An accurate method to study the Rayleigh-Bénard problem in a rotating layer saturated by a Newtonian nanofluid

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1 INTRODUCTION

The nanofluid is considered as a homogeneous fluid containing colloidal suspensions of nano-sized particles named nanoparticles in the base fluid (water, ethylene glycol, oil). The nanoparticles used in nanofluids are generally prepared of metals, oxides, carbides, or carbon nanotubes. The purpose of using nanofluids is to obtain a higher value of heat transfer coefficient compared with that of the base fluid, this remarkable properties make them potentially useful in many practical applications, for example in modern science and engineering including rotating machineries like nuclear reactors, petroleum industry, biochemical and geophysical problems.

In the recent years, the problem of natural convection in a confined medium filled of a Newtonian nanofluid layer has been studied in different situations by several authors [1-7]. When the volumetric fraction of nanoparticles is constant at the horizontal walls limiting the layer, they found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on the relative distribution of nanoparticles between the top and bottom walls.

Today, the problem of natural convection for the nanofluids is studied by some authors [9-14] using a new type of boundary conditions for the nanoparticles which combines the contribution of the Brownian motion and the thermophoresis of nanoparticles instead to impose a nanoparticle volume fraction at the boundaries of the layer. The new model of boundary conditions assumes that the nanoparticle flux must be zero on the impermeable boundaries. D.A. Nield and A.V. Kuznetsov [9] are considered as the first ones who were used this type of boundary conditions for the nanoparticles. Until now, the precedent boundary conditions are used to study only the problem of natural convection in a porous (Darcy or Darcy-Brinkman model) or non-porous medium saturated by a nanofluid using the Galerkin weighted residuals method based only on some test functions.

Our work consists of studying the Rayleigh-Bénard problem in a rotating medium filled of a Newtonian nanofluid layer in the free-free, rigid-free and rigid-rigid cases where the nanoparticle flux is assumed to be zero on the boundaries, our problem will be solved with a more accurate numerical method based on analytic approximations (power series method).
In this investigation we assume that the effect of the rotation in the momentum equation is restricted to the Coriolis force and also the centrifugal acceleration is negligible compared to the buoyancy force.

The used method gives results with an absolute error of the order of $10^{-6}$ to the critical values characterizing the onset of the convection. To show the accuracy of our method in this study, we will check some results treated by Chandrasekhar [8] concerning the study of the convective instability of the regular fluids in a rotating medium.

2  MATHEMATICAL FORMULATION

We consider an infinite horizontal layer of an incompressible Newtonian nanofluid characterized by a low concentration of nanoparticles, heated uniformly from below and confined between two identical horizontal surfaces where the temperature is constant and the nanoparticle flux is zero on the boundaries, this layer will be subjected to a uniform rotation characterized by an angular velocity $\Omega = \Omega \hat{e}_z$ and also acted upon by the gravity force $g = -\hat{g} \hat{e}_z$ (Fig 1). The thermophysical properties of nanofluid (viscosity, thermal conductivity, specific heat) are assumed constant in the vicinity of the temperature of the cold wall $T_c$ except for the density variation in the momentum equation which is based on the Boussinesq approximations. The asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks).

\[
\vec{V}^*, \vec{T}^* = 0
\]

\[
\rho_0 \left[ \frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^* \cdot \vec{V}^*) \vec{V}^* \right] = -\vec{V}^* p^* - 2 \rho_0 \Omega \times \vec{V}^* + \left\{ \rho_0 [1 - \beta(T^* - T_c)](1 - \chi^*) + \rho_0 \chi^* \right\} \vec{g} + \eta \nabla^2 \vec{V}^*
\]

\[
(\rho_c) \left[ \frac{\partial \vec{T}^*}{\partial t^*} + (\vec{V}^* \cdot \vec{V}^*) \vec{T}^* \right] = k \nabla^2 T^* + (\rho_c) \left[ D_b \nabla^2 \chi^* \vec{V}^* T^* + \frac{D_T}{T_c} \vec{V}^* \vec{T}^* \nabla^2 \vec{T}^* \right]
\]

\[
\frac{\partial \chi^*}{\partial t^*} + (\vec{V}^* \cdot \vec{V}^*) \chi^* = D_b \nabla^2 \chi^* + \frac{D_T}{T_c} \vec{V}^* \nabla^2 T^*
\]

Where $\vec{V}^*$ is the vector differential operator.

