# Implementation of a new approach for modeling and determining the electrical parameters of solar cells 

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#### Abstract

A new approach is presented in this work, to extract electrical parameters of a photovoltaic cell, using the double exponential model. The equivalent circuit parameters of this model are the photocurrent ( $l_{\text {ph }}$ ), ideality factor ( $\eta$ ), diffusion current ( $l_{\text {od }}$ ), recombination current ( $l_{\text {or }}$ ), series resistance $\left(R_{s}\right)$ and the shunt resistance ( $R_{\text {sh }}$ ). Several research studies have been performed to extract these parameters. The majority of these developed methods are limited on several levels. In this work the proposed technique is based on the equalization of the electric model of photovoltaic cells, and a polynomial model equivalent. The comparison of these two models at $\mathrm{I}=0$, allows representing the electrical parameters with the polynomial model coefficients. This method is tested on a monocrystalline solar panel and obtained results show the advantage of this technique in level of speed, convergence and precision.


KEYWORDS: Photovoltaic cell; double exponential model; parameters extraction; polynomial model; optimization.

## 1 Introduction

The Photovoltaic electricity is experiencing a potential interest in recent years on the scientific and economic levels. This interest is due to the growing demand of energy in most industrial sectors, and also to environmental obligations. The photovoltaic solar cells are in the heart of the electricity production chain. Competition on optimizing and increasing the efficiency of photovoltaic cells (PVC), leads researchers to find methods to determine the intrinsic parameters of these cells. These electrical parameters are ( $R_{s}$ : series resistance, $R_{s h}$ : shunt resistance, $I_{o r}$ and $I_{o d}$ : the saturation currents and ideality factor $\eta$ ). There have a very important role in electric modeling of PV junctions, they can give informations about the performance of these cells, and they can provide indications of degradation of those junctions during electrical operation [1]. The accuracy of these parameters is very requested for a proper analysis of this element. In literature, several studies were the subject of the development of methods to extract these parameters [2]. The existence of implicit current and complexity of electric models, limits the reliability of the majority of these methods. In this work, another simple polynomial model is proposed to solve this problem. This is to represent the PVC voltage V , depending with his current I by this polynomial model. The intrinsic electrical parameters are determined with the coefficients of this polynomial model. The validation of this method is made by a monocrystalline solar panel. An electrical test (I-V) of this panel is made with an automated electronic system. The analysis and interpretation of results are based on comparing the statistical indicators errors, which are calculated for both models.

## 2 BASIC ELECTRICAL MODELLING

A photovoltaic cell (PVC) can be studied by several models: analytical, electrical or numerical [3]. The supplied electric current by the cell is given by:

$$
\begin{equation*}
\mathrm{I}_{\mathrm{PVC}}=\mathrm{I}_{\mathrm{ph}}-\mathrm{I}_{\mathrm{J}} \tag{1}
\end{equation*}
$$

$I_{\text {PVC }}, I_{\text {ph }}$ and $I_{J}$ are respectively, the electrical current supplied by PVC (A), Photoelectric current (A) and the Loss current (A).
To simplify the study of the cell characteristic, we limited to study $\mathbf{I}_{\mathbf{J}}$ current, instead of $\mathbf{I}_{\text {pvc }}$. The characteristic is reversed and then it is shifted (Figure 1).


Fig. 1. I-V characteristic reversed and shifted
Among the most popular models in this area, the electric model with a single diode (Figure 2-a), and that of double diodes (Figure 2-b) [4]. In the first electrical model, the electrical current in PV junctions is described by the following equation:

$$
\begin{equation*}
\mathrm{I}=\mathrm{I}_{\mathrm{J}}=\mathrm{I}_{0} \cdot\left[\exp \left(\frac{\left(\mathrm{~V}-\mathrm{R}_{\mathrm{s}} \cdot \mathrm{I}\right)}{\eta \cdot \mathrm{V}_{\mathrm{th}}}\right)-1\right]+\frac{\left(\mathrm{V}-\mathrm{R}_{\mathrm{s}} \cdot \mathrm{I}\right)}{\mathrm{R}_{\mathrm{sh}}} \tag{2}
\end{equation*}
$$

$\mathrm{KT} / \mathrm{q}=\mathrm{V}_{\mathrm{th}}$ is the thermal voltage.
Those models is shown schematically by the following equivalent circuits:


