# Spatial resource allocation: multi task, multi competences case 

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ABSTRACT: In this paper we consider a problem of spatial allocation of agents "Technicians" with multi competences, for maintenance of distributed equipment requiring multi skills tasks. The objective is to ensure optimal partitioning of resource allocation taking into account the multi competences constraints and resources displacement. The approach to the problem is performed using the techniques of cellular automata and Voronoi diagrams. We first consider a simplified case of seven equipments and tree technicians with two competences. An algorithm is proposer for the considered case, and a simulation is presented. The technician displacement is not treated in this paper. A generalization to $m$ equipments $p$ technicians and n competences is proposed.

Keywords: Cellular Automata, Voronoi diagram, industrial, maintenance, modelling, process.

## 1 Introduction

The maintenance department by putting in good operating conditions production equipments and preventing failures is, conspicuously, influencing on companies productivity. The human factor appears to be the best asset to carry out any project. Its impact on the functioning of the company now seems indisputable. Optimizing the use of human resources becomes an aspect of performance that manufacturers are rediscovering and trying to master to exploit these resources. Such recognition has led to the emergence of a new model of management called the competence [1].

Each activity is associated with required competences. Solving assignment problem, will, therefore, lead to find the right resource and the best time to carry out the activity. The assignment of tasks and the organization of the activity of human resources of the maintenance service allow the service to improve considerably its efficiency.

Many studies have focused on the problems of human resource allocation taking into account the competences. Some consider the difference between the qualification levels of the operators as a factor that directly impact the operational performance. Authors, in other studies, have shown that the real duration of a task varies linearly according to the rate of competence [2] or according to the productivity [3] of the resource. [4] assume that the execution time of a task varies by a certain pre-set percentage according to the level of expertise on the resource. The integration of competences in the formulation of the problems of assignment aroused the interest of many researchers. Various approaches and methods of resolution were proposed. We plan, within the framework of this work, to enrich this base by putting forward the impact of the joint introduction of competences and partitioning problem for optimal resource allocation on the performance of an organization. The formulation and the approach of resolution of the problem of assignment which we propose in the following section, is a generalization of previous works of [5].

The aim of this paper is to develop a general approach of optimization and visual simulation by cellular automata based on the integration of spatio-temporal factors in a resources allocations problem (Location of equipment's and facilities, technician displacements) We study the particular case of a manufacturing company consisting of several equipments and facilities, geographically distributed, subject to wide kinds of maintenance interventions requiring different competences. We are particularly interested in the process of assigning interventions of preventive maintenance tasks. Consider a set of
equipment spread over a space and a set of technicians with competences. Maintenance technicians must perform different types of interventions. These maintenance activities are beings managed by allocating tasks to the most appropriate resource available and the closest spatially.

## 2 Problem Statement

The model must take into account various constraints: tasks duration, significant displacement time between the equipment and facilities of the site, optimal scenarios of intervention tasks, precedence between tasks of the same entity and competence required to each task. We have to consider, also, unplanned interventions due to corrective maintenance. The number of interventions that can be processed simultaneously depends on the technician's skills level and on their competences. The response time is conditioned by the choice of the resource and its proximity to the equipment or installation. However, all technicians must be engaged in a uniform manner, to avoid work overload and optimize transport and travel.

## System model:

We consider:

- A set of equipments noted $E$ with $M$ elements. We assume that each equipment with index $j$ for $1 \leq j \leq M$ requires intervention with $k$ competences $1 \leq k \leq I$ where $I$ is the total number of different competences required for all equipment. We note $\mathbf{E}_{j}^{k}$ the equipment index $j$ and requiring $k$ competence;
- A set of technicians noted $\mathbf{T}$ compound $P$ technician. We assume that each technician with index $i$ technician, for $1 \leq i \leq P$ has $n$ qualifications with $1 \leq n \leq Q$ where $Q$ is the total number of different skills of all technicians. We note $\mathbf{T}_{i}^{n}$ the technician index $i$ with $n$ competences.

Also we assume that each equipment $\mathbf{E}_{j}^{k}$ requires preventive" interventions. For each equipment $\mathbf{E}_{j}^{k}$, each intervention $k$ should be performed after $T_{k}^{I}$ (Mean Time Between Failures) and it lasts $T_{k}^{R}$ (Mean Time To Repair).

We consider the following problem : How to allocate technician's to the different maintenance tasks taking into account required skill, technicians competences and their spatial positions in order to reach optimal performance as : less equipment downtime, Better use of in-shop maintenance personnel , less technician displacement ... etc ...

