# Learning and Forgetting Effects of Flexible Flow Shop Scheduling 

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#### Abstract

In this paper we consider learning and forgetting effect of workers for flexible flowshop scheduling problem with sequence dependent setup times. The objective is to minimize the weighted sum of maximum completion time and maximum tardiness. The learning effect occurs when operator's (workers) skill increases after repeating similar job causing the decrease of processing time. On the other hand, forgetting effect occurs when an operator relearns the process after an interruption for a batch setup, machine maintenance or operator condition recovery, causing the increase of processing time.


KeYwords: flexible flow shop Scheduling, learning effect, Forgetting effect, Makespan(Total completion time), Tardiness.

## 1 Introduction

The general flow shop scheduling problem is a production problem where a set of $n$ jobs have to be processed with identical flow pattern on $m$ machines. The idea of flowshop sequencing is given by Johonson in 1954. A flexible flowshop (FFS) is a generalization of the flowshop and the parallel processor environments. A flexible flowshop is alternatively called a hybrid flowshop or multiprocessor flowshop. In the most general setting of a flexible flowshop environment, there are multiple stages ( T - stages), each of which consists of $\mathrm{m}(\mathrm{t})(\mathrm{t}=1,2,3, \ldots, \mathrm{~T}$ ) parallel processors).In Flexible Flow Shop (FFS) problem there are some stages. In each stage there is at least one machine and at least one stage must contain two or more parallel machines. Production flow of jobs is from stage 1 to the last stage and it is possible for a job to skip any number of stages. Flexible flow shops (FFS) are common manufacturing environments in which a set of njobs is to be processed in a series of $m$ stages for optimizing a given objective function. There are a number of variantsand almost all the following characteristics are common in all variants:

The number of processing stages $m$ should be at least 2 .

- Each stage k has $\mathrm{M}_{\mathrm{t}}>=1$ machines in parallel and at least one of the stage $\mathrm{M}_{\mathrm{t}}>1$.
- All jobs are processed the following same production flow: stage 1 , stage $2, \ldots$, stage $m$. A job might skip any number of stages provided it is processed in at least one of them.
- Each job $j$ requires a processing time $P_{t}$ in stage $t$. We shall refer to the processing of job $j$ in stage $t$ as operation $\mathrm{O}_{\mathrm{t}}[1]$.

FFS happens in real world areas like electronic, and textile industries. There are also some cases in servicing areas like civil engineering, architecture and systems of transportation and information technology([ 3],[8],[13],[16],[17],[18],[21],[22], [24], [25], [34]).In this research, flexible flow shop problem and manpower with learning and forgetting effects will be considered. The objective is to find the sequence of jobs on machines and the sequence of workers for doing the setup of machines to minimize the weighted sum of makespan and maximum tardiness.

The first study on of learning effect in scheduling was introduced by Biskup[5], Moshio[11] demonstrated the minimization of makespan on a single machine with considering learning effect will be solved as polynomial. Moshio and Sidny[10] studied on single machine when the learning effect was not identical for all of jobs. Lee et al[30] studied bi-criteria optimization on single machine and presented a heuristic approach for solving them to find good solutions. Lee and Wu [31] proposed a meta-heuristic algorithm for minimizing the maximum completion time in the two machine problem by learning effect that is dependent to jobs. Ern and Guner [27] studied single machine problem based on the bi-criteria to minimize the sum of completion time and maximum tardiness.Another studies are done on single machine with learning effects such as: Cheng and Wang [28], Kuo and Yang [32], Lee [29] ,Biskup and Simons[4], Moshio and Sidny[9].a little studies were done on forgetting effect in the literature. Yang and Chand [31] considered learning and forgetting effects on single machine with group setup time. Their objective was to minimize the maximum completion time of jobs. Nembhard and Uzumeri [6] considered forgetting the effect of an organization that applied new technologies and approaches on period of time. Jabera et al [20] called learning and forgetting effects to be consistent. Globerson [26] ,Shtub et al[1] called power models were suitable for considering forgetting effects.