If we consider the following dimensionless variables:

\[
(x^*; y^*; z^*) = h(x; y; z) ; \quad t^* = \frac{h^2}{a} ; \quad \vec{V}^* = \frac{a}{h} \vec{V} ; \quad \vec{P}^* = \frac{h^3}{a} P ; \quad T^* - T_c = (T_h - T_c) T ; \quad \chi^* - \chi_0^* = \chi_0 \chi
\]

Then, we can get from the equations (1)-(4) the following adimensional forms:

\[
\vec{V} \cdot \vec{V} = 0
\]

\[
P_r^{-1} \left[ \frac{\partial \vec{P}}{\partial t} + (\vec{V} \cdot \vec{P}) \vec{V} \right] = -\vec{V} (P + R_M \chi) + \sqrt{T_c A(x \vec{e}_x - \vec{w} \vec{e}_y)} + \nabla^2 \vec{V} + [(1 - \chi_0) R_s T - R_N \chi - \chi_0 R_s T \chi] \vec{e}_z
\]
\[
\frac{\partial T}{\partial t} + (\bar{V} \cdot \nabla)T = \bar{V}^2T + N_B L_e^{-1} \nabla \chi \cdot \bar{V} + N_A N_B L_e^{-1} \nabla \bar{V} \cdot \bar{V}
\]

(7)

\[
\frac{\partial \chi}{\partial t} + (\bar{V} \cdot \nabla)\chi = L_e^{-1} \chi^2 \bar{V} + N_A N_B L_e^{-1} \bar{V}^2 T
\]

(8)

Such that:

\[
P_f = \frac{\eta}{\rho_0 \alpha} ; \quad L_e = \frac{\alpha}{D_B} ; \quad N_B = \frac{(\rho c)_p}{(p c)} \chi_0^* ; \quad T_A = \left( \frac{2 \rho c \Omega h^2}{\eta} \right)^2 ; \quad \alpha = \frac{\kappa}{(p c)} ; \quad R_a = \frac{\rho_R g \Omega h^3 (T_h - T_c)}{\eta \alpha}
\]

\[
R_M = \left[ \frac{\rho_p (1 - \chi_0^*) + \rho_B \chi_0^*}{\eta \alpha} \right] ; \quad R_N = \left( \frac{\rho_p - \rho_0}{\rho_0} \right) \chi_0^* g h^3 \eta \alpha ; \quad N_A = \frac{D_T}{D_B T_c} \left( \frac{T_h - T_c}{\chi_0^*} \right)
\]

Where \( \chi_0^* \) is the reference value for nanoparticle volume fraction.

### 2.1 Basic Solution

The basic solution of our problem is a quiescent thermal equilibrium state, it’s assumed to be independent of time where the equilibrium variables are varying in the z-direction, therefore:

\[
\bar{V}_b = 0
\]

(9)

\[
T_b = 1 ; \quad \frac{d \chi_b}{dz} + N_A \frac{d T_b}{dz} = 0 \quad \text{at} \quad z = 0
\]

(10)

\[
T_b = 0 ; \quad \frac{d \chi_b}{dz} + N_A \frac{d T_b}{dz} = 0 \quad \text{at} \quad z = 1
\]

(11)

If we introduce the precedent results into equations (6)-(8), we obtain:

\[
\bar{V}(\rho_b + R_M \Delta V) = [(1 - \chi_0^*) R_A T - R_N \chi_b^* R_A T \chi_b] \bar{e}_z
\]