We will formulate this problem as a cellular automaton dynamical system. Cellular Automata [6] and Voronoi diagram [7] will be the basic tools to be implemented in the present approach. The use of evolutionary methods in spatial optimization problems such as the ones outlined here is called for by their complexity and nonlinearity. Additionally, a particular characteristic of these problems is the relation between local interactions and global system behaviour. We recall the Cellular Automata and Voronoi diagram definitions and properties.

### 2.1 Cellular Automata

Cellular automata date back to von Neumann. Their fundamental importance was demonstrated by Wolfram (2002). They have been used as a background for modelling a great diversity of natural, as well as social and economic systems. Also, numerous applications have been presented for the spatial analysis.

## Definition: A cellular automaton is a quadruple $\mathbf{A}=(\mathbf{L}, \mathbf{S}, N, f)$ where:

- L is a lattice defined by a regular grid of cells denoted by $C_{i_{1} i_{2} \cdots i_{d}}$, on a dimensional domain $\Omega$,
- $\mathbf{S}$ a finite set of states given as a commutative ring in which modular arithmetic will be used,
- $N$ a neighbourhood of the cell $c$,
- $\quad f$ a set of local transition rules which update the state of cell $c$. It can be given by a transition function :

$$
f: \begin{array}{ll}
\mathbf{S}^{N(c)} & \rightarrow \mathbf{S} \\
s_{t}(N(c)) & \rightarrow \\
s_{t+1}(c)
\end{array}
$$

### 2.2 Voronoi diagram adapted to lattice

Consider a lattice $\mathbf{L}$ of $p \times q$ cells and a set of cells

$$
\mathbf{C}=\left\{C_{i j} \in \mathbf{L},(i, j) \in \Delta\right\}
$$

Where $\Delta=\left\{(i, j): 1 \leq i \leq p^{\prime} ; 1 \leq j \leq q^{\prime}\right\}$ and $p, q \in N$

Definition: we define:

- The Voronoi cell $V\left(C_{i j}\right)$ of $C_{i j}$ as the subset of L defined by:

$$
V\left(C_{i j}\right)=\left\{C \in \mathrm{C}: d\left(C, C_{i j}\right) \leq d\left(C, C_{l k}\right) \text { for all }(i, j) \neq(l, k) ; \text { and }(l, k) \in \Delta\right\}
$$

Where $d\left(C, C_{i j}\right)$ is a distance from $C$ to $C_{i j}$.
And the Voronoi diagram of $\mathbf{C}$ is the set $\operatorname{Vor}(\mathbf{C})=\bigcup_{(i, j) \in \Delta} V\left(C_{i j}\right)$

- The cells $C_{i j},(i, j) \in \Delta$ are called seed (site, or generator) the Voronoi cell.


## Remark:

The Voronoi cell of seed $C_{i j}$ is the region of all Cellular Automaton cells closer to that seed than to any other. We will use this property to approach our problem.

We recall some important properties of Voronoi diagram that we will use in our approach.

## Proposition:

- The intersection of two Voronoi cells is either empty or an edge
- The intersection of three Voronoi cell is either empty or a Cellular Automata cell called Voronoi vertex
- A Voronoi cell is convex.


### 2.3 CONCEPTUAL FRAMEWORK

To simplify the notations and present clearly our work, a hypothetical problem is considered in order to illustrate, without loss of generality, we assume that $M=Q=2$. At the end of the paper we give the algorithm in the case Where $M$ and / or $Q$ is greater than 2

In this case we get $T_{k}^{I}$ and $T_{k}^{R}$ for $k=1.2$. These times are in general random, and in the case they may be deterministic and $T_{k}^{I}$ decreasing, $T_{k}^{R}$ is growing according to "age" or "running time" of the equipment. We do not consider the random case in the present work but it will be considered in future work.

To apply the cellular automata approach to the resource allocation problem, we start by defining the corresponding cellular automaton $\mathbf{A}=(\mathbf{L}, \mathbf{S}, N, f)$.
 represents the total number of cells $L$. We assume that our domain is bounded.
$\mathbf{S}$ a finite set of states of cells. A cell is either empty, contains equipment operational or equipment down or an assigned for maintenance or available, busy or assigned technician:

Equipments are indexed in a set $\mathbf{m}, i=\{1, \ldots, E\}$ and technicians in a set $\mathbf{P}=\{1, \cdots, p\}$.
The equipment, noted $\mathbf{E}_{j}^{k}$ for $k=1,2$, can be Operational or Down. In the case it fails it may need the Intervention 1 or 2 or Both 1 and 2 and if any technician is available he can be assigned. So we can assign to each equipment a state values:

$$
\theta\left(\mathbf{E}_{j}^{k}\right)= \begin{cases}3 & \text { equipment } \mathbf{E}_{j}^{k} \text { operational } \\ -1 & \text { need intervention 1 } \\ -0.5 & \text { is assigned for intervention 1 } \\ -2 & \text { need intervention 2 } \\ -1.5 & \text { is assigned for intervention 2 } \\ -3 & \text { need intervention } 1 \text { and } 2 \\ -2.5 & \text { is assigned for intervention } 1 \text { and } 2 \\ -2.1 & \text { need intervention } 1 \text { and } 2 \text { and assigned 1 } \\ -2.2 & \text { need intervention } 1 \text { and } 2 \text { and assigned } 2\end{cases}
$$

Each equipment $\mathbf{E}_{j}^{k}$ need intervention after running time (Mean Time Between Failures) $T_{k}^{I}$. The intervention lasts (Mean Time To Repair) $T_{k}^{R}$.

The technician, noted $\mathrm{T}_{i}^{n}$ for $n=1,2$, has a state $\varphi$ (.) defined by:

$$
\varphi\left(T_{i}^{n}\right)=\left\{\begin{array}{cc|}
4 & \text { if the technician } T_{i}^{n} \text { is available and have two competencies 1 and 2 } \\
-4 & \text { if } T_{i}^{n} \text { is assigned to an equipment for intervention requring both comptencies 1 and 2 } \\
3.1 & \text { if the technician } T_{i}^{n} \text { is available and have competency 1 } \\
3.2 & \text { if the technician } T_{i}^{n} \text { is available and have competency } 2 \\
-3.1 & \text { if the technician } T_{i}^{n} \text { is assigned to an equipment requiring competency1 } \\
-3.2 & \text { if the technician } T_{i}^{n} \text { is assigned to an equipement requiring competency } 2
\end{array}\right.
$$

Then the state space is given by: $\mathbf{s}=\{3,-1,-0.5,-2,-1.5,-3,-2.5,-2.1,-2.2,4,-4,3.1,3.2,-3.1,-3.2\}$ where the state of each cell is:

- $e\left(c_{i j}\right)=0$ if the cell is empty.
- $e\left(c_{i j}\right)=3$, if the cell is occupied by an operational equipment ;
- $e\left(c_{i j}\right)=-1$ if the cell is occupied by a down equipment requiring competence 1 ;
- $e\left(c_{i j}\right)=-0.5$ if the cell is occupied by an assigned down equipment for intervention with competence 1 ;
- $e\left(c_{i j}\right)=-2$ if the cell is occupied by a down equipment requiring competence 2 ;
- $e\left(c_{i j}\right)=-1.5$, if the cell is occupied by an assigned down equipment for intervention with competence 2 ;
- $e\left(c_{i j}\right)=-3$ if the cell is occupied by a down equipment requiring both competence 1 and 2 ;
- $e\left(c_{i j}\right)=-2.5$ if the cell is occupied by an assigned down equipment for intervention with both competence 1 and 2 ;
- $e\left(c_{i j}\right)=-2.1$ if the cell is occupied by an assigned down equipment for intervention with competence 1 ;
- $e\left(c_{i j}\right)=-2.2$, if the cell is occupied by a down equipment requiring competence 2 ;
- $e\left(c_{i j}\right)=4$ if the cell is occupied by an technician not available ;
- $e\left(c_{i j}\right)=-4$ if the cell is occupied by an technician available with two competences 1 and 2 ;
- $e\left(c_{i j}\right)=3.1$ if the cell is occupied a technician available with competence 1 ;
- $e\left(c_{i j}\right)=3.2$ if the cell is occupied by a technician available with competence 2 ;
- $e\left(c_{i j}\right)=-3.1$, if the cell is occupied by an assigned technician to 1 ;
- $e\left(c_{i j}\right)=-3.2$ if the cell is occupied by an assigned technician to 2 ;
- Neighbourhood: We consider the Von Neumann or the Moore neighbourhoods which are characterized by a radius $r$ and defined by: $N(c)=\left\{c^{\prime} \in \mathbf{L} \mid\left\|c^{\prime}-c\right\|_{i} \leq r\right\}$ where $\|c\|_{i}, i=1, \infty$ indicates the sum and the maximum respectively, of the absolute value of the components of cell $c$.
- Transition rules: The evolution of the state of each cell $c_{i j}$, depends on its state and those of its neighbourhood and the technicians displacement. It is summarized as follow:

$$
e_{t}\left(c_{i j}\right) \rightarrow e_{t+1}\left(c_{i j}\right)
$$

if $e_{t_{i}}\left(c_{i j}\right)=0$ then:

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
4 \text { or } 3.1 \text { or } 3.2 & \text { if occupaied by, respectively, an available technician with } 1 \text { and } 2, \text { or } 1 \text { or } 2 \\
-4 \text { or }-3.1 \text { or }-3.2 & \text { if occupaied by, respectively, an assigned technician for } 1 \text { and } 2, \text { for } 1 \text { or for } 2 \\
0 & \text { otherwise }
\end{array}\right.
$$

if $e_{t_{i}}\left(c_{i j}\right)=3$ then:

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
3 \text { if the equipementoperational } & \\
-1 \text { or }-2 \text { or }-3 & \text { if the equipement requires } 1 \text { or } 2 \text { or }(1 \text { and } 2) \\
0 & \text { if the equipment is removed }
\end{array}\right.
$$

if $e_{t_{i}}\left(c_{i j}\right)=-3$ then

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
-2.5 \text { or }-0.5 \text { or }-2.5 & \text { if assigned respectively for }(1 \text { and } 2) \text { or } 1 \text { or } 2 \\
-3 & \text { if not assigned } \\
0 & \text { if the equipment is removed }
\end{array}\right.
$$

if $e_{t_{i}}\left(c_{i j}\right)=-1$ or -2 then

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
-0.5 \text { or }-1.5 & \text { if assigned } \\
-1 \text { or }-2 & \text { if not assigned }
\end{array}\right.
$$

if $e_{t_{i}}\left(c_{i j}\right)=-1.5($ or -2.5 or -0.5$)$ then :

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
4 & \text { Equipement fixed } \\
-1,5 & \text { Si } c_{i j} \text { Equipement not yet fixed }
\end{array}\right.
$$

if $e_{t_{i}}\left(c_{i j}\right)=-2.1$ or -2.2 then:

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
-2.5 & \text { if assigned for } 1 \text { and } 2 \\
-2.1 \text { or }-2.2 & \text { if no new assignment } ? \\
-2 \text { or }-1 & 1 \text { fixed and need } 2 \text { or } 2 \text { fixed and need } 1
\end{array}\right.
$$

if $e_{t_{i}}\left(c_{i j}\right)=4$ or 3.1 or 3.2 then:

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
4 & \text { technician still available } \\
-4 \text { or }-3.1 \text { or }-3.2 & \text { if the technician is assigned to } 1 \text { and } 2 \text { or to } 1 \text { or to } 2
\end{array}\right.
$$

$$
\text { if } e_{t_{i}}\left(c_{i j}\right)=-4 \text { or }-3.1 \text { or }-3.2 \text { then : }
$$

$$
e_{t_{i+1}}\left(c_{i j}\right)=\left\{\begin{array}{cc}
-4 \text { or }-3.1 \text { or }-3.2 & \text { technician still repairing } \\
4 \text { or } 3.1 \text { or } 3.2 & \text { finished fixing the equipment and become available? } \\
0 & \text { the technician left the cell }
\end{array}\right.
$$

## 3 Problem Approach

The problem consists in keeping all (or maximum) equipments operational. In case where some interventions of technicians are required, the problem is to perform them with an optimal way: minimal cost, energy, displacements.... etc;

### 3.1 Resource allocation Problem

As we have assumed that each equipment may require both interventions with competences 1 and 2 then we consider two cases: The case where these two interventions are independent and the case where these interventions are prioritized (eg 1 before 2 if the machine requires both at the same time).

1- Case where Interventions are independents.

## Algorithm:

- We divide all the down equipments into three subsets $\mathbf{E}_{1}, \mathbf{E}_{2}$ and $\mathbf{E}_{12}$ equipment requiring respectively the intervention with skill 1 , skill 2 and the both 1 and 2 at the same time.

Note that $\mathbf{E}=\mathbf{E}_{1} \cup \mathbf{E}_{2} \cup \mathbf{E}_{12} \cup\{$ operational equipments $\}$.

- We divide all technicians available in three subsets $\mathbf{T}_{1}, \mathbf{T}_{2}$ and $\mathbf{T}_{12}$ technicians with the competences respectively 1,2 and both 1 and 2.

Note that $\mathbf{T}=\mathbf{T}_{1} \cup \mathbf{T}_{2} \cup \mathbf{T}_{12} \cup\{$ Technicians assigned $\}$.
Remark Priority is given to equipments requiring a highest number of skills in our case both 1 and 2.

- We consider a first Voronoi diagrams relatively to $\mathbf{E}_{12}$. In the case there's only one technician available with the required competences in each Voronoi cell, we assign it.

