International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 32 No. 2 Sep. 2017, pp. 251-264

http://www.ijisr.issr-journals.org/

Reduced Order Model Based GPC for a Binary Distillation Column

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ABSTRACT: In this paper, an optimal Generalized Predictive Control (GPC) is developed for binary distillation column using first principle model and linearized models. The nonlinear multivariable binary distillation column process is simulated with first principle differential equations and its linearized 16th order and reduced 5th order models were obtained. The GPC is designed based on original linearized model, reduced model and nonlinear first principle model. The performance of GPC with the above three considered model structures were compared. The response of GPC with reduced 5th order model shows similar characteristics of linearized 16th order model. Hence, the computation complexity can be reduced using a reduced order model for a binary distillation column process, without compromising on the performance.

KEYWORDS: Binary distillation column, multivariable, reduced order, modeling, linearization, generalized predictive controller.

1 INTRODUCTION

Distillation is most commonly used separation method in the petroleum and chemical industries for purification of final products. The design and efficient control of distillation column is a challenging task due to high nonlinearity and dynamic behavior. Model Predictive Control (MPC) methods which are based on model based control strategies and are widely used to control parameters in various industrial process applications. Distillation column consists of a vertical column, where plates or trays are used to increase the component separations. Distillation column is separated into two sections, mainly stripping section and rectification section. The trays above the feed tray is called stripping section and the trays below the feed tray are called rectification section. Reboiler and condenser are used as heat duties. Condenser is used to condense distillate vapor and reboiler is used to provide heat for the necessary vaporization from the bottom of the column. Condensed vapor is collected in reflux drum and required amount of it is used as a reflux [6][7]. The vertical column is designed for 14 trays and the list of process parameters considered is shown in Fig. 1.

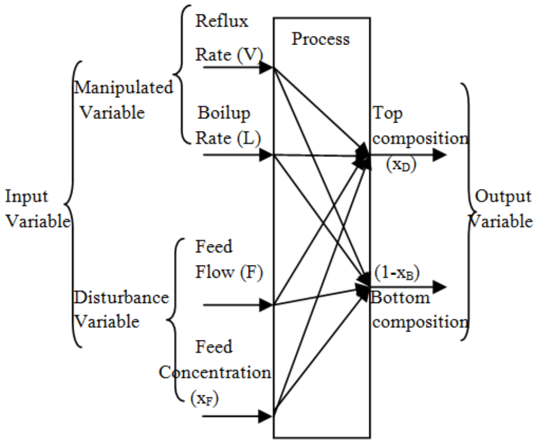

Fig.1 Process parameters of Distillation Column

2 Mathematical Modeling

2.1 Assumptions

The various assumptions considered for the distillation column modeling [4] are given below:

a) The relative volatility α is constant throughout the column. This means the vapor liquid equilibrium relationship can be expressed by,

$$y_n = \frac{\alpha x_n}{1+(\alpha-1)x_n} \tag{1}$$

x_n is liquid composition on n^{th} stage,

y_n is vapor composition on n^{th} stage,

α is the relative volatility.

b) The overhead vapor is totally condensed in a condenser.

c) The liquid holdups on each tray, condenser, and the re-boiler are constant and perfectly mixed

d) The holdup of vapor is negligible throughout the system.

e) The molar flow rates of the vapor and liquid through the stripping and rectifying Sections are constant.

f) The column is numbered from bottom (n=1 for the re-boiler, n=2 for the first tray, n=f for the feed tray, n=N+1 for the top tray and n=N+2 for the condenser)

2.2 Dynamic model of distillation column process

Based on the assumptions described in section 1.1, the dynamic models of distillation process are expressed by the following component material balance equations:

Condenser (n=16):

$$M_D\dot{x}_n = (V+V_F)\,y_{n-1} - Lx_n - Dx_n \tag{2}$$

Tray n (n=10 to 15):

$$M\dot{x}_n = (V+V_F)(y_{n-1} - y_n) + L(x_{n+1} - x_n) \tag{3}$$

Tray above the feed flow (n=9):

$$M\dot{x}_n = V(y_{n-1} - y_n) + L(x_{n+1} - x_n) + V_F(y_F - y_n) \tag{4}$$

Tray below the feed flow (n=8):

$$M\dot{x}_n = V(y_{n-1} - y_n) + L(x_{n+1} - x_n) + L_F(x_F - x_n) \qquad (5)$$

Tray n (n=2 to 7):

$$M\dot{x}_n = V(y_{n-1} - y_n) + (L + L_F)(x_{n+1} - x_n) \qquad (6)$$

Re-boiler (n=1):

$$M_B\dot{x}_1 = (L + L_F)x_2 - Vy_1 - Bx_1 \qquad (7)$$

3 PROCESS DATA FOR DISTILLATION COLUMN UNDER NOMINAL OPERATING CONDITION

The process data are based on a real petroleum project from Petro Vietnam Gas Company reported in [8]. The input feed consist of LPG and Naphtha. The relative volatility α under operating conditions is 5.68. The properties and variations of the feed includes molar weight, liquid density [2], feed composition of feed under operating conditions are listed in Table 1.