Fig. 2. Electric models, (a) a single diode and (b) a double diodes
In the second electrical model of the photovoltaic cell (Figure-b), the electrical current (1) is described by the following expression:

$$
\begin{equation*}
I=I_{J}=I_{\text {or }}\left(e^{\frac{\left(V-I R_{s}\right)}{\eta \cdot V_{t h}}}-1\right)+I_{\text {od }}\left(e^{\frac{\left(V-I R_{s}\right)}{V_{t h}}}-1\right)+\frac{V-I R_{s}}{R_{\text {sh }}} \tag{3}
\end{equation*}
$$

$I_{\text {or }}$ and $I_{\text {od }}$ are respectively, the diffusion current and the recombination current. This model, known as the double exponential model [5, 6]. Intrinsic parameters of this model are associated with extrinsic parameters such as; the shortcircuit current $\mathbf{I}_{\mathrm{cc}}$, which represents the current value when the voltage to the cell is zero $(\mathrm{V}=0)$. It's approximated by:

$$
\begin{equation*}
\mathrm{I}_{\mathrm{cc}}=\mathrm{I}_{\mathrm{ph}} \cdot \frac{\mathrm{R}_{\mathrm{sh}}}{\left(\mathrm{R}_{\mathrm{s}}+\mathrm{R}_{\mathrm{sh}}\right)} \tag{4}
\end{equation*}
$$

After extracting $R_{s}$ and $R_{s h}$, the intrinsic parameter $I_{p h}$ will be deducted with these parameters and current $I_{c c}$ (equation 4).

## 3 Parameters extraction methods

Several methods have been developed to extract these parameters [7]. Among the most widely used methods, we can mention the lateral and vertical optimization [8], which are based on minimizing the sum of squares of the relative differences between the measured and the fitting values of current. The method of Lambert [9], uses the Lambert's function to find the exact analytical solution of the current. There are many numerical methods [10] that use an iterative algorithm to extract the parameters.

### 3.1 Graphical method

The parameter $\mathrm{R}_{\mathrm{s}}$ is only effective in the non-linear region of the forward $\mathrm{I}-\mathrm{V}$ characteristics at sufficiently high applied voltage, but parameters such as the ideality factor and saturation current are effective in both the linear and non-linear regions of these characteristics[16]. So we obtain the series resistance (Rs) from the slope of the linear region of the curve $\mathrm{dV} / \mathrm{dl}$ versus $1 / \mathrm{l}$ in high voltage, and the shunt resistance (Rsh) can be obtained from the slope of the low voltage region:


Fig. 3. Graphical method to extracting Rs and Rsh

Measuring angles " $a$ " and " $b$ ", the resistances $R_{s}$ and $R_{s h}$ are determined by:

$$
\begin{equation*}
\mathrm{Rs}=1 / \tan (\mathrm{a}) \quad ; \quad \mathrm{Rsh}=1 / \tan (\mathrm{b}) \tag{5}
\end{equation*}
$$

From the curve $\ln \left(I_{\mathrm{d}}\right)$ versus ( $\mathrm{Y}=\mathrm{V}-\mathrm{I} . \mathrm{Rs}$ ) we can obtained the parameters $\eta$ and $I_{0}$ from the slope and intercept of linear region on the curve $\ln \left(I_{d}\right)-Y$.

The graphical methods generally does determine that one or sometimes up to three parameters only five or six parameters of different models. This explains their validity for certain structures when the missing parameters are negligible. They produce values that are often very different from each other which is much more serious.

### 3.2 ANALYtical method using Lambert function

In the case of a single diode model, the implicit form of $I(V)$ cannot be constructed from just common elementary functions, but solution for this equation can be found in terms of the Lambert function, and the equation of model can be transformed to a simplified form by this transformation:

$$
\begin{equation*}
y e^{y}=x \Leftrightarrow y=W_{k}(x) \tag{6}
\end{equation*}
$$

And the explicit solution of $\mathrm{I}(\mathrm{V})$ is :

$$
\left.I=\frac{n V_{t h}}{R_{s}} W_{0}\left(n V_{t h} I_{0} e^{\left(\frac{\frac{V}{n V_{t h}-\left(V V_{t h}\right) R_{s}}}{n V_{t h} R_{s h}\left(1+\frac{R_{s}}{R_{s h}}\right)}\right.} \begin{array}{l}
n V_{s s h}\left(1+\frac{R_{s}}{R_{s h}}\right. \tag{7}
\end{array}\right)\right)+\frac{V}{R_{s h}\left(1+\frac{R_{s}}{R_{s h}}\right)}-\frac{I}{\left(1+\frac{R_{s}}{R_{s h}}\right)}
$$

The Lambert function is implemented in Mathematica under the name ProductLog or Matlab under the name Lambert. The presented method assumes as the most published methods, that all parameters are bias independent. Next limit of the method is connected with semiconductor property. For doping levels below approximately $10^{18}$ per $1 \mathrm{~cm}^{3}$, the carrier transport occurs via thermionic emission over the barrier given by model equation, while for larger doping levels tunnel or field emission through the then narrower depletion layer is predominant.

