Analysis of Voltage Stability using HOPF Bifurcation in Power System

Ruchika Khare¹ and Arunima Verma²

¹M.Tech Scholar, IET, Lucknow, India
²Associate Prof, IET, Lucknow, India

ABSTRACT: In this paper an attempt has been made for analysis of voltage stability using Hopf bifurcation in power system and a new formula for computing and loading of any power system is proposed with the help of using Bifurcation technique using VST Toolbox. Minimum Voltage of the feeders can also be maintained by allowing the feeders to take load growth up to a specific period of time. The critical values of total real power load (TPL) and total reactive power load (TQL) for constant power, constant current, constant impedance, composite as well as exponential load modeling is derived out for the substation voltage of power system. When any parameter changes, the stable Equilibrium points may lose its dynamic stability at the feasibility boundary, which is caused by one of three different local bifurcation the singularity induced bifurcation, saddle-node and Hopf bifurcation. After solving the Differential Algebraic Equation is calculated for different power condition using iterative process. Those results will help to find types of bifurcation, and after that to using MATLAB coding to analyzing HOPF bifurcation static and dynamic process with the help of two simple indices we detected oscillatory problems in power systems, based on the system state matrix as well as an augmented system matrix.

KEYWORDS: Voltage stability, Static voltage stability, Dynamic voltage stability, voltage collapse, VST Toolbox.

1 INTRODUCTION

Generation, Transmission and Distribution systems are the main components of an electric power system. Generating stations and distribution systems are connected through transmission lines. Normally, transmission line implies the bulk transfer of power by high-voltage links between main load centers. Electric power is generated in the range of 11 kV to 25 kV, which is increased by stepped up transformers to the main transmission voltage. The network formed by these very high voltage lines is sometimes called as the Super Grid. This grid, in turn, feeds a sub-transmission network operating at 132 kV or less. In our country, networks operate at 132 kV, 66 kV, 33 kV, 11 kV or 6.6 kV and supply the final consumer feeders at 400 volt three phase, giving 230 volt per phase. The distribution system objectives are distributing electric energy to various consumers, is measured in terms of voltage regulation, flexibility, security of supply efficiency and cost. But all this power distribution system some problems are introduced such as voltage collapse, blackout and voltage stability. Voltage stability analysis problems are removed from for analysis of voltage stability using Hopf bifurcation in power system and a new formula for computing and loading of any power system is proposed with the help of using Bifurcation technique using VST Toolbox.

2 VOLTAGE STABILITY

In the recent years, the analysis of voltage stability has assumed importance, mainly due to several documented incidents of voltage collapse in France, Japan, Belgium and Florida. Voltage stability is commonly analyzed by employing two techniques, namely time-domain (dynamic) simulation and steady-state analysis. Depending on the stability phenomenon or phenomena under investigation, one or both of these techniques may be applied. At any point of time, a power system operating condition should be stable, meeting various operational criteria, and it should also be secure in the event of any...
credible contingency. Present day power systems are being operated closer to their stability limits due to economic and environmental constraints. Maintaining a stable and secure operation of a power system is therefore a very important and challenging issue. Voltage instability has been given much attention by power system researchers and planners in recent years, and is being regarded as one of the major sources of power system insecurity. Voltage instability phenomena are the ones in which the receiving end voltage decreases well below its normal value and does not come back even after setting restoring mechanisms such as VAR compensators, or continues to oscillate for lack of damping against the disturbances. Voltage collapse is the process by which the voltage falls to a low, unacceptable value as a result of an avalanche of events accompanying voltage instability. Once associated with weak systems and long lines, voltage problems are now also a source of concern in highly developed networks as a result of heavier loading.

3 CLASSIFICATION OF VOLTAGE STABILITY

The time span of a disturbance in a power system, causing a potential voltage instability problem, can be classified into short-term and long-term. Automatic voltage regulators, excitation systems, turbine and governor dynamics fall in this short-term or ‘transient’ time scale, which is typically a few seconds. Induction motors, electronically operated loads and HVDC interconnections also fall in this category. If the system is stable, short-term disturbance dies out and the system enters a slow long-term dynamics.