Table 1 Properties of feed

Properties	LPG	Naphtha
Molar weight	54.4-55.6	84.1-86.3
Liquid density (kg/m^3)	570-575	725-735
Feed composition (vol %)	38-42	58-62

Table 2 Operating conditions of distillation column process

Stream	Feed	LPG	Naphtha
Temperature (^{o}C)	118	46	144
Pressure (atm)	4.6	4	4.6
Density (kg/m^3)	670	585	727
Volume flow rate (m^3/h)	22.76	8.78	21.88
Mass flow rate (kg/h)	15480	5061	10405
Plant capacity (ton/year)	130000	43000	87000

Table 3 Nominal values for process parameters of distillation column process

Variable	Stream	Molar flow	Unit
M_B	Liquid holdup in the column base	31.11	kmole
M	Liquid holdup on a tray	5.8	kmole
M_D	Liquid holdup in the reflux drum	13.07	kmole
V_F	Vapor rate in feed	98.5152	kmole/h
L_F	Liquid rate in feed	104.2491	kmole/h
V	Internal vapor rate	66.3407	kmole/h
L	Internal liquid rate	75.638	kmole/h
D	Distillate flow rate	92.7597	kmole/h
B	Bottoms flow rate	110.9235	kmole/h

The operating conditions of distillation column process and nominal values of process parameters reported in [8] are summarized in Table 2 and Table 3 respectively.

4 PROCESS SIMULATION AND LINEARIZATION

The process is simulated using Eq. (2) to (7) and nominal parameter values reported in Table I to III. The variation in concentration of top and bottom product on each tray for nominal operating conditions of the binary distillation column is shown in Fig.2

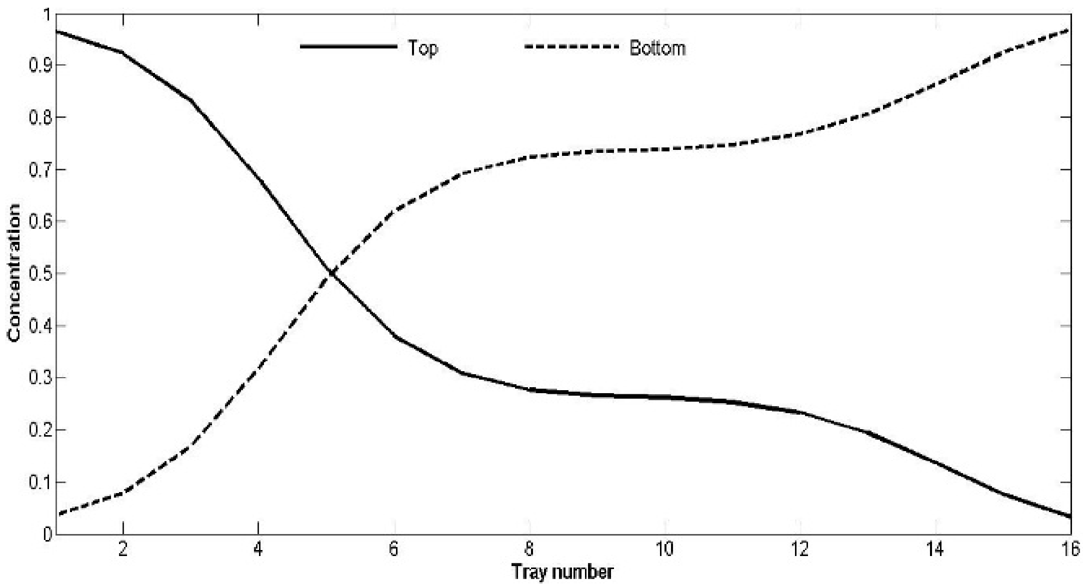

Fig.2 concentration of top and bottom product with respect to different tray

From Fig.2, it is observed that simulation with the nominal values of stream, the purity of the distillate product is 96.45% and the impurity of the bottoms product is 3.13%.

4.1 LINEARIZATION OF NONLINEAR DIFFERENTIAL EQUATION

Multivariable binary distillation column is a nonlinear process, which is linearized to perform a simulation and stability analysis. A linearized model of order 16 is obtained using Taylor's series expansion and Jacobian linearization process. The linearized 16^{th} order state space model is obtained as given in Eq.8, where x represents the states of the concentration at each stages ranging from 1 to 16, L and V represents the internal liquid rate and internal vapor rate respectively.

$$\begin{aligned} \dot{x} &= \mathbf{A}x+\mathbf{B}u \\ y &= \mathbf{C}x+\mathbf{D}u \end{aligned} \tag{8}$$

where,

matrices **A, B, C, D** having size (16×16), (16×2), (2×16), (2×2) respectively.