(12)

\[
\frac{d^2 T_b}{dz^2} + N_B L_e^{-1} \left( \frac{d \chi_b}{dz} \frac{d T_b}{dz} \right) + N_A N_B L_e^{-1} \left( \frac{d \chi_b}{dz} \right)^2 = 0
\]

(13)

\[
\frac{d^2 \chi_b}{dz^2} + N_A \frac{d^2 T_b}{dz^2} = 0
\]

(14)

After using the boundary conditions (10) and (11), we can integrate the equation (14) between 0 and z for obtaining:

\[
\chi_b = N_A (1 - T_b) + \chi_0
\]

(15)

Where \( \chi_0 \) is the relative nanoparticle volume fraction at \( z = 0 \), such that:

\[
\chi_0 = \frac{\chi_0^* (0) - \chi_0^*}{\chi_0^*}
\]

If we take into account the expression (15), we can get after simplification of the equation (13):

\[
\frac{d^2 T_b}{dz^2} = 0
\]

(16)

Finally, we obtain after an integrating of the equation (16) between 0 and z:

\[
T_b = 1 - z
\]

(17)

\[
\chi_b = N_A z + \chi_0
\]

(18)

### 2.2 Stability Analysis

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows:

\[
T = T_b + T' ; \quad \bar{V} = \bar{V}_b + \bar{V}' ; \quad P = P_b + P' ; \quad \chi = \chi_b + \chi'
\]

(19)
In the framework of the Oberbeck-Boussinesq approximations, we can neglect the terms which are coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (6), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (19) into equations (5)-(8) the following linearized equations:

\[ \nabla \cdot \mathbf{V}' = 0 \]  
\[ P_r^{-1} \frac{\partial \mathbf{V}'}{\partial t} = -\mathbf{V}' + \nabla T' + (R_A T' - R_N N_L) \nabla x' + \nabla^2 \mathbf{V}' \]  
\[ \frac{\partial T'}{\partial t} - w' = \nabla^2 T' - N_A N_b L_e^{-1} \frac{\partial T'}{\partial z} - N_b k_e^{-1} \frac{\partial \chi'}{\partial z} \]  
\[ \frac{\partial \chi'}{\partial t} + N_A w' = N_A L_e^{-1} \nabla^2 T' + L_e^{-1} \nabla^2 \chi' \]

After application of the curl operator twice to the equation (21) and using the equation (20), we obtain the following equations:

\[ P_r^{-1} \frac{\partial F'}{\partial t} = \nabla^2 F' + \nabla \cdot \mathbf{V}' \]  
\[ P_r^{-1} \frac{\partial}{\partial t} \nabla^2 w' = \nabla^4 w' + R_A \nabla^2 T' - R_N \nabla^2 \chi' - \nabla \cdot \mathbf{V}' \]

Where:

\[ \nabla^2 = \left( \frac{\partial^2}{\partial x^2} \right) + \left( \frac{\partial^2}{\partial y^2} \right); \quad F' = \left( \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right) \]

Analyzing the disturbances into normal modes, we can simplify the equations (22) - (25) by assuming that the perturbation quantities are of the form:

\[ (w', T', \chi', F') = (w(z), T(z), \chi(z), F(z)) \exp[i(k_x x + k_y y) + \alpha t] \]

After introducing the expressions (26) into equations (22) - (25), we obtain:

\[ P_r^{-1} \alpha F' = (D^2 - k^2) F' + \nabla \cdot \mathbf{V}' \]  
\[ P_r^{-1} \nabla (D^2 - k^2) w' = (D^2 - k^2)^2 w' - k^2 R_e T' + k^2 R_b \chi - \sqrt{T_A} D F' \]  
\[ \alpha T' - w' = (D^2 - k^2) T' - N_A N_b L_e^{-1} D T' - N_b k_e^{-1} D \chi \]  
\[ \alpha \chi' + N_A w' = N_A L_e^{-1} (D^2 - k^2) T' + L_e^{-1} (D^2 - k^2) \chi' \]