Otherwise, if heir's more than one technician we consider a second Voronoi diagram $\mathbf{T}_{12}$. The intersection between the two diagrams is either empty, in that case no assignment can be performed or contain a technician then this one is assigned.

- We eliminate the assigned equipments and technicians and go back to 1.

Note $\mathbf{E}(n o)_{12}$ and $\mathbf{T}(n o)_{12}$ respectively all equipments and technicians not assigned.

- As it was assumed that interventions are independent, equipments and technicians assigned for both 1 and 2 are removed we consider then the sets $\mathbf{E}_{1} \cup \mathbf{E}(n o)_{12}$ and $\mathbf{T}_{1} \cup \mathbf{T}(n o)_{12}$. We go back to 1 by replacing the sets $\mathbf{E}_{12}$ and $\mathbf{T}_{12}$ by $\mathbf{E}_{1} \cup \mathbf{E}(n o)_{12}$ and $\mathbf{T}_{1} \cup \mathbf{T}(n o)_{12}$, respectively;

2- In case interventions are prioritized We do the same thing for interventions with skills 2.

## Generalization $\mathrm{n}>\mathbf{2}$ :

$i=j=n$ Do while there is down equipments and not assigned : Draw Voronoi diagram for down equipments necessitating the skills $j$; If in the Voronoi cell, of seed (site, or generator) $\mathbf{E}_{j}$, there is only one technician having competences jassign it;

If there is more than one technician do: Draw Voronoi diagram for available technicians; Assign technician $\mathbf{T}_{j}$ to equipment $\mathbf{E}_{j}$ if they are in the intersection of the two Voronoi cells with seed (site, or generator) $\mathbf{E}_{j}$ and $\mathbf{T}_{j}$ respectively.

$$
j=j-1
$$

Remark 1) Given the properties of the Voronoi diagram, we have two extreme cases:

- A down equipment (respectively an available technician) is midway between two available technicians (respectively two equipments down);
- A down equipment (respectively an available Technician) is at a scabies distance of three available technicians (respectively three down equipment);
- In those cases the rules of priority are to be specified. In this work we consider the following rules: The technician who is in the EAST takes priority and if there's more than one the one in the NORTH. This rule allows us to take in account all extreme cases considered above.
- If the interventions are prioritized we change the order in the algorithm (eg, interventions 2 before 1 ).


### 3.2 Problem of technician's displacements

For the displacement of the technicians we consider the approach proposed by Ouardouz et al. [5] which we will remind the principle.

Displacement rules: Respect to the Voronoi diagram, the following traffic rules are adopted:

- If two technicians cross in perpendicular directions: the rule of the priority to the right will be adopted.
- If two technicians target the same empty cell with opposite directions, we perform, for at least one of them, a $\pi / 2$ rotation to the right of its direction
- If a technician goes to a cell containing equipment a clockwise rotation of the angle $\pi / 2$ is performed


## 4 Simulation And Results

### 4.1 Initial Conditions

For the simulation we consider the initial conditions represented in fig1 a company layout with:

- 5 down equipments of 7 and 3 available technicians.
- Tree competences Mechanical (1), Electrical (1) and Electromechanical (12)
- $T_{k}^{I}=3$ and $T_{k}^{R}=2$


Fig1. : Initial conditions

### 4.2 Simulation

The snapshots of the simulation are represented in fig2.


Fig2. : Simulation result
We first draw the Voronoi diagram considering the two down equipments requiring the highest number of competences E red in our case (Grid1 fig2). We assign after that the available technician with both the two competences T12 to the first equipment (Grid2 fig2), and the two technicians with respectively, competence 1 and 2 to the second equipment (Grid3 fig2).

We eliminate the assigned equipments and technicians and we draw a new Voronoi diagram (Grid4 fig2). We continue the algorithm until all down equipment are assigned.

## 5 Conclusion

In this paper, we considered the problem of spatial resource allocation in the multi task and multi competences cases. This work is an extension of the previous work done by Ouardouz et al. [5] who considered only one task and one competence. We propose an approach to the problem based on an algorithm for finding the optimal partition resource allocation. The approach is based on the known Cellular automata and Voronoi diagram techniques.

The contribution of our work is an efficient algorithm that combines multi task and multi resource competences allocation with the underlying spatial information for retrieving the final result.

Our approach is illustrated in the case where the equipments require tree tasks and technicians with two competences. Some simulations are presented.

We have considered in this paper that failure and interventions are deterministic however in practice such situations are random. It will be very interesting to extend this work to random cases. Such problems are under investigation.

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