corresponds to dairy, x_4 corresponds to grains, and x_5 corresponds to proteins.
p_i = Market price per unit per food item consumed,
a_{ij} = The i^{th} nutrient content pf the j^{th} food item,
b_i = Recommended dietary allowance of the i^{th} nutrient,
I = corresponds to the six nutrients constraints (iron, calories, protein, calcium, vitamin A, vitamin C),
j = 1, 2,…,5 food item.

The information for the *A* matrix were obtained from the weighted nutrients amount of each food group (Food and Nutrition Encyclopedia, pp.802-989). We are assuming two assumptions that under the *'A'* matrix: all the items were to be consumed by the individuals in the quantities indicated, and the data used in the consumptions were both relevant and reliable, therefore the *'A'* matrix should be generally applicable (Eckstein, 1983, p257).

Therefore, the matrices *A*, *X* and *b*, for our case are given by:

$$A(*) = \begin{bmatrix} 144.65 & 229.74 & 676.94 & 1171.43 & 804.22 \\ 7.38 & 3.6 & 21.78 & 29.3 & 104.14 \\ 97.92 & 50.86 & 21.77 & 59.44 & 78.09 \\ 61.94 & 7.47 & 9.45 & 0.03 & 5.26 \\ 98.91 & 60.38 & 1.7 & 0 & 0 \\ 2.70 & 1.19 & 0.79 & 4.69 & 5.35 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}; b = \begin{bmatrix} 2350 \\ 40 \\ 1300 \\ 15 \\ 700 \\ 65 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(*) The values in the matrix A are the average of nutrient amounts converted from milligram and microgram to pound to match the measurement unit of the food items.

4 RESULTS

In this study, we found that for a daily cost of 13$ (see output in table 2), a healthy diet that respects all the daily nutrients and calories intake could be attained. Moreover, this value means that with $13.22 per day, a young girl can be provided by 0.23 lbs. of vegies and 21 lbs. of grains from the larger choice that is offered by the vegetables and grains food lists (see tables 2,3,4,5, and 6).

Table 2. List of most popular vegies in US (x_1)

Item	Price/lb	Calories	Protein	Calcium	Vitamin A	Vitamin C	Iron
Potato	0.56	77	2	12	2	19.7	0.8
Tomato	1.24	18	0.9	10	833	13.7	0.3
Onion	1.04	40	1.1	23	2	7.4	0.2
Carrot	0.74	41	0.9	33	1670	5.9	0.3
Lettuce	1.21	15	1.4	36	7405	9.2	0.9
Broccoli	1.64	34	2.8	47	623	89.2	0.7
Bell Pepper	1.41	20	0.9	10	370	80.4	0.3
Celery	1.11	16	0.7	40	449	3.1	0.2
Cucumbers	1.3	16	0.7	16	105	2.8	0.3
Corn	2.69	365	9	7	0	0	2.7
Mushrooms	3.41	22	3.1	3	0	2.1	0.5
Sweet Potato	0.92	86	1.6	30	1418	2.4	0.6
Spinach	3.83	23	2.9	99	9377	28.1	2.7
Green Cabbage	0.58	25	1.3	40	98	36.6	0.5
Green Beans	2.14	31	1.8	37	108	16.3	1
Cauliflower	1.23	25	1.9	22	0	48.2	0.4
Asparagus	3.21	20	2.2	24	756	5.6	2.1
MEAN	1.409429	31.89231	1.626462	21.58877	1365.647059	21.805882	0.595749

Table 3. List of most popular fruits in The U.S. (x_2)

Item	Price/lb	Calories	Protein	Calcium	Vitamin A	Vitamin C	Iron
Banana	0.53	89	1.1	5	64	8.7	0.3
Apples	1.57	52	0.3	6	54	4.6	0.1
Grapes	2.09	67	0.6	14	100	4	0.3
Strawberries	2.36	33	0.7	16	12	58.8	0.4
Oranges	1.04	47	0.9	40	225	53.2	0.1
Watermelon	0.33	30	0.6	7	569	8.1	0.2
Blueberries	4.73	57	0.7	6	54	9.7	0.3
Peaches	1.59	39	0.9	6	326	6.6	0.3
Cantaloupe	0.54	34	0.8	9	3382	36.7	0.2
Avocados	2.24	160	2	12	146	10	0.6
Pineapple	0.63	50	0.5	13	58	47.8	0.3
Cherries	3.59	50	1	16	1283	10	0.3
Pears	1.46	57	0.4	9	25	4.3	0.2
Raspberries	6.98	53	1.2	25	33	26.2	0.7
Blackberries	5.77	43	1.4	29	214	21	0.6
Plums	1.83	46	0.7	6	345	9.5	0.2
Nectarine	1.76	44	1.1	6	332	5.4	0.3
Grapefruit	3.59	42	0.8	22	1150	31.2	0.1
Mean	1.732154	50.65213	0.793672	11.2139	164.7396	13.31268	0.262604