## 2 Problem definition

We investigates flexible flow shop problem with rule of workers to process jobs. Each worker has learning and forgetting coefficient. The processing of jobs aredone automatically by machines. In this study, we determine the sequence of jobs on each machine, assign workers to each machine and determine the route of each worker to prepare of machines for processing of jobs based on learning and forgetting effects of workers. The objective is to minimize the weighted sum of maximum completion time and maximum tardiness. The structure of problem graphically illustrated in Figure 1.


Fig. 1. The flexible flow shop environment
The assumptions, indices, parameters and decision variables are introduced before developing the proposed mathematical model.

## A. Assumptions

- Machines are available any time on the horizon.
- Jobs are available any time on the horizon.
- The processing time of jobs is pre-known.
- Each machine can process only one job at the time.
- Each job processed only by one machine at the time.
- Each worker can be set only one machine at a time.
- The process of jobs done automatically by machines and workers prepare only machines for processing of jobs.
- Workers have different skills for setting machines to process jobs (it relates to expert; age; degree ;...).
- Learning and forgetting effects of workers are based on iteration.
- Setup times of jobs on machines by workers are pre-defined.


## Indices

$i, j, k$ : Indices of jobs
$s, s^{\prime}$ : Indices of sequences
$l, \dot{l}:$ Indices of number of jobs that are done by each worker
$w, w$ : Indices of workers
$m, m$ : Indices of machines
$t, t$ : Indices of stages

$$
\begin{aligned}
& i, j, k=(1,2, \ldots, n) \\
& s, s^{\prime}=(1,2, \ldots, n) \\
& I^{\prime} I^{\prime}=(1,2, \ldots, n) \\
& w, w^{\prime}=(1,2, \ldots, n w) \\
& m, m^{\prime}=\left(1,2, \ldots, M_{t}\right) \\
& t, t^{\prime}=(1,2, \ldots, n s)
\end{aligned}
$$

## Parameters

$P_{\text {imt }}$ : Processing time of jobion machine mat stage $t$
$S e_{\text {wimt }}$ : Setup time of machine $m$ at stage $t$ for processing job $i$ by worker $w$
$\operatorname{Setup}_{i j w m t}$ : Setup time of machine $m$ at stage $t$ between jobs $i$ and $j$ by worker $w$
$a_{i w}$ : Learning coefficient of job $i$ by worker $w$
$b_{i w}$ : Forgetting coefficient of job $i$ by worker $w$
$d_{i}$ : Due date of job $i$
$M_{t}$ : Number of parallel identical machines at stage $t$
$n w$ : Number of workers on shop floor
$n s$ : Number of stages
$n$ : Number of jobs for processing
$M$ : A positive big number

## Decision variables

$x_{i s m l w t}=\left\{\begin{array}{c}1 \text { if jobithat issthjob on machine mat staget, }, l t h j o b \text { that is done by workerw } \\ 0\end{array}\right.$
$S_{i t}$ : Starting time of the worker when the machine should preparefor processing of jobiat staget
$F_{i t}$ : Leaving time of the worker when the machine is preparedfor processing of jobiat stage $t$
$C_{i t}$ : The completion time of jobiat staget
$C_{\max }$ : Maximum of completion time(Makespan)
$r_{i t}$ : The number of setups on the same machine that are done before of jobiat stagetby the same worker
$h_{i t}$ : The number of setups on the different machine that are done before of jobiat stagetbythe same worker $T_{i}$ : Tardiness of job $i$
Tmax: Maximum of tardiness

According to the above mentioned parameters and variables, the mathematical model is suggested as a mixed integer formulation as follows:

## 3 ProposedMathematical model

$\operatorname{Minz}=\alpha \operatorname{Tmax}+(1-\alpha) C \max$

$$
\begin{array}{ll}
\sum_{s=1}^{n} \sum_{m=1}^{M t} \sum_{l=1}^{n} \sum_{w=1}^{n w} x_{i s m l w t}=1 & \forall i, t \\
\sum_{i=1}^{n} \sum_{l=1}^{n} \sum_{w=1}^{n w} x_{i s m l w t} \leq 1 & \forall m, s, \tag{3}
\end{array}
$$

$$
\begin{align*}
& \sum_{i=1}^{n} \sum_{l=1}^{n} \sum_{m=1}^{M t} x_{i s m l w t} \leq 1  \tag{4}\\
& \sum_{i=1}^{n} \sum_{l=1}^{n} \sum_{w=1}^{n w} x_{i s m l w t} \leq \sum_{i=1}^{n} \sum_{l=1}^{n} \sum_{w=1}^{n w} x_{i s-1 m l w t} \forall s>1, m, t \\
& \sum_{i=1}^{n} \sum_{s=1}^{n} \sum_{m=1}^{M t} x_{i s m l w t} \leq \sum_{i=1}^{n} \sum_{s=1}^{n} \sum_{m=1}^{M t} x_{i s m l-1 w t} \forall l>1, w, t  \tag{5}\\
& x_{i s m l w t} \geq-M . x_{j s^{\prime} m l^{\prime} w t} \forall w, m, i, j, t, s^{\prime}>s, l^{\prime}<l \tag{6}
\end{align*}
$$

$x_{i s m l w t} \leq-M .\left(1-x_{j s^{\prime} m l^{\prime} w t}\right) \forall w, m, i, j, t, s^{\prime}>s, l^{\prime}<l$

$$
\begin{array}{ll}
S_{i t} & \geq \sum_{j=1, j \neq i}^{n} \sum_{m=1}^{M t} \sum_{m^{\prime}=1}^{M t} \sum_{l=2}^{n} \sum_{s=1}^{n} \sum_{s^{\prime}=1}^{n} \sum_{w=1}^{n w}\left(F_{j t} \cdot x_{j s^{\prime} m^{\prime} l-1 w t} \cdot x_{i s m l w t}\right) \\
S_{i t} \geq \sum_{j=1, j \neq i}^{n} \sum_{m=1}^{M t} \sum_{l^{\prime}=1}^{n} \sum_{l=1}^{n} \sum_{s=2}^{n} \sum_{w^{\prime}=1}^{n w} \sum_{w=1}^{n w}\left(C_{j t} \cdot x_{j s-1 m l^{\prime} w^{\prime} t} \cdot x_{i s m l w t}\right)
\end{array}
$$

$$
\left.S_{i t} \geq \sum_{m=1}^{M t} \sum_{m^{\prime}=1}^{M t} \sum_{l^{\prime}=1}^{n} \sum_{l=1}^{n} \sum_{w^{\prime}=1}^{n w} \sum_{w=1}^{n w} \sum_{s^{\prime}=1}^{n} \sum_{s=1}^{n} C_{i t-1} . x_{i s m l w t} \cdot x_{i s^{\prime} m^{\prime} l^{\prime} w^{\prime} t}\right) \forall i, t>1
$$

$$
S_{i t} \geq \sum_{m=1}^{M t} \sum_{m^{\prime}=1}^{M t} \sum_{s^{\prime}=1}^{n} \sum_{s=1}^{n} \sum_{l^{\prime}=1}^{n} \sum_{w=1}^{n w}\left(F_{j t^{\prime}} \cdot x_{j s^{\prime} m^{\prime} \iota^{\prime} w t^{\prime}} \cdot x_{i s m 1 w t}\right) \forall i, j, t^{\prime}<t, t>1
$$

$$
r_{i t}=\sum_{s=1}^{n} \sum_{m=1}^{M t} \sum_{w=1}^{n w} \sum_{l=1}^{n}\left(\sum_{j=1, j \neq i}^{n} \sum_{s^{\prime}=1}^{s} \sum_{l^{\prime}=1}^{l} x_{j s^{\prime} m l^{\prime} w t}\right) \cdot x_{i s m l w t} \forall i, t
$$