Where:

\[ k = \sqrt{k_x^2 + k_y^2} \quad ; \quad D = \frac{d}{dz} \]

The equations (27) - (30) will be solved subject to the following boundary conditions:

- For the rigid-rigid case:
  \[ \omega = D \omega = T = D(\chi + N_A F) = F = 0 \quad \text{at} \quad z = 0 ; 1 \]  

- For the free-free case:
  \[ \omega = D^2 \omega = T = D(\chi + N_A F) = DF = 0 \quad \text{at} \quad z = 0 ; 1 \]  

- For the rigid-free case:
  \[ \omega = D \omega = T = D(\chi + N_A F) = F = 0 \quad \text{at} \quad z = 0 \]  
  \[ \omega = D^2 \omega = T = D(\chi + N_A F) = DF = 0 \quad \text{at} \quad z = 1 \]
2.3 Method of Solution

In this study we assume that the principle of exchange of stability is valid, as we are interested in a stationary stability study ($\sigma = 0$), then the equations (27)-(30) become:

\[
\sqrt{T_A}D\nu + (D^2 - k^2)\mathcal{F} = 0
\]  
\[
(D^2 - k^2)^2\nu - k^2R_\nu T + k^2R_N\chi - \sqrt{T_A}D\mathcal{F} = 0
\]  
\[
\nu + (D^2 - k^2)T - N_A N_B L_e^{-1}D\mathcal{T} - N_B L_e^{-1}D\chi = 0
\]  
\[
N_A\nu - N_A L_e^{-1}(D^2 - k^2)\mathcal{T} - L_e^{-1}(D^2 - k^2)\chi = 0
\]

We can solve the equations (35)-(38) which are subjected to the conditions (31)-(34), by making a suitable change of variables that makes the number of variables equal to the number of boundary conditions to obtain a set of ten first order ordinary differential equations which we can write it in the following form:

\[
\frac{d}{dz}u_i(z) = a_{ij}u_j(z) ; \quad 1 \leq i, j \leq 10
\]  

With:

\[a_{ij} = a_{ij}(k, R_\nu, T_A, N_B, L_e, R_N, N_A)\]

The solution of the system (39) in matrix notation can be written as follows:

\[U = BC\]  

Where:

\[B = \left( b_{ij}(z) \right)_{1\leq i, j \leq 10} \]  

\[U = \left( u_i(z) \right)_{1\leq i \leq 10}^T \]  

\[C = \left( c_j \right)_{1\leq j \leq 10}^T \]

If we assume that the matrix $B$ is written in the following form:

\[B = \left( u_i^j(z) \right)_{1\leq i, j \leq 10} \]

Therefore, the use of five boundary conditions at $z = 0$, allows us to write each variable $u_i(z)$ as a linear combination for five functions $u_i^j(z)$, such that:

\[b_{ij}(0) = u_i^j(0) = \delta_{ij}\]  

Where $\delta_{ij}$ is the Kronecker delta symbol.

After introducing the new expressions of the variables $u_i(z)$ in the system (39), we will obtain the following equations:

\[
\frac{d}{dz}u_i^j(z) = a_{il}u_l^j(z) ; \quad 1 \leq i, l, j \leq 10
\]

For each value of $j$, we must solve a set of ten first order ordinary differential equations which are subjected to the initial conditions (42), by approaching the variables $u_i^j(z)$ with power series defined in the interval $[0,1]$ and truncated at the order $N$, such that:

\[u_i^j(z) = \sum_{p=0}^N d_{ip}^j z^p\]
A linear combination of the solutions $\mathbf{u}(z)$ satisfying the boundary conditions (31), (32) or (33) and (34) at $z = 1$ leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_a,k,T_A,N_B,L_e,N_R,N_A) = 0$$ (45)