### 3.3 Numerical method using Levenberg-Marquardt algorithm

This method uses an algorithm developed based on iterations Levenberg Marquardt, to estimate the model parameters [11]. Modification of parameters is done according to the relationship:

$$
\begin{gather*}
P_{j}=P_{j-1}-\left[H_{j}+\lambda_{j-1}\right]^{-1} \cdot \nabla J_{j-1}  \tag{8}\\
\nabla \mathbf{J}=\operatorname{grad}(\mathbf{J}(\mathrm{P})) \tag{9}
\end{gather*}
$$

With $\mathrm{P}, \mathrm{J}$ and H are respectively the vector of electrical parameters to estimate, the Jacobean and the Hessian. The iteration step is $\left(1 / \lambda_{j-1}\right)$. The calculation of the Jacobean matrix is based on derivatives voltage equation $\mathrm{V}(\mathrm{I})$ in relation to electrical parameters:

$$
\begin{equation*}
\frac{\mathrm{dV}(\mathrm{I})}{\mathrm{dR}_{\mathrm{s}}} ; \frac{\mathrm{dV}(\mathrm{I})}{\mathrm{dR}_{\mathrm{sh}}} ; \frac{\mathrm{dV}(\mathrm{I})}{\mathrm{dI}_{\mathrm{od}}} ; \frac{\mathrm{dV}(\mathrm{I})}{\mathrm{dI}_{\mathrm{or}}} ; \frac{\mathrm{dV}(\mathrm{I})}{\mathrm{d} \eta} \tag{10}
\end{equation*}
$$

## 4 PROPOSED NUMERICAL METHOD

The proposed method in this work is already tested with a single diode model, and it has shown good results [11]. In case of the double exponential model, the voltage (V) of PVC was replaced by a polynomial model of the current (I). So we use a polynomial voltage to solve the problem of the implicit current, as:

$$
\begin{equation*}
\tilde{\mathrm{V}}(\mathrm{I})=\sum_{\mathrm{i}=\mathrm{o}}^{+\infty} \mathrm{b}_{\mathrm{i}} \mathrm{I}^{\mathrm{i}}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{I}+\ldots+\mathrm{b}_{\mathrm{n}} \mathrm{I}^{\mathrm{n}}+\operatorname{Err}(\mathrm{I}) \tag{11}
\end{equation*}
$$

With $\operatorname{Err}(\mathrm{I})=\sum_{\mathrm{i}=\mathrm{n}+1}^{+\infty} \mathrm{b}_{\mathrm{i}} \mathrm{I}^{\mathrm{i}}$,
The estimator must verify the equ. 3 as:

$$
\begin{equation*}
I=I_{o r}\left(e^{\frac{q\left(\tilde{V}(I)-I R_{s}\right)}{\eta K T}}-1\right)+I_{o d}\left(e^{\frac{q\left(\tilde{V}(I)-I R_{s}\right)}{K T}}-1\right)+\frac{\tilde{V}(I)-\mathrm{IR}_{s}}{R_{\text {sh }}} \tag{12}
\end{equation*}
$$

It uses that the derivatives at the origin of the current, are proportional to the coefficients $\left(b_{i}\right)$ of the estimator $\tilde{\mathrm{V}}(\mathrm{I})$, such as:

$$
\begin{gather*}
\tilde{\mathrm{V}}(0)=\mathrm{b}_{0} ; \quad \frac{\mathrm{d} \tilde{\mathrm{~V}}(0)}{\mathrm{dI}}=\mathrm{b}_{1} ; \quad \frac{\mathrm{d}^{3} \tilde{\mathrm{~V}}(0)}{\mathrm{dI}^{3}}=6 \mathrm{~b}_{3} \\
\frac{\mathrm{~d}^{\mathrm{n}} \tilde{\mathrm{~V}}(0)}{\mathrm{dI}}=\mathrm{n}!\mathrm{b}_{\mathrm{n}} \tag{13}
\end{gather*}
$$