Table 4. List of most popular Dairy in The U.S. (x_3)

Item	Price/lb	Calories	Protein	Calcium	Vitamin A	Vitamin C	Iron
Fluid Milk	0.41	42	3.4	125	47	0	0
Cheese	3.1	402	25	721	1002	0	0.7
Ice Cream	0.84	207	3.5	128	421	0.6	0.1
Butter	2.48	717	0.9	24	2499	0	0
Cottage Cheese	1.98	98	11	83	140	0	0.1
Yogurt	1.01	59	10	110	4	0	0.1
Sour Cream	2.99	193	2.1	110	623	0.9	0.2
Eggnog	0.77	88	4.6	130	206	1.5	0.2
MEAN	0.980954	149.24	4.805764	117.4246	208.4225	0.375	0.175

Table 5. List of most popular grains in The U.S. (x_4)

Item	Price/lb	Calories	Protein	Calcium	Vitamin A	Vitamin C	Iron
Wheat Flour		364	10	15	0	0	1.2
Rice	0.697	130	2.7	10	0	0	0.2
White Flour	0.52	364	10	15	2	0	4.6
MEAN	0.60203	258.255	6.463304	13.10371	0.66	0	1.03353

Table 6. List of most popular proteins in The U.S. (x_5)

Item	Price/lb	Calories	Protein	Calcium	Vitamin A	Vitamin C	Iron
Chicken	1.43	165	31	15	21	0	1
Beef	4.083	250	26	18	0	0	2.6
Pork	3.155	145	21	8	0	0	0
Turkey	1.558	189	29	14	39	0	1.1
Eggs	2.808	155	13	50	520	0	1.2
MEAN	2.40578	177.3034	22.96038	17.21513	116	0	1.18

The optimal choice: 0.23 lbs. of vegies and 21 lbs. of grains with no amount of other food groups, is a consequence of considering the mean of retail price of each group, and the mean of the nutrients, that have been used in the objective function "*f*", the matrices "*A*" and "*b*" calculation to simplify the linear programming application despite the own price or amount value of each element. The average price measures only the central tendency in a data and it is well-known that it is very sensitive to extreme values, which explains our results of zeros in fruits, dairy and proteins groups, and also in the daily cost amount.

It is believable that a vegan diet could seem healthy but a diet of 21 lbs. of grains is not reasonable, and studies show that cereal-based diets that are low in animal products, vegetables, and fruits cannot meet the nutritional recommendations for children (Brown 1991). That is, to fulfil the daily nutrients and calories intake requirement, a diet needs to contain elements from each of the five food groups.

5 CONCLUSION

The results of this study showed that linear programming can be used to assist in the formulation of nutritional recommendations with the use of data from local food consumption surveys. The findings confirms that with a daily cost of $13, a healthy diet could be affordable for young American girls.

The applicability of linear programming depends on the validity of the nutritional constraints introduced into individual models ensuring that optimized diets meet the nutrient needs of most people in the population (Darmon N,2002). However, these constraints were based on assumptions that can be braved, such as: the population is healthy, nutrient requirements are independent, and the food-composition database is accurate.

The applicability and validity of linear programming in nutrition studies is determined by the availability of data for defining food consumption. In addition, for this study, we faced a lack of updated data and information of food consumption habits to ensure the palatability of the optimized diets, even though that the use of data may not be essential. For example, in Colombia, linear programming was used to identify a “food basket” providing a low-cost diet that supplied recommended energy and protein intakes for the average family where simple information on family food purchases was used to design this diet (Lareo, 1990). Similar pragmatic approaches could also be used, such as interviewing key informants or measuring the food consumption of a limited number of young girls from low-income families to quickly identify foods and the maximum quantities effectively eaten by young girls (Darmon, 2002).

By addressing young girls diet problem, this paper illustrates the linear programing method through one of its most famous application examples, that permits to represent a wide range of real-world situations in an easy, flexible and simple way. However, we believe that these optimal results can be improved by many ways such as: (1) each food element could be taken apart instead of considering the food group average, (2) evaluate the set of recommendations by using different sets of nutritional constraints based on recommendations from different committees to compare results, and (3) narrow the population targeted to a smaller age group.

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