If we give to each control parameter $(T_A, N_B, L_e, N_R, N_A)$ its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number $R_a$ which corresponds to a fixed wave number $k$ and verifies the dispersion relation (45). After that, we will find a set of points $(k, R_a)$ which help us to plot our curve and find the critical value $(k_c, R_{ac})$ which characterizes the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

### 2.4 Validation of the Method

The main aim in this investigation is to study the influence of a uniform rotation on the convective instability of the Newtonian nanofluids in a confined medium filled of a Newtonian nanofluid layer for different cases of boundary conditions: free-free, rigid-free and rigid-rigid cases. Our study shows that the thermal stability of Newtonian nanofluids depends on five parameters: $T_A, N_B, L_e, N_A$ and $N_R$.

The truncation order $N$ which corresponds to the convergence of our method is determined, when the five digits after the comma of the critical thermal Rayleigh number $R_{ac}$ for a Newtonian nanofluid ($N_B = 0.01, L_e = 100, N_R = 1, N_A = 0.1$) remain unchanged (Tables 1 and 3).

To validate our method, we compared our results with those obtained by Chandrasekhar [8] concerning the Rayleigh-Bénard problem in a rotating medium filled of a regular fluid layer (Tables 4 and 5). To make this careful comparison, we must take into consideration the restrictions $L_e^{-1} = N_R = N_A = N_B = 0$ in the governing equations of our problem.

<table>
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<tr>
<th>N</th>
<th>$T_A = 100$</th>
<th>$T_A = 400$</th>
<th>$T_A = 1000$</th>
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### Table 2. The stationary instability threshold of the Newtonian nanofluid for different values of the Taylor number in the rigid-free case

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**Exact value**  
2.84015  1188.87685  3.19644  1454.44486  3.62144  1866.87873

### Table 3. The stationary instability threshold of the Newtonian nanofluid for different values of the Taylor number in the rigid-rigid case

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**Exact value**  
3.15251  1740.79627  3.27497  1880.55963  3.47890  2136.89453
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<p>| Table 4. The comparison of critical values of Rayleigh number and the corresponding wave number with Chandrasekhar [8] for different values of the Taylor number in the free-free and rigid-rigid cases |</p>
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<th>T&lt;sub&gt;A&lt;/sub&gt;</th>
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<tr>
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<td>1275</td>
<td>2.27756</td>
<td>1274.56710</td>
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<tr>
<td>1000</td>
<td>3.710</td>
<td>1676</td>
<td>3.71043</td>
<td>1676.11802</td>
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</table>

<p>| Table 5. The comparison of critical values of Rayleigh number and the corresponding wave number with Chandrasekhar [8] for different values of the Taylor number in the rigid-free case |</p>
<table>
<thead>
<tr>
<th>T&lt;sub&gt;A&lt;/sub&gt;</th>
<th>Chandrasekhar rigid-free case</th>
<th>Present study rigid-free case</th>
</tr>
</thead>
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<tr>
<td></td>
<td>k&lt;sub&gt;c&lt;/sub&gt;</td>
<td>R&lt;sub&gt;ac&lt;/sub&gt;</td>
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<td>187.5</td>
<td>2.975</td>
<td>1291.7</td>
</tr>
</tbody>
</table>

According to the above results, we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. Briefly, the convergence of the results depends greatly on the truncation order N of the power series and also of the Taylor number T<sub>A</sub>. Finally, to ensure the accuracy of our obtained critical values for the studied nanofluids, we will take as truncation order:

- N = 22 : for the free-free case.
- N = 27 : for the rigid-free case.
- N = 30 : for the rigid-rigid case.