The choice set of polynomial estimator is that it is infinitely differentiable, and that error and its derivative of order ( n ) are zero at the origin, as follows:

$$
\begin{align*}
& \operatorname{Err}(\mathrm{I})=\sum_{\mathrm{i}=\mathrm{n}+1}^{+\infty} \mathrm{b}_{\mathrm{i}} \mathrm{I}^{\mathrm{i}}  \tag{14}\\
& \mathrm{I}=0  \tag{15}\\
&=0 \\
&\left.\frac{\mathrm{~d}^{n} \operatorname{Err}(\mathrm{I})}{d I^{n}}\right|_{\mathrm{I}=0}=0
\end{align*}
$$

The equality of these two models, at origin current $(I=0)$, gives an analytical solution of the electrical parameters based on polynomial coefficients of the estimated voltage. Furthermore the coefficients $b_{n}$ are easy to estimate using either the function Polyfit of the library Matlab, or using the function FindFit of the library Mathematica [12]. We combine the two models, using the estimator derived from the voltage $\tilde{\mathrm{V}}(\mathrm{I})$ relative to I . The derivative up to order 5 is sufficient to determine a system of equations with the five unknowns ( $\eta, I_{o r}, I_{o d}, R_{s}$ and $R_{s h}$ ). The following notations will be used later to indicate the four derivatives with respect to $I$ :

$$
\begin{align*}
& \frac{d \tilde{V}(I)}{d I}=\tilde{V}^{\prime}  \tag{16}\\
& \frac{d^{2} \tilde{V}(I)}{d I^{2}}=\tilde{V}^{\prime \prime}  \tag{17}\\
& \frac{d^{3} \tilde{V}(I)}{d^{3}}=\tilde{V}^{\prime \prime \prime}  \tag{18}\\
& \frac{d^{n} \tilde{V}(I)}{d I^{n}}=\tilde{V}^{(n)} \tag{19}
\end{align*}
$$

and

The derivatives of order less than 5 relative to the current, yields a system of nonlinear equations, grouping the electrical parameters and polynomial coefficients. We obtain the system in equation (20). At $\mathrm{I}=0$, and taking account of this system and equations (11-14), we obtain a new system of five nonlinear equations and five unknowns. We find an analytic solution of system of equations (21), the numerical resolution is feasible by several methods [13] (fsolve function of Matlab).