$$x=[x_1, x_2 ..., x_{16}]^T$$

$$u=[L,V]^T$$

$$
\mathbf{A}=\begin{bmatrix}
-12.7805 & 5.7823 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
49.4273 & -66.25 & 31.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 35.2349 & -55.0955 & 31.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 24.0805 & -48.8275 & 31.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 17.8125 & -45.9154 & 31.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 14.9003 & -44.664 & 31.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 13.649 & -44.1413 & 31.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 13.1263 & -43.9229 & 13.041 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.9078 & -43.744 & 13.041 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30.703 & -40.0547 & 13.041 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 27.0136 & -33.9266 & 13.041 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20.8855 & -27.0765 & 13.041 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14.0355 & -22.506 & 13.041 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.465 & -19.7888 & 13.041 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.7447 & -18.7598 & 13.041 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.5378 & -12.8843
\end{bmatrix}
$$

$$
\mathbf{B}=\begin{bmatrix}
0.0025 & -0.005 \\
0.0105 & -0.0284 \\
0.0098 & -0.0264 \\
0.0066 & -0.0178 \\
0.0035 & -0.0094 \\
0.0016 & -0.0043 \\
0.0007 & -0.0019 \\
0.0018 & -0.0008 \\
0.0056 & -0.002 \\
0.0124 & -0.0057 \\
0.0225 & -0.0103 \\
0.0296 & -0.0136 \\
0.0257 & -0.0118 \\
0.0155 & -0.0071 \\
0.0074 & -0.0034 \\
-0.0738 & 0.0754
\end{bmatrix}
$$

$$
\mathbf{C}=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
\mathbf{D}=\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
$$

4.2 MODEL ORDER REDUCTION (MOR)

The requirement of model order reduction is that the reduced order model obtained should retain the important and key qualitative and quantitative properties such as stability, transient and steady state response etc. of the original system[13]. Three different model order reduction techniques namely Balanced Truncation, Singular Perturbation, Hankel Norm approximation are attempted and it is observed that a 5th order reduced model obtained using Hankel Norm captures the majority of behavior of the system [1].

Based on the reduced order model the simulation is carried out for nominal operating values. Model response of linearized model and reduced order model for top and bottom product is shown in Fig.3 and Fig.4 respectively.

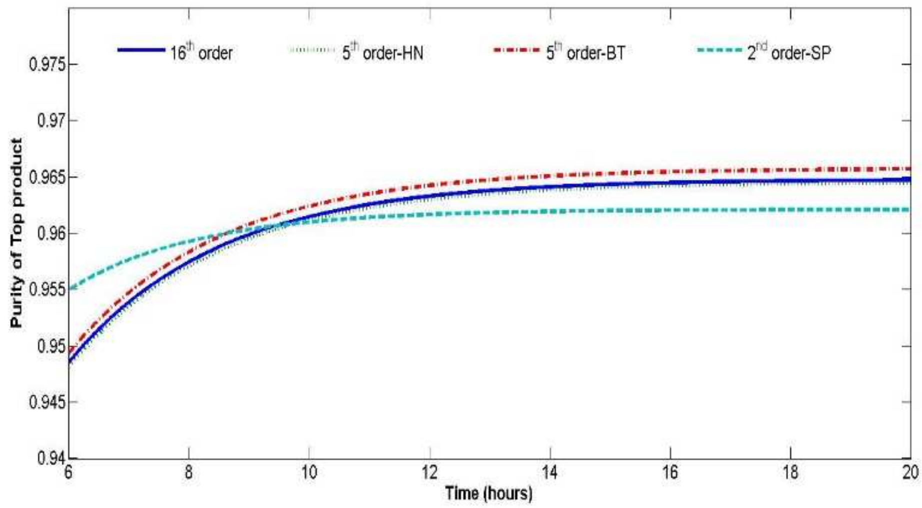

Fig.3 Comparison of linearized model and reduced order models for top product quality

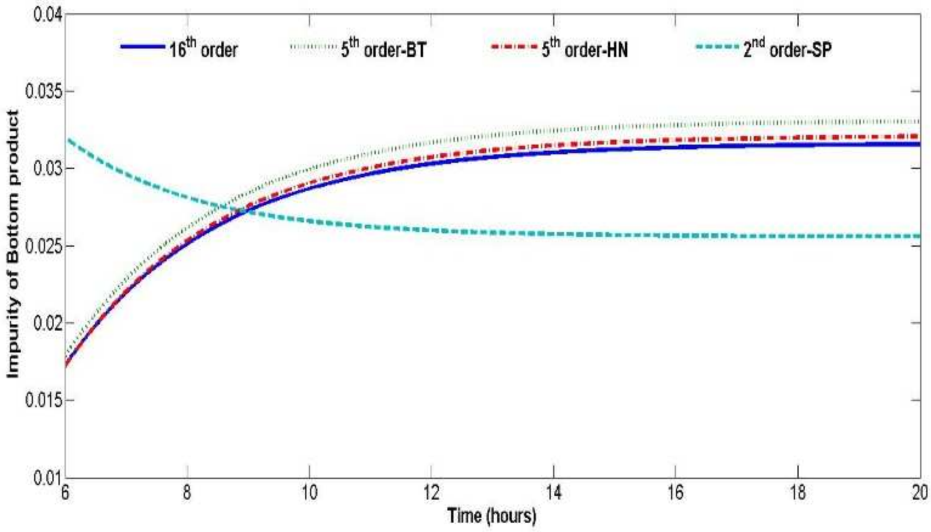

Fig.4 Comparison of linearized model and reduced order models for bottom product quality

Based on the response of the reduced order model based on Hankel norm approximation captures the majority of the input output behavior of the system.