$$
\begin{equation*}
h_{i t}=\max \left\{\sum_{s=1}^{n} \max _{s^{\prime}=1: s}\left(s^{\prime} .\left(\sum_{m=1}^{M t} \sum_{l=1}^{n} \sum_{l^{\prime}=1}^{l} \sum_{w=1}^{n w} \sum_{j=1, j \neq i}^{n} x_{j s^{\prime} m l^{\prime} w t} \cdot x_{i s m l w t}\right)\right), 1\right\} \quad \forall i, t \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
F_{i t} \geq S_{i t}+\sum_{m=1}^{M t} \sum_{l=1}^{n} \sum_{w=1}^{n w}\left(s e_{w i m t} \cdot x_{i 1 m l w t}\right) \tag{15}
\end{equation*}
$$

$$
\left.\begin{array}{l}
F_{i t} \geq \sum_{k=1, k \neq i}^{n} \sum_{j=1, j \neq i}^{n} \sum_{m=1}^{M t} \sum_{s=2}^{n} \sum_{l=1}^{n} \sum_{l+1}^{n} \sum_{l^{\prime}=1 l^{\prime \prime}=1, l^{\prime \prime}<l^{\prime}}^{n} \sum_{w=1}^{n w} \sum_{w^{\prime}=1}^{n w}\left(S_{i t}+\left[( r _ { i t } ) ^ { a _ { i w } } \cdot \left(l-l^{\prime \prime}\right.\right.\right. \\
\forall i, t, a_{i w}<0, b_{i w} \geq 1
\end{array}\right] . x_{j s-1 m l^{\prime} l^{\prime \prime} t \cdot x_{k h_{i t} m l^{\prime \prime} w t} \cdot x_{i s m l w t}}^{F_{i t} \geq \sum_{j=1, j \neq i}^{n} \sum_{m=1}^{M t} \sum_{s=2}^{n} \sum_{l=1}^{n} \sum_{l^{\prime}=1}^{n} \sum_{w=1}^{n w} \sum_{w^{\prime}=1}^{n w}\left(S_{j t}+\left[\left(r_{i t} a_{i w}\right) \cdot \operatorname{setup}_{j i w m t}\right]\right) \cdot x_{j s-1 m l^{\prime} w^{\prime \prime} t} \cdot x_{i s m l w t}} \begin{array}{|}
\forall i, t, a_{i w}<0 \\
C_{i t}=\sum_{l=1}^{n} \sum_{m=1}^{M t} \sum_{w=1}^{n w} \sum_{s=1}^{n}\left(F_{i t}+p_{i m t}\right) \cdot x_{i s m l w t} \forall i, t
\end{array}
$$

$$
\begin{equation*}
C \max \geq C_{i n} \forall i \tag{19}
\end{equation*}
$$

$T_{i} \geq C_{i n}-d_{i} \forall i$
$T_{i} \geq 0 \quad \forall i$
$T m a x \geq T_{i} \forall i$
$S_{i t}, F_{i t}, C_{i t}, C \max , T_{i}, T \operatorname{Tmax}, h_{i t}, r_{i t} \geq 0$
$x_{\text {ismlwt }}=\{0,1\}$

Minimization of the weighted sum of makespan and maximum tardiness are shown in $\mathrm{Eq}(1) . \mathrm{Eq}(2)$ guarantieswhich worker do each job at each stage Eq.(3) explains that in each position on each machine at each stage can be only one job processed. $\mathrm{Eq}(4)$ ensures that in each position by each worker at each stage can be only one job processed.Eq (5) ensures that if in the one sequence on one machine at one stage is not any job for processing by a worker that is assigned to this sequence therefore it is not possible to process one job to the next sequence on the same machine at the same stage (it causes to produce a feasible sequence on each machine at each stage). Eq(6) ensures that if in the route of each worker at each stage is not any job for processing by a worker that is assigned to this route therefore it is not possible to process one job to the next sequence by the same worker at the same stage (it causes to produce a feasible route on each worker at each stage). $\mathrm{Eq}(7)$ and (8) generate a feasible sequence on each machine done by workers in the solution space.Eq(9)-(12) calculate starting time of each job at each stage by worker that prepare machine for processing them. Eq (13) determines the number of setups on the same machine that are done before of job $i$ at stage $t$ by the same worker. Eq (14) determines the number of setups on the different machine that are done before of job $i$ at stage $t$ by the same. $\mathrm{Eq}(15)-(17)$ calculate leaving time of each job at each stage by a worker that is prepared machine for processing them. Eq (18) calculates the completion time of each job at each stage. Eq (19) determines the maximum completion time (Makespan). Esq (20) and (21) determines tardiness of each job. Maximum of tardiness calculated by Eq (22)