![Figure 1 Classification of Voltage Stability](image)

4 VOLTAGE STABILITY ANALYSIS

Midterm stability programs that can simulate the system up to a few minutes can be utilized for this purpose provided that they can model the load characteristics accurately and also include the dynamics of OLTC and over excitation limiters. Load representation should include not only static loads which are voltage dependent but also dynamic loads such as...
induction motors and thermostatic loads. All the reactive compensation devices: switched shunt reactors, capacitors, SVC (with limiting action) need to be represented adequately. In addition, special protection schemes such as under voltage load shedding, OLTC blocking, reactor tripping and generator runback should be modeled. Voltage stability is commonly analyzed by employing two techniques, namely time-domain (dynamic) simulation and steady-state analysis. Depending on the stability phenomenon or phenomena under investigation, one or both of these techniques may be applied. For example, if steady-state analysis reveals that voltages at the buses at or near induction motor loads drop by more than 10% of their pre-disturbance value, time-domain (dynamic) analysis should be undertaken to assess the potential for motor stalling (steady-state analysis will not directly yield this information). This may involve extending the model to incorporate aggregate induction motor models at lower voltage buses as necessary. The two most common methods employed within the electric utility industry for analyzing power system stability are static and dynamic analysis.

## 5 Static Analysis

Static analysis (also referred to as load-flow or steady-state analysis) reveals equilibrium points of a system under study. The power flow equations employed in static analysis assume constant system frequency; in other words, generation output equals load demand plus losses. Voltage stability studies are frequently undertaken through the use of static analysis. A common use of this is the development of P-V curves. The graph is obtained in power-flow simulation by monitoring a voltage at a bus of interest and varying the power in small increments until power-flow divergence is encountered. Each equilibrium point shown represents a steady-state operating condition. In other words, each point may be considered as representing a system that has been in a stable operating point for over ½ hour. This means that the generation real-power dispatch and all voltage support equipment have been established such that the system meets the NERC/WECC reliability criteria for each operating point on the graph up to and including the operating limit point indicated on the graph. Beyond the operating limit, further increase in power may result in a breach of one or more of the WECC reliability criteria. A series of curves can be produced, each one as shown in Figure 2, with each curve depicting one or more transmission outages.

![Figure 2 Curve depicting Transmission Outage](image)

## 6 Dynamic Analysis

Dynamic analysis is commonly employed in the study of power system stability to reveal system trajectory after a disturbance. In contrast to static analysis in which equilibrium points of a P-V curve are not time-dependent, dynamic analysis method reveals the transient and/or the longer-term stability of a power system under study. The graph below adopted from the “Undervoltage Load Shedding Guidelines” document published by WECC in 1999 provides an example of dynamic analysis undertaken to study voltage stability. Voltage Stability Index the formulation of a voltage stability load index at a load bus using voltage equations. The technique uses measurements of voltage phasor and no-load voltage at the bus to calculate the voltage stability L-index. The complete mathematical derivation of the L-index is presented in the Appendix. The index gives the distance of the bus to the voltage stability limit. The voltage stability L-index is given by the equation: voltage phasor at the load bus for a given topology of the system. Since the no-load voltage is dependent upon the system topology and operating point, it varies as the system topology or operating point changes. In practice it is difficult to obtain no-load voltage at a bus. The proposed Multilayer Perceptron (MLP) approach capable of estimating the L-index without directly obtaining the no-load voltage overcomes this limitation of and facilitates on-line determination and use of L-index. The electric power grid is rapidly growing and demanding new technologies for efficient and rapid control in order to ensure reliable and secure power networks carries out a study of the impact of Plug-in Electric Vehicles (PEVs) parking lots (Smart
Parks) on the stability of a power system. In particular, the study shows voltage characteristics following changes in power demand of PEVs. When PEVs discharge into the power network, system voltage support is enhanced, while charging action is accompanied by voltage drop in the load area. The method for estimation of voltage stability L-index developed in this study is applied for monitoring voltage stability in a power system with Smart Parks include.