Table 4 ISE, IAE, MSE performance indices for original and reduced order model

Product	Order	ISE	IAE	MSE
Bottom composition	5^{th} order HN	2.5×10^{-3}	2.02	8.3×10^{-8}
	2^{nd} order SP	2.54	41.45	8.4×10^{-5}
	5^{th} order BT	1.31	48.02	4.3×10^{-5}
Top composition	5^{th} order HN	2.02×10^{-4}	0.61	6.7×10^{-9}
	2^{nd} order SP	5.11×10^{-1}	18.13	1.7×10^{-5}
	5^{th} order BT	8.10×10^{-1}	383.18	2.6×10^{-3}

From the Table 4, it is observed that the reduced order model obtained using Hankel norm has minimum ISE, IAE and MSE values for both top and bottom product quality. The linearized reduced order state space model using Hankel norm is obtained as given below,

$$\begin{aligned}\dot{x}_h &= A_h x + B_h u \\ y_h &= C_h x + D_h u\end{aligned} \tag{9}$$

where

$$x_h = [x_1, x_2 ..., x_5]^T$$

$$u = [L, V]^T$$

$$\mathbf{A}_h = \begin{bmatrix} -20.69 & 4.391 & -4.548 & 1.85 & 1.562 \\ 0 & 12.62 & -1.194 & -1.375 & 1.621 \\ 0 & 0 & -6.285 & -1.983 & 1.667 \\ 0 & 0 & 0 & -1.378 & -0.205 \\ 0 & 0 & 0 & 0 & -0.397 \end{bmatrix}$$

$$\mathbf{B}_h = \begin{bmatrix} 0.05684 & -0.09297 \\ -0.1871 & 0.1725 \\ -0.113 & 0.1035 \\ -0.03752 & 0.09185 \\ 0.0309 & 0.002422 \end{bmatrix}$$

$$\mathbf{C}_h = \begin{bmatrix} -0.03884 & -0.009042 & 0.007691 & -0.08733 & 0.006092 \\ -0.09442 & 0.2501 & 0.1421 & 0.0944 & -0.03215 \end{bmatrix}$$

$$\mathbf{D}_h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5 GPC FOR BINARY DISTILLATION COLUMN

Composition control diagram of a multivariable binary distillation column process is shown in Fig.5. In this control configuration, the vapor flow rate V and the liquid flow rate L are the control inputs to maintain the specification of the product concentration outputs X_B and X_D (controlled variable) due to disturbance F (feed flow) and X_F (feed concentration)[3].

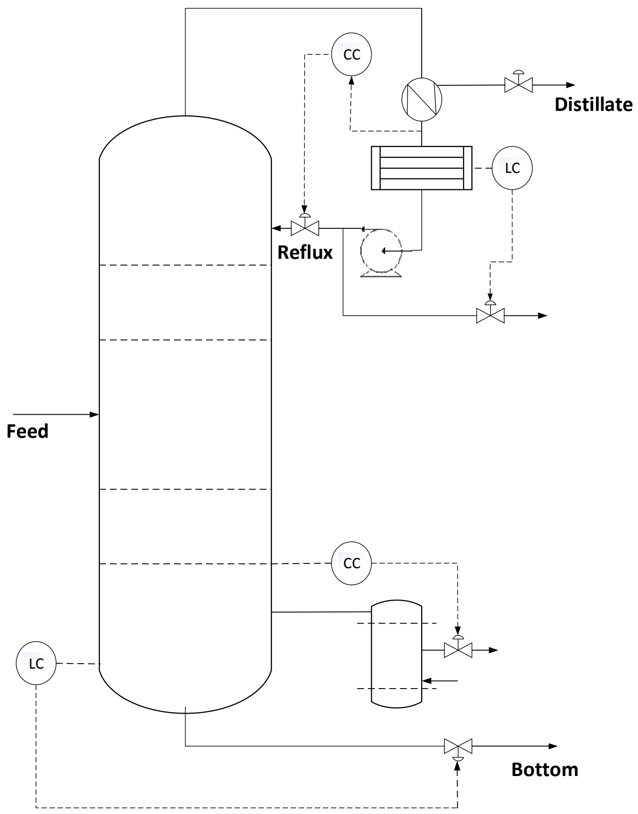

Fig. 5 Composition control of binary distillation column

5.1 MODEL PREDICTIVE CONTROLLER

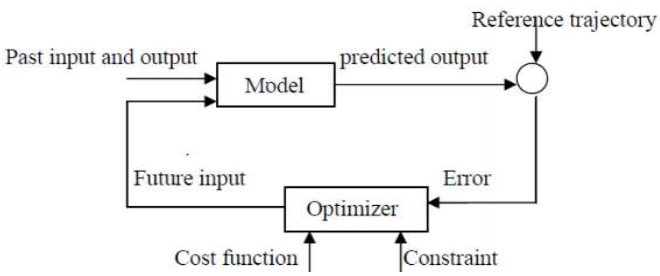

Fig.6 Model predictive control

MPC structure is shown in Fig.6. MPC is the family of controllers, makes the explicit use of model to obtain control signal. The reason for its popularity in industry and academia is its capability of operating without expert intervention for long periods. There are various control design methods based on model predictive control concepts.