An example is provided and the results of the execution with details of schedule of jobs on machines are presented. Suppose that the weight of makespan is 0.2 and there are 6 jobs, 3 workers, the processing times, setup times, due dates, are shown in Tables1-5.

Table1. The processing time of example

|  | Job 1 | Job 2 | Job 3 | Job 4 | Job 5 | Job 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Stage 1 | 4 | 6 | 10 | 4 | 4 | 8 |
| Stage 2 | 3 | 5 | 8 | 11 | 4 | 5 |

Table2. The sequence dependent set up time between jobs at stage 1 of example

|  |  |  | Job 1 | Job 2 | Job 3 | Job 4 | Job 5 | Job 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stage 1 | Worker 1 | Job 1 | 0 | 6 | 6 | 6 | 7 | 6 |
|  |  | Job 2 | 7 | 0 | 4 | 7 | 6 | 6 |
|  |  | Job 3 | 5 | 6 | 0 | 6 | 6 | 7 |
|  |  | Job 4 | 6 | 6 | 7 | 0 | 4 | 8 |
|  |  | Job 5 | 7 | 8 | 7 | 5 | 0 | 7 |
|  |  | Job 6 | 4 | 5 | 3 | 6 | 6 | 0 |
|  | Worker 2 | Job 1 | 0 | 5 | 7 | 6 | 6 | 4 |
|  |  | Job 2 | 7 | 0 | 6 | 8 | 5 | 4 |
|  |  | Job 3 | 2 | 6 | 0 | 7 | 7 | 3 |
|  |  | Job 4 | 5 | 2 | 7 | 0 | 7 | 5 |
|  |  | Job 5 | 6 | 8 | 8 | 1 | 0 | 6 |
|  |  | Job 6 | 4 | 5 | 5 | 9 | 8 | 0 |
|  | Worker 3 | Job 1 | 0 | 4 | 6 | 5 | 4 | 3 |
|  |  | Job 2 | 6 | 0 | 7 | 3 | 5 | 5 |
|  |  | Job 3 | 7 | 5 | 0 | 6 | 9 | 4 |
|  |  | Job 4 | 8 | 6 | 5 | 0 | 7 | 3 |
|  |  | Job 5 | 4 | 7 | 4 | 6 | 0 | 3 |
|  |  | Job 6 | 3 | 8 | 7 | 8 | 5 | 0 |