3 RESULTS AND DISCUSSION

To study the effect of a parameter (T<sub>A</sub>, N<sub>B</sub>, L<sub>e</sub>, R<sub>N</sub>, N<sub>A</sub>) on the onset of the convective instability in a rotating medium filled of a Newtonian nanofluid layer, we must plot in Figs 2-5 the variation of the critical thermal Rayleigh number R<sub>ac</sub> as a function of the Taylor number T<sub>A</sub> for different values of this parameter and compare the obtained results with those of the regular fluids which are characterized by:

\[ L_{e}^{-1} = R_{N} = N_{A} = N_{B} = 0 \]

Generally the variation in the critical thermal Rayleigh number R<sub>ac</sub> with the Taylor number T<sub>A</sub> is an increasing function whatever the value taken for the parameters N<sub>B</sub>, L<sub>e</sub>, R<sub>N</sub> and N<sub>A</sub>, so the presence of the Coriolis forces allows us to reduce the effect of the buoyancy forces, hence the Taylor number T<sub>A</sub> has a stabilizing effect. The precedent figures confirm that the presence of the friction on the horizontal walls is a factor producing the thermal stability of the system, such that:

\[ R_{ac}^{ff} > R_{ac}^{rf} > R_{ac}^{ff} \]
**Fig. 2.** Plot of $R_{ac}$ as a function of $T_A$ for different values of $N_B$ in the case where $L_e=100$, $R_N=1$, and $N_A=0.1$

**Fig. 3.** Plot of $R_{ac}$ as a function of $T_A$ for different values of $L_e$ in the case where $N_B=0.01$, $R_N=1$, and $N_A=0.1$
An accurate method to study the Rayleigh-Bénard problem in a rotating layer saturated by a Newtonian nanofluid

Fig. 4. Plot of $R_{ac}$ as a function of $T_A$ for different values of $R_N$ in the case where $N_B=0.01$, $L_e=100$ and $N_A=0.1$

Fig. 5. Plot of $R_{ac}$ as a function of $T_A$ for different values of $N_A$ in the case where $N_B=0.01$, $L_e=100$ and $R_N=1$
The Fig 2 shows that the modified particle-density increment $N_B$ has almost no effect on the convective instability of the nanofluids, this result may be explained by its low value ($N_B \sim 10^{-3} - 10^{-1}$) which appears only in the perturbed energy equation (22) as a product with the inverse of the Lewis number ($L_e \sim 10^2 - 10^3$) near the temperature gradient and the volume fraction gradient of nanoparticles, so the effect of this parameter on the onset of convection in nanofluids will be very small which we can neglect it.

From the Figs 3 and 4 we conclude that an increase either in the Lewis number $L_e$ or in the concentration Rayleigh number $R_N$ allows us to accelerate the onset of the convection, hence they have a destabilizing effect. Therefore, to ensure the stability of the system, we can use the nanofluids which are having a less thermal diffusivity or containing less dense nanoparticles.

In this investigation, we find also that an increase in the volume fraction of nanoparticles destabilizes the nanofluids, because an increase in this parameter, increases also the Brownian motion and the thermophoresis of nanoparticles, which cause the destabilizing effect, this result confirm that the regular fluids are more stable than the nanofluids.

When the modified diffusivity ratio $N_A$ increases, the temperature difference between the horizontal plates also increases. The Fig 5 shows that an increase in the modified diffusivity ratio $N_A$ allows us to decrease the critical thermal Rayleigh number $R_{ac}$, this result can be explained by the increase in the buoyancy forces which destabilizes the system.

4 Conclusions

In this paper, we have examined the effect of a uniform rotation on the onset of convection in a confined medium filled of a Newtonian nanofluid layer, heated uniformly from below and cooled from above for free-free, rigid-rigid and rigid-free boundaries in the case where the nanoparticle flux is zero on the boundaries. The contribution of the Brownian motion and the thermophoresis of nanoparticles in the equation expressing the buoyancy effect coupled with the conservation of nanoparticles have a major effect on the onset of convection compared with their contributions in the thermal energy equation.