The most widely used MPC control strategies are Dynamic Matrix Control (DMC), Model Algorithmic Control (MAC), Predictive Functional Control (PFC), Extended Prediction Self-Adaptive Control (EPSAC), Extended Horizon Adaptive Control (EHAC) and GPC [4].

5.2 GENERALIZED PREDICTIVE CONTROL

GPC is one of the most popular predictive control algorithms developed by D. W. Clarke in 1987. GPC caters for offsets (since it uses integrated Controlled Auto Regressive Moving Average (CARIMA) model), feed-forward signals, and multivariable plant without detailed prior knowledge of structural indices[14]. Basic principle of GPC is shown in Fig7.

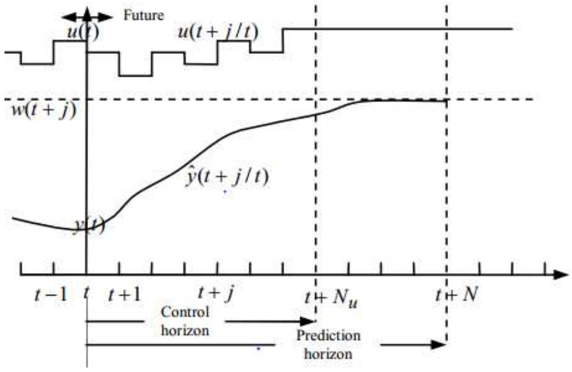

Fig.7 Moving horizon strategy

A CARIMA model is used to obtain good output predictions and optimize a sequence of future control signals to minimize a multistage cost function defined over a prediction horizon. The inclusion of disturbance is necessary to deduce the correct controller structure.

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + C(z^{-1})\frac{e(t)}{\Delta} \tag{10}$$

$\Delta = 1 - z^{-1}$ is the backward shift operator.

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na} \tag{11}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{nb} z^{-nb} \tag{12}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{nc} z^{-nc} \tag{13}$$

$$R = r \times \mathrm{eye}(2 \times N_c) \quad Q = q \times \mathrm{eye}(2 \times N_c) \tag{14}$$

$$J = \sum_{j=N_{p1}}^{N_{p2}} R\left[\hat{y}(t+j/t) - w(t+j)\right]^2 + \sum_{j=1}^{N_u} Q\left[\Delta u(t+j-1)\right]^2 \tag{15}$$

where, $\hat{y}(t+j/t)$ is the *j* step ahead prediction of the system on data upto time *t,*

$w(t+j)$ is the future reference trajectory,

N_{p1} is the minimum value for prediction horizon,

N_{p2} is the maximum value for prediction horizon,

N_c is the control horizon,

Q and *R* are weighting matrices.

The optimal input is given by, $\Delta u = K(w-f)$

K is the first row of matrix $(G^T G + \lambda I)^{-1} G^T$

where, G is the step response

The current control is given by, $u(t) = u(t-1) + K(w-f)$

Whereas for $(w\text{-}f)=0$, there is no control move. GPC depends on the integration of assumption of CARIMA model, recursion of Diophantine equation, consideration of weighting of control increments in cost function and the choice of a control horizon. The advantages of GPC is that it is applicable to nonminimum phase, open loop unstable and having variable dead time. Also it is capable of considering both constant and varying future set points and It is unaffected (unlike pole-placement strategies) if the plant model is over parameterized [6].

6 RESULTS AND DISCUSSIONS

6.1 CLOSED LOOP PERFORMANCE WITH GPC

6.1.1 GPC BASED ON LINEARISED 16TH ORDER MODEL

Servo response of GPC with linearized 16th order model for bottom product impurity and top product purity is shown in Fig.8a and Fig.8b respectively.