Figure 3 Impact of Increasing Load and LTC Operation

7 Bifurcation Analysis

In any system with the change in parameters there is some change in the dynamic behavior. Most of the time, these changes are only quantitative in nature. But there may also be situations, where a small change in parameters may result qualitative change in steady state behavior of a dynamic system. Such events are called Bifurcations. The number of attractors in a non-linear dynamic system can change when the system parameter is changed. This change is called Bifurcation. It is accomplished by a change of stability of an attractor. In a bifurcation point, at least one Eigen value of the Jacobian matrix gets a zero real part. It is defined as the sudden change in the behavior of the system as a parameter passes through a critical value called a bifurcation point.

Global Bifurcation- Global bifurcation often occurs when larger invariant sets of the system collide with each other or with equilibrium of the system. They cannot be detected purely by a stability analysis of the equilibrium.

Local Bifurcation- Local bifurcation can be analyzed entirely throughout changes in the local stability property of equilibrium, periodic orbits or other invariant sets as parameters cross through critical thresholds.

Figure 4 Bifurcation Process
Then further Local Bifurcation is divided into three different types –

a) **Saddle-Node Bifurcation** - A saddle-node bifurcation is a local bifurcation in which two fixed point (or equilibrium) of a dynamical system collide and annihilate each other.

b) **Hopf Bifurcation** - A Hopf bifurcation is a local bifurcation in which a fixed point of a dynamic system loses stability as a pair of the complex conjugate Eigen values of the linearization around the fixed point cross the imaginary axis of the plane.

c) **Singularity Induced Bifurcation** - It is a generic property of this encounter that one Eigen value of the linear stability mapping associated with the equilibrium will pass form one half of the complex plane to the other without passing through the imaginary axis. This is known as singularity induced bifurcation. While this property is generic it is shown how more than one Eigen value can diverge in an analogous manner, with applications in electrical power systems.

![Bifurcation graph](image)

**Figure 5 Bifurcation graph: (A) Hopf bifurcation, (B) Saddle node bifurcation, (C) Singularity induced bifurcation**

8 **HOPF BIFURCATION ANALYSIS**

A Hopf-bifurcation is a local bifurcation in which a fixed point of a dynamic system loses stability as a pair of the complex conjugate Eigen values of the linearization around the fixed point cross the imaginary axis of the complex plane. To develop Jacobian matrix J we use the following equation which is provided by the company. the necessary condition for HB is the existence of purely imaginary Eigen values. In this presents the application of the bifurcation analysis to study the behavior of active distribution systems and detects the critical points for unbalanced distribution systems. Some voltage collapse problems are not directly associated to bifurcations, but are generated by voltage control devices such as under-load tap changers or AVRs. Some early work on the Hopf bifurcation related instability of power systems was done. A fundamental platform for solving practical stability related problems in large constrained nonlinear systems such as power systems was developed in. In our work, we study power system stability related to Hopf bifurcations of the oscillatory modes involved in real power-angle dynamics. In the past, significant efforts have been made in analyzing the oscillatory instabilities. the coherent oscillations between weakly connected and geographically scattered clusters in a large power system. Oscillations are related to local instabilities of operating points of the system through a study of feasibility boundaries and sustained coherent inter-cluster oscillations are explained as Hopf bifurcations which occur at certain operating conditions.