Table3. The sequence dependent set up time between jobs at stage 2 of example


Table 4. The set up time of jobs at stage 1 and 2 of example

|  |  |  | Job 1 | Job 2 | Job 3 | Job 4 | Job 5 | Job 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stage 1 | Worker 1 | Machine 1 | 524 | 6 | 1 | 7 | 2 | 6 |
|  |  | Machine 2 |  | 8 | 4 | 6 | 6 | 8 |
|  |  | Machine 3 |  | 9 | 6 | 7 | 4 | 7 |
|  | Worker 2 | Machine 1 <br> Machine 2 <br> Machine 3 | 628 | 8 | 5 | 7 | 8 | 9 |
|  |  |  |  | 9 | 4 | 6 | 9 | 8 |
|  |  |  |  | 7 | 1 | 4 | 8 | 7 |
|  | Worker 3 | Machine 1 <br> Machine 2 <br> Machine 3 | $\begin{aligned} & 6 \\ & 7 \\ & 8 \end{aligned}$ | 9 | 6 | 9 | 9 | 7 |
|  |  |  |  | 9 | 5 | 7 | 7 | 8 |
|  |  |  |  | 9 | 4 | 6 | 8 | 6 |
| Stage 2 | Worker 1 | Machine 1 <br> Machine 2 <br> Machine 3 <br> Machine 4 | 8976 | 9 | 8 | 9 | 8 | 1 |
|  |  |  |  | 2 | 9 | 7 | 9 | 4 |
|  |  |  |  | 7 | 2 | 8 | 7 | 5 |
|  |  |  |  | 7 | 9 | 9 | 9 | 10 |
|  | Worker 2 | Machine 1 | 8978 | 8 | 7 | 9 | 8 | 2 |
|  |  | Machine 2 |  | 7 | 2 | 7 | 6 | 6 |
|  |  | Machine 3 |  | 5 | 7 | 6 | 7 | 4 |
|  |  | Machine 4 |  | 9 | 8 | 9 | 8 | 9 |
|  |  | Machine 1 | 9 | 8 | 9 | 8 | 5 | 9 |
|  | Worker 3 | Machine 2 | 8 | 1 | 6 | 6 | 6 | 7 |
|  |  | Machine 3 | 9 | 4 | 8 | 8 | 8 | 8 |
|  |  | Machine 4 | 7 | 5 | 8 | 2 | 10 | 10 |

Table 5. The due date of example

|  | Job 1 | Job 2 | Job 3 | Job 4 | Job 5 | Job 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Due date | 24 | 27 | 18 | 27 | 20 | 18 |

Table 6. The learning coefficient

|  |  | Job 1 | Job 2 | Job 3 | Job 4 | Job 5 | Job 6 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Learning | Worker 1 | -1 | -2 | -3 | -2 | -2 | -1 |
| coefficient | Worker 2 | -1 | -2 | -2 | -2 | -1 | -3 |
|  | Worker 3 | -1 | -1 | -1 | -1 | -1 | -3 |

Table7. The forgetting coefficient

|  |  | Job 1 | Job 2 | Job 3 | Job 4 | Job 5 | Job 6 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Forgetting | Worker 1 | 1 | 2 | 1 | 2 | 1 | 1 |
| coefficient | Worker 2 | 2 | 1 | 1 | 3 | 3 | 2 |
|  | Worker 3 | 1 | 1 | 1 | 1 | 2 | 2 |



## Fig.2.Presentation of solution graphically

The objective function is to minimize the weighted sum of makesapn and maximum tardiness. Figure 2 and Table 3 illustrate the solution obtained of solution space in details from the mathematical model

Completion times of jobs at stage 1 are as follows:
$C_{11}=\left\{\operatorname{Max}\left\{0, F_{31}\right\}+\operatorname{Se}_{2121}+P_{11}\right\}=\{\operatorname{Max}\{0,1\}+1+4\}=6 ;$
$C_{21}=\left\{\operatorname{Max}\left\{C_{41}, F_{21}\right\}+\operatorname{Setup}_{4211} *\left(r_{21}\right)^{a_{21}}+P_{21}\right\}=\{\operatorname{Max}\{14+1+5\}=12 ;$
$C_{31}=\left\{\operatorname{Se}_{2331}+P_{31}\right\}=\{1+10\}=11 ;$
$C_{41}=\left\{\operatorname{Max}\left\{C_{51}, F_{11}\right\}+\operatorname{Setup}_{5421} *\left(h_{41}\right)^{b_{41}}+P_{41}\right\}=\left\{\operatorname{Max}\{6,2\}+1 *(2)^{+2}+4\right\}=14 ;$
$C_{51}=\left\{S e_{5111}+P_{51}\right\}=\{2+4\}=6 ;$
$C_{61}=\left\{\operatorname{Max}\left\{C_{11}, F_{51}\right\}+\operatorname{Setup}_{5421} *\left(h_{61}\right)^{b_{61}}+P_{61}\right\}=\{\operatorname{Max}\{6,2\}+1+8\}=15 ;$
Completion times of jobs at stage 2 are as follows:
$C_{12}=\left\{\operatorname{Max}\left\{C_{11}, F_{42}\right\}+\operatorname{Setup}_{2132}+P_{12}\right\}=\{\operatorname{Max}\{6,16\}+1+3\}=20 ;$
$C_{22}=\left\{\operatorname{Max}\left\{C_{21}, F_{32}\right\}+\operatorname{Se}_{1222} *\left(h_{22}\right)^{b_{22}}+P_{22}\right\}=\{\operatorname{Max}\{20,10\}+2+5\}=27 ;$
$C_{32}=\left\{\operatorname{Max}\left\{C_{31}, F_{61}\right\}+\operatorname{Se}_{1332}+P_{32}\right\}=\{\operatorname{Max}\{11,5\}+2+8\}=19 ;$
$C_{42}=\left\{C_{41}+S e_{3442}+P_{42}\right\}=\{14+2+11\}=27 ;$