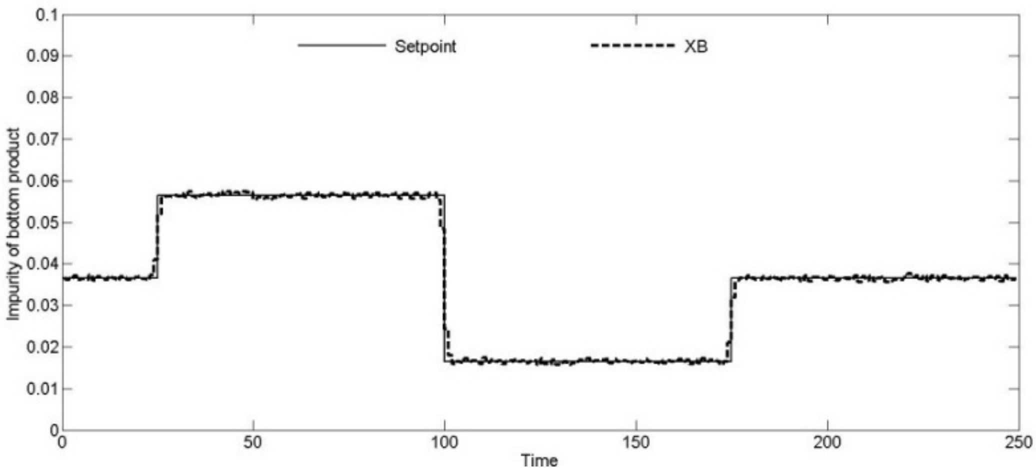

Fig.8a Servo response of GPC for X_B based on 16th order linearized model

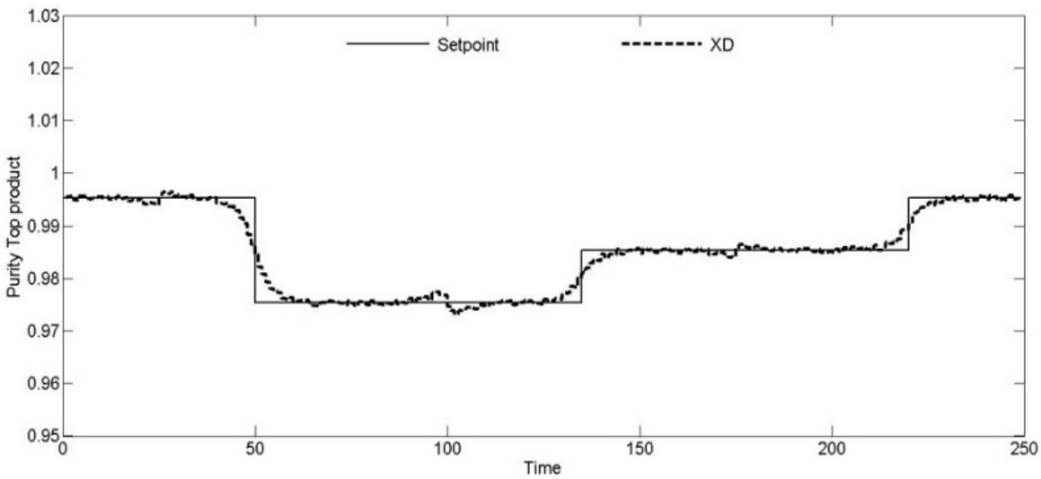

Fig.8b Servo response of GPC for X_D based on 16th order linearized model

The optimally tuned GPC is able to track set point with minimum overshoot and settling time. The control action for the given multiple positive and negative step changes to the process for the top and bottom component using a full order linearized model is as shown in Fig.9.

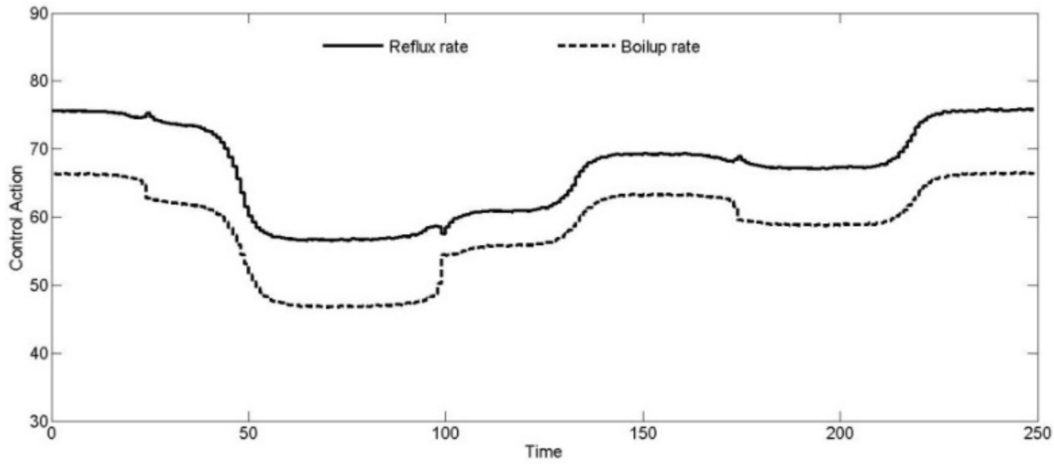

Fig.9 Controller output of GPC based on 16th order linearized model of the process for reflux & boilup rate

6.1.2 GPC BASED ON REDUCED 5TH ORDER MODEL

Servo response of GPC with linearized 5th order model for bottom product impurity and top product purity is shown in Fig.10a and 10b.