The resulting eigenvalue problem is solved analytically and numerically using the power series method. The behavior of various parameters like the Taylor number $T_A$, the modified particle-density increment $N_B$, the Lewis number $L_e$, the concentration Rayleigh number $R_N$ and the modified diffusivity ratio $N_A$ on the onset of convection has been analysed. The results can be summarized as follows:

I. The presence of the Coriolis forces allows us to stabilize the Newtonian nanofluids, such that an increase in the Taylor number $T_A$ induces also an increase in the critical thermal Rayleigh number $R_{ac}$. 

II. The presence of the friction on the horizontal walls is a factor producing the thermal stability of the system, where the rigid-rigid case is the more stable case compared with the rigid-free and free-free cases, such that:

$$R_{ac}^{ff} > R_{ac}^{rf} > R_{ac}^{ff}$$

III. To ensure the stability of the system, we can use the nanofluids which are having a less thermal diffusivity, a low concentration of nanoparticles or consisting of less dense nanoparticles.

IV. An increase either in the volume fraction of nanoparticles, in the buoyancy forces, in the Brownian motion or in the thermophoresis of nanoparticles allows us to destabilize the nanofluids.

V. The regular fluids are more stable than the nanofluids.

VI. The used method to solve the convection problem gives more accurate results, because the absolute error of the obtained critical values which characterize the onset of the convection is of the order of $10^{-6}$, Hence, we can used our results as a reference to validate other results of the similar problems.
NOMENCLATURE

Symbols :
- $D_B$ Brownian diffusion coefficient (m$^2$/s)
- $D_T$ Thermophoretic diffusion coefficient (m$^2$/s)
- $g$ Acceleration due to gravity (m/s$^2$)
- $h$ Layer depth (m)
- $\kappa$ Thermal conductivity of Nanofluid (W/K m)
- $k_x^*$ Wave number in $x^*$ direction (m$^{-1}$)
- $k_y^*$ Wave number in $y^*$ direction (m$^{-1}$)
- $k_c^*$ Critical wave number (m$^{-1}$)
- $L_e$ Lewis number
- $N_A$ Modified diffusivity ratio
- $N_B$ Modified particle-density increment
- $P^*$ Pressure (Pa)
- $Pr$ Prandtl number
- $Ra$ Thermal Rayleigh number
- $R_{ac}$ Critical Rayleigh number
- $R_M$ Density Rayleigh number
- $R_N$ Concentration Rayleigh number
- $\vec{V}^*$ Velocity vector (m/s)
- $T_A$ Taylor number
- $T^*$ Temperature (K)
- $t^*$ Time (s)
- $u^*, v^*, w^*$ Velocity components (m/s)
- $x^*, y^*, z^*$ Cartesian coordinates (m)

Greek symbols :
- $\alpha$ Thermal diffusivity of nanofluid (m$^2$/s)
- $\beta$ Thermal expansion coefficient of base fluid (K$^{-1}$)
- $\Omega$ Angular velocity (rad. s$^{-1}$)
- $\eta$ Viscosity of nanofluid (Pa.s)
- $\rho$ Nanofluid density (kg/m$^3$)
- $\rho_0$ Fluid density at reference temperature (kg/m$^3$)
- $(\rho c_p)$ Heat capacity of nanofluid (J/m$^3$. K)
- $(\rho c_p)_p$ Heat capacity of nanoparticles (J/m$^3$. K)
- $\sigma^*$ Growth rate of disturbances (s$^{-1}$)
- $\chi^*$ Volume fraction of nanoparticles

Superscripts :
- * Dimensional variable
- ' Perturbation variable
- ff Free - Free case
- rf Rigid - Free case
- rr Rigid - Rigid case

Subscripts :
- c Cold
- h Hot
- ac Critical number
- b Basic solution
- f Base fluid
- p Nanoparticle
REFERENCES