$$
\begin{aligned}
& C_{52}=\left\{\operatorname{Max}\left\{C_{51}, F_{12}\right\}+\operatorname{Setup}_{1532} *\left(r_{52}\right)^{a_{52}}+P_{52}\right\}=\{\operatorname{Max}\{6,12\}+4+4\}=20 ; \\
& C_{62}=\left\{{\left.\operatorname{Max}\left\{C_{61}, F_{22}\right\}+\operatorname{Se}_{2612}+P_{62}\right\}=\{\operatorname{Max}\{15,11\}+2+5\}=22 ;}^{2}\right\}
\end{aligned}
$$

Tardiness of jobs is calculated as follows:

$$
\begin{aligned}
& T_{1}=\operatorname{Max}\left\{0, C_{12}-d_{1}\right\}=\operatorname{Max}\{0,20-24\}=0 ; \\
& T_{2}=\operatorname{Max}\left\{0, C_{22}-d_{2}\right\}=\operatorname{Max}\{0,27-27\}=0 ; \\
& T_{3}=\operatorname{Max}\left\{0, C_{32}-d_{3}\right\}=\operatorname{Max}\{0,19-18\}=1 ; \\
& T_{4}=\operatorname{Max}\left\{0, C_{42}-d_{4}\right\}=\operatorname{Max}\{0,27-27\}=0 ; \\
& T_{5}=\operatorname{Max}\left\{0, C_{52}-d_{5}\right\}=\operatorname{Max}\{0,20-20\}=0 ; \\
& T_{6}=\operatorname{Max}\left\{0, C_{62}-d_{6}\right\}=\operatorname{Max}\{0,22-18\}=4 ; \\
& \operatorname{Tmax}=\operatorname{Max}\{0,0,1,0,0,4\}=4
\end{aligned}
$$

Table 8. Decision variable of example

| Number of job | Completion time at stage 1 | Completion time at stage 2 | Tardiness |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Job 1 | 6 | 20 | 0 |
| Job 2 | 12 | 27 | 0 |
| Job3 | 11 | 19 | 1 |
| Job4 | 14 | 27 | 0 |
| Job5 | 6 | 20 | 0 |
| Job6 | 15 | 22 | 4 |

The objective value is equal to $\left(0.2 * 27+0.8^{*} 4\right)=8.6$.

## 4 CONCLUSION AND FUTURE WORKS

In this paper a new mathematical model is presented for scheduling flexible flow shop problem with learning and forgetting effects. Manpower has the significant rule for doing setup on machines and their skill will be increased when they do similar jobs and it is possible for them to prepare machines to process of jobs with higher speed (Learning effect). On the other hand, when workers do a number of new jobs on different machines therefore they will be forget the setup on before machines (Forgetting effect). In this mathematical model we consider both learning and forgetting effects of workers so as to minimize the objective function.

The jobs are automatically processed and manpower work is preparing of machine to process the jobs.
Flexible flowshop problem is Np -hard, therefore it is worthwhile to apply other search methods or meta-heuristic algorithms (Neural Network, Genetic Algorithm, Ant Colony Optimization, etc.) to find the optimal solution in the solution space. The transportation time between stages may be also applied for future research.

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