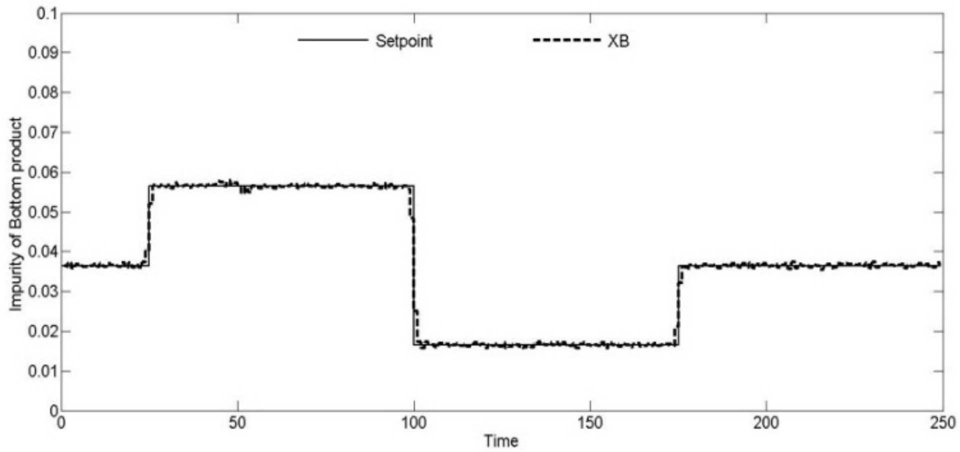

Fig.10a Servo response of GPC for X_B based on 5th order linearized model

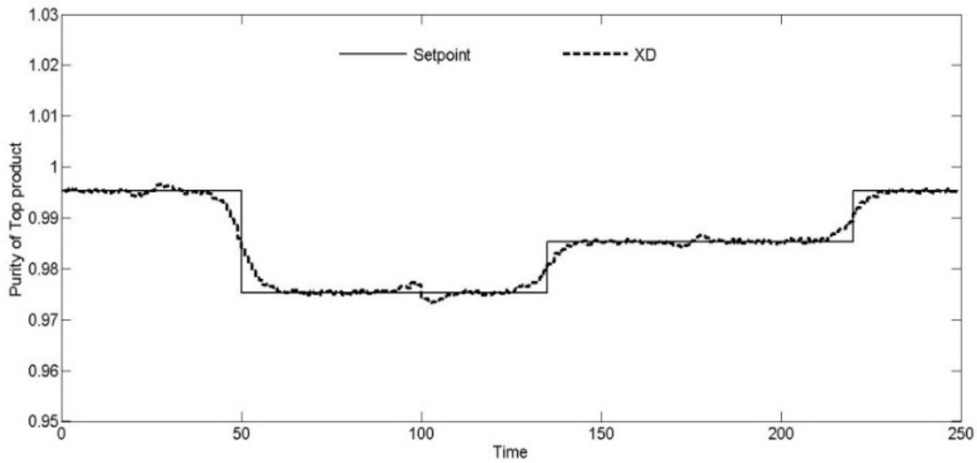

Fig.10b Servo response of GPC for X_D based on 5th order linearized model

The optimally tuned GPC is able to track set point with minimum overshoot and settling time.

The control action for the given multiple positive and negative step changes to the process for the top and bottom component using a reduced order linearized model is as shown in Fig.11. It is observed that the GPC control for linearized reduced order model has similar characters and tracks the step inputs quickly when compared to that of linearized 16th order model.

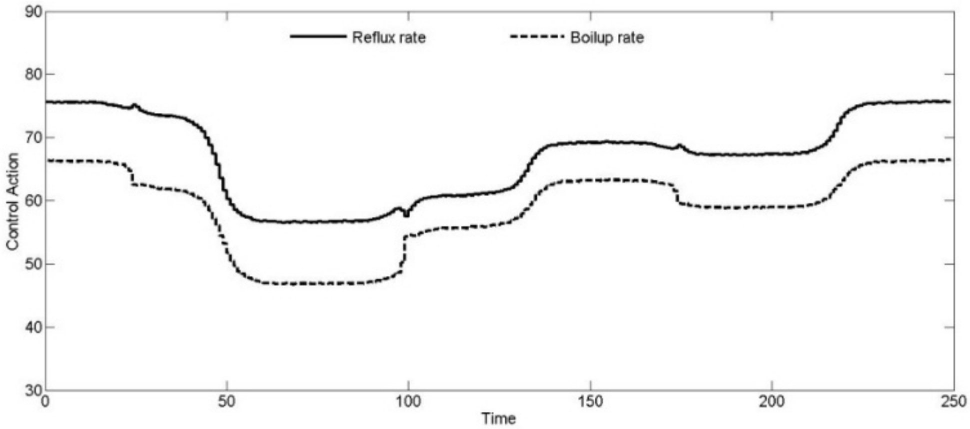

Fig. 11 Controller output of GPC based on 5th order linearized model for reflux & boilup rate

6.1.3 GPC BASED ON FIRST PRINCIPLE MODEL

Servo response of GPC with simulated process model for bottom product impurity and top product purity is shown in Fig.12a and 12b.