If the Hopf bifurcation is supercritical, locally there exist stable limit cycles around the equilibrium. As illustrated on the branching diagram, an exchange of stability takes place between the periodic branch and the stationary branch. The amplitude of these periodic solutions, denoted by [x], grows continuously as the parameter value gets away from the bifurcation point. Thus, a soft loss of stability (soft generation of limit cycles) takes place through supercritical Hopf bifurcation. If the Hopf bifurcation is subcritical, unstable limit cycles encircle the equilibrium, however we can observe stable periodic solutions after the periodic branch turns back and gains stability globally at a turning point, a saddle-node bifurcation of periodic orbits, Thus, a hard loss of stability (hard generation of limit cycles) can be experienced due to a subcritical Hopf bifurcation. In order to study these phenomena in relation with inter cluster oscillations, we start our analysis with small examples of power system models and continue with systems involving power flow distribution (weakly connected) and coherency based clusters.
The methods of bifurcation theory can be effectively used to analyze various types of stability problems in power systems, such as voltage stability and collapse and oscillatory phenomena. This paper describes a new Matlab-based voltage stability toolbox (VST) that uses visualization capabilities of Matlab and integrates the symbolic and numeric computations to investigate voltage stability and bifurcation issues in power systems. VST is an open source simulation tool and is freely available at http://power.ece.drexel.edu, the website of the Center for Electric Power Engineering (CEPE) of Drexel University, Philadelphia. VST was first designed to have a computational tool that supports the ongoing research on the voltage stability analysis at CEPE. Table 1 gives a comparison of the currently available Matlab-based tools for power system analysis and VST. The features illustrated in the table are: load flow (LF), voltage stability analysis (VSA), small-signal stability analysis (SSA),

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SOFTWARE CONFIGURATION

![Figure 7 Steps in Software Configuration](image)

The main features and application modules of VST can be summarized as follows –

- Use of Matlab’s visualization capability to create graphical user interface and visualize output data.
- Use of stand-alone MEX-files to generate classical power system model equations.
- Use of symbolic toolbox to generate Jacobian and second order derivative matrices.
- Voltage stability analysis: identification of local static and dynamic bifurcation points, such as SN, Hopf, and SI bifurcations.
- Small-signal stability analysis.
- Dynamic (time-domain) simulations.

Power System Stability and Control, a graduate course, mainly focuses on bifurcations and voltage stability, small-signal stability, transient stability, and control of active and reactive power. For this course, the bifurcation analysis and TD simulation applications of VST are used for class demonstration to illustrate to students the concept of multiple equilibria, loss of equilibria, small-signal stability features of the equilibrium, and local bifurcations. The PV curves for a load increase pattern as illustrated in combines the above concepts into a single graph, helping students easily graph the idea. During the class, students are asked to choose a load increase scenario for the IEEE 118-bus system to observe the change of equilibrium and stability with respect to the load increase to determine local bifurcations. They are also asked to run the TD simulation around stable and unstable operating points along the PV curve to investigate the dynamic response of the system. The graduate students are given several assignments for which the capabilities of VST could be used. The typical assignments are as follows –
Find a load/generation increase pattern so that the entire lower voltage part of the PV curve is unstable (i.e., only the SN bifurcation is observed).

Using the right and left eigenvectors information at the SN bifurcation point, determine the buses that are most sensitive to loading and find remedial action information useful for corrective control.

Analyze the participation factors to determine which components (state variables) are most involved in the various system modes (Eigen values).

Initialize TD simulations around the Hopf bifurcation point and determine the frequency of the oscillations.

**Figure 8** (a) GUI for dynamic simulations, (b) Simulation result for generator 2.

Voltage profile of IEEE 14-bus system: (a) Voltage magnitude and (b) phase angle.

**Figure 9** Results (a) PV curves whose lower part is unstable. (b) Hopf bifurcation.

### 10 Conclusion and Future Work

For control of static voltage stability, the shunt capacitor and tap-changer were used. From the simulation result, it was shown that the control of the shunt capacitor and the tap-changer can increase the stability margin for static voltage stability and the increase of the stability margin improves the overall voltage profile of the power system, preventing a severe voltage decline. It is shown that the static voltage stability criterion developed in this paper is a useful tool in preventing a severe voltage drop. A voltage stability simulation tool in Matlab is presented in this paper. The developed VST is the outcome of the ongoing research conducted at Drexel University on bifurcation theory and its implementation into voltage stability analysis.
VST integrates numeric and symbolic computations with powerful graphical interfaces for load flow, small-signal and transient stability, and bifurcation analysis. Simulation results show that it is a powerful and promising tool for voltage stability studies, and very helpful to understand voltage stability phenomena. Based on its modular programming structure, other element models, such as nonlinear dynamic load models and more sophisticated generator models, can be easily included in the toolbox, which will definitely improve its usage. Future work will concentrate on such issues.

**References**

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