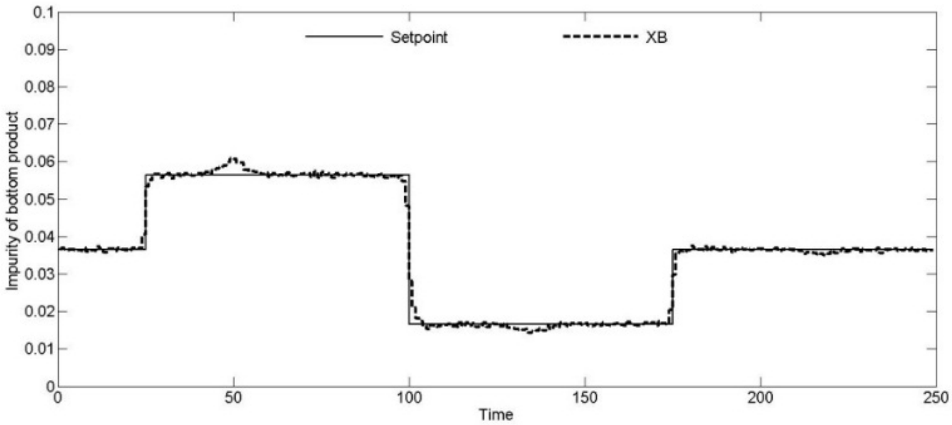

Fig.12a Servo response of GPC for X_B based on first principle model

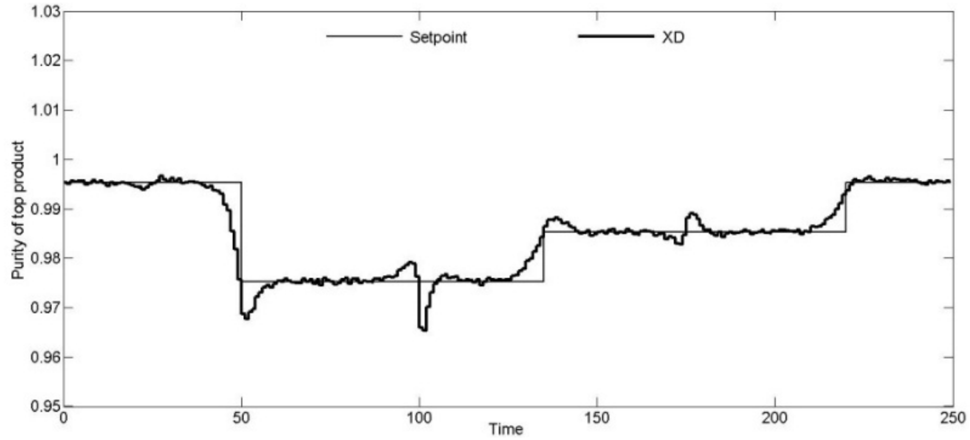

Fig.12b Servo response of GPC for X_D based on first principle model

The optimally tuned MPC is able to track set point with minimum overshoot and settling time. The control action for the given multiple positive and negative step changes to the process for the top and bottom component using first principle model is as shown in Fig.13. It is observed that the GPC algorithm for linearized models tracks the multiple reference inputs quickly to that of a GPC control using a first principle model.

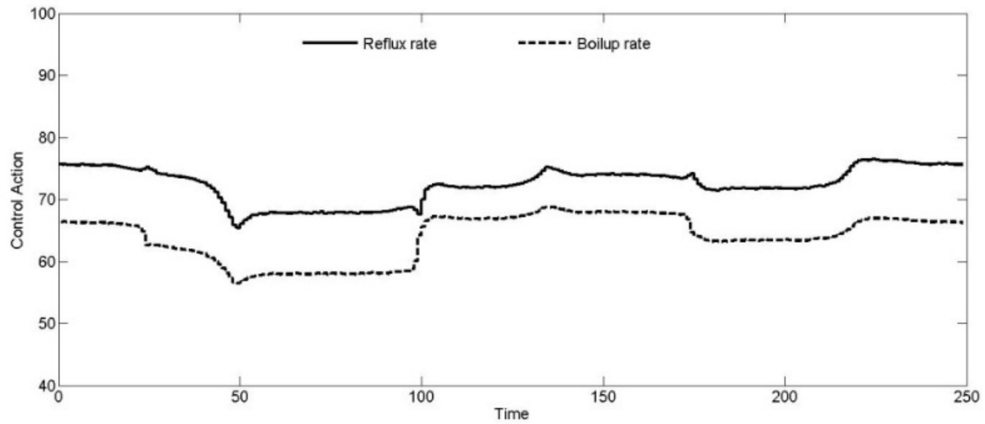

Fig.13 Controller output of GPC based on first principle model for reflux & boilup rate

From the Table 5, it is observed that the response of linearised 5th order model based GPC has almost close values to that of linearised 16th order model based GPC control and matches the dynamics of the nonlinear multivariable process. Hence, the computation complexity can be very much reduced using reduced order model.

Table 5 Performance comparison of linearized models and first principle model

	Linearized 16th order		Reduced 5th order		Nonlinear first principle model	
	Top product	Bottom product	Top product	Bottom product	Top product	Bottom product
ISE	247.66	0.3329	247.65	0.3328	247.65	0.3312
IAE	248.8	9.1201	248.8	9.121	248.83	9.12
MSE	0.9906	0.0013	0.9906	0.0013	0.9906	0.0013

7 CONCLUSIONS

The first principle model of binary distillation column was developed using governing equations and parameter values. The simulated distillation column was validated under nominal and steady state operating conditions.

Generalized Predictive Controller was implemented with three different model structures for binary distillation column i) Linearized 16^{th} order model, ii) Linearized 5^{th} order model and iii) Nonlinear first principle model using MATLAB. A comparative study was carried out by evaluating performance indices such as Integral Square Error (ISE), Integral Absolute Error (IAE) and Mean Square Error (MSE). The response of predictive control with linearised 5^{th} order model is almost close to the linearised 16^{th} order model, thereby reducing the computational complexity in implementing GPC for binary distillation column process.

ACKNOWLEDGMENT

The authors gratefully acknowledge Anna University, Chennai for providing financial support to carry out this research work under Anna Centenary Research Fellowship (ACRF) scheme.

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