$$
\left\{\begin{array}{c}
I_{0 r}\left(e^{\frac{\tilde{V}-I R_{s}}{\eta V_{t h}}}-1\right)+I_{0 d}\left(e^{\frac{\tilde{V}-I R_{s}}{V_{t h}}}-1\right)+\frac{\tilde{V}-I R_{s}}{R_{s h}}=I  \tag{20}\\
I_{0 r} \frac{\tilde{V}^{\prime}-R_{s}}{\eta V_{t h}} e^{\frac{\tilde{V}-I R_{s}}{\eta V_{t h}}}+I_{0 d} \frac{\tilde{V}^{\prime}-R_{s}}{V_{t h}} e^{\frac{\tilde{V}-I R_{s}}{V_{t h}}}+\frac{\tilde{V}^{\prime}-R_{s}}{R_{s h}}=1 \\
I_{0 r} e^{\frac{\tilde{V}-I R_{s}}{\eta V_{t h}}}\left(\left(\frac{\tilde{V}^{\prime}-R_{s}}{\eta V_{t h}}\right)^{2}+\frac{\tilde{V}^{\prime \prime}}{\eta V_{t h}}\right)+I_{0 d} e^{\frac{\tilde{V}-I R_{s}}{V_{t h}}}\left(\left(\frac{\tilde{V}^{\prime}-R_{s}}{V_{t h}}\right)^{2}+\frac{\tilde{V}^{\prime \prime}}{V_{t h}}\right)+\frac{\tilde{V}^{\prime \prime}}{R_{s h}}=0 \\
I_{0 r} e^{\frac{\tilde{V}-I R_{s}}{\eta V_{t h}}}\left(\left(\frac{\tilde{V}^{\prime}-R_{s}}{\eta V_{t h}}\right)^{3}+\frac{3\left(\tilde{V}^{\prime}-R_{s}\right) \tilde{V}^{\prime \prime}}{(\eta V t h)^{2}}+\frac{\tilde{V}^{\prime \prime \prime}}{\eta V_{t h}}\right)+ \\
I_{0 d} e^{\frac{\tilde{V}-I R_{s}}{V_{t h}}}\left(\left(\frac{\tilde{V}^{\prime}-R_{s}}{V_{t h}}\right)^{3}+\frac{3\left(\tilde{V}^{\prime}-R_{s}\right) \tilde{V}^{\prime \prime}}{(V t h)^{2}}+\frac{\tilde{V}^{\prime \prime \prime}}{V_{t h}}\right)+\frac{\tilde{V}^{\prime \prime \prime}}{R_{s h}}=0 \\
I_{0 r} e^{\frac{\tilde{V}-I R_{s}}{\eta V_{t h}}}\left(\left(\frac{\tilde{V}^{\prime}-R_{s}}{\eta V_{t h}}\right)^{4}+\frac{3 \tilde{V}^{\prime \prime 2}}{(\eta V t h)^{2}}+\frac{6 \tilde{V}^{\prime \prime}\left(\tilde{V}^{\prime}-R_{s}\right)^{2}}{(\eta V t h)^{3}}+\frac{4\left(\tilde{V}^{\prime}-R_{s}\right) \tilde{V}^{\prime \prime \prime}}{(\eta V t h)^{2}}+\frac{\tilde{V}^{\prime \prime \prime \prime}}{\eta V_{t h}}\right)+ \\
I_{0 d} e^{\frac{\tilde{V}-I R_{s}}{n V_{t h}}}\left(\left(\frac{\tilde{V}^{\prime}-R_{s}}{V_{t h}}\right)^{4}+\frac{3 \tilde{V}^{\prime, 2}}{(V t h)^{2}}+\frac{6 \tilde{V}^{\prime \prime}\left(\tilde{V}^{\prime}-R_{s}\right)^{2}}{(V t h)^{3}}+\frac{4\left(\tilde{V}^{\prime}-R_{s}\right) \tilde{V}^{\prime \prime \prime}}{(V t h)^{2}}+\frac{\tilde{V}^{\prime \prime \prime \prime}}{V_{t h}}\right)+\frac{\tilde{V}^{\prime \prime, ' \prime}}{R_{s h}}=0
\end{array}\right.
$$

After many simplifications, and thereafter, the replacement of equation (3) in this system can give the following equations:

$$
\begin{aligned}
& \int \quad I_{0 r}\left(e^{\frac{b_{0}}{\eta V_{\text {th }}}}-1\right)+I_{0 d}\left(e^{\frac{b_{0}}{V_{\text {th }}}}-1\right)+\frac{b_{0}}{R_{\text {sh }}}=0 \\
& I_{0 r} \frac{b_{1}-R_{s}}{\eta V_{\text {th }}} e^{\frac{b_{0}}{\eta V_{\text {th }}}}+I_{0 d} \frac{b_{1}-R_{s}}{V_{\text {th }}} e^{\frac{b_{0}}{V_{\text {th }}}}+\frac{b_{1}-R_{s}}{R_{\text {sh }}}=1 \\
& I_{0 r} e^{\frac{\mathrm{b}_{0}}{\eta V_{\mathrm{th}}}}\left(\left(\frac{\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}}{\eta \mathrm{~V}_{\mathrm{th}}}\right)^{2}+\frac{2 \mathrm{~b}_{2}}{\eta \mathrm{~V}_{\mathrm{th}}}\right)+\mathrm{I}_{0 \mathrm{~d}} \mathrm{e}^{\frac{\mathrm{b}_{0}}{\mathrm{~V}_{\mathrm{th}}}}\left(\left(\frac{\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}}{\mathrm{~V}_{\mathrm{th}}}\right)^{2}+\frac{2 \mathrm{~b}_{2}}{\mathrm{~V}_{\mathrm{th}}}\right)+\frac{2 \mathrm{~b}_{2}}{\mathrm{R}_{\mathrm{sh}}}=0 \\
& \mathrm{I}_{0 \mathrm{r}} \mathrm{e}^{\frac{\mathrm{b}_{0}}{\eta \mathrm{~V}_{\mathrm{th}}}}\left(\left(\frac{\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}}{\eta \mathrm{~V}_{\mathrm{th}}}\right)^{3}+\frac{6\left(\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}\right) \mathrm{b}_{2}}{(\eta \mathrm{Vth})^{2}}+\frac{6 b_{3}}{\eta \mathrm{~V}_{\mathrm{th}}}\right)+ \\
& I_{0 d} e^{\frac{b_{0}}{V_{\text {th }}}}\left(\left(\frac{b_{1}-R_{s}}{V_{\text {th }}}\right)^{3}+\frac{6\left(b_{1}-R_{s}\right) b_{2}}{(V t h)^{2}}+\frac{6 b_{3}}{V_{t h}}\right)+\frac{6 b_{3}}{R_{\text {sh }}}=0 \\
& I_{0 r} e^{\frac{b_{0}}{\eta V_{\mathrm{th}}}}\left(\left(\frac{b_{1}-R_{s}}{\eta V_{t h}}\right)^{4}+\frac{12 b_{2}^{2}}{(\eta \mathrm{Vth})^{2}}+\frac{12 b_{2}\left(b_{1}-R_{s}\right)^{2}}{(\eta \mathrm{Vth})^{3}}+\frac{24\left(\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}\right) \mathrm{b}_{3}}{(\eta \mathrm{Vth})^{2}}+\frac{24 \mathrm{~b}_{4}}{\eta \mathrm{~V}_{\mathrm{th}}}\right)+ \\
& \mathrm{I}_{\mathrm{Od}} \mathrm{e}^{\frac{\mathrm{b}_{0}}{\mathrm{~V}_{\mathrm{th}}}}\left(\left(\frac{\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}}{\mathrm{~V}_{\mathrm{th}}}\right)^{4}+\frac{12 \mathrm{~b}_{2}{ }^{2}}{(\mathrm{Vth})^{2}}+\frac{12 \mathrm{~b}_{2}\left(\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}\right)^{2}}{(\mathrm{Vth})^{3}}+\frac{24\left(\mathrm{~b}_{1}-\mathrm{R}_{\mathrm{s}}\right) \mathrm{b}_{3}}{(\mathrm{Vth})^{2}}+\frac{24 \mathrm{~b}_{4}}{\mathrm{~V}_{\mathrm{th}}}\right)+\frac{24 \mathrm{~b}_{4}}{\mathrm{R}_{\text {sh }}}=0
\end{aligned}
$$

## 5 RESULTS AND DISCUSSIONS

### 5.1 EXTRACTED PARAMETERS

The numerical resolution of the latter system is possible, but the analytical resolution requires the introduction of a new equation to eliminate the exponential term which contains the ideality factor. The number of parameters is five; it takes five equations to find the exact expression of these parameters. In this case, we happen to find the expression of four parameters, and the last parameter will be found numerically. This is the root of a polynomial of degree six, as a result:

$$
\begin{gather*}
\eta=\frac{b_{2}\left(b_{1}-R_{s}\right)^{4}+3\left(b_{1}-R_{s}\right)^{2}\left(2 b_{2}{ }^{2}+b_{3}\left(R_{s}-b_{1}\right)\right) V_{t h}}{3 V_{t h}\left(\left(-2 b_{2}{ }^{2}+b_{3}\left(b_{1}-R_{s}\right)\right)\left(b_{1}-R_{s}\right)^{2}-4\left(5 b_{2}^{3}+b_{4}\left(b_{1}-R_{s}\right)^{2}+5 b_{2} b_{3}\left(R s-b_{1}\right)\right) V_{t h}\right)} \\
R_{s h}=\frac{3 V_{t h}\left(-2 b_{2}{ }^{2}+b_{3}\left(b_{1}-R_{s}\right)\left(b_{1}-R_{s}\right)^{5}-12\left(b_{1}-R_{s}\right)^{3}\left(5 b_{2}^{3}+b_{4}\left(b_{1}-R_{s}\right)^{2}+5 b_{2} b_{3}\left(R_{s}-b_{1}\right)\right)\right.}{\left(-4 b_{2}{ }^{2}+3 b_{3}\left(b_{1}-R_{s}\right)\right)\left(b_{1}-R_{s}\right)^{4}-6 V_{t h}\left(b_{1}-R_{s}\right)^{2}\left(8 b_{2}{ }^{3}+2 b_{4}\left(b_{1}-R_{s}\right)^{2}+9 b_{2} b_{3}\left(R_{s}-b_{1}\right)\right)-6\left(8 b_{2}^{4}-3 b_{3}{ }^{2}\left(b_{1}-R_{s}\right)^{2}+4 b_{2} b^{4}\left(b_{1}-R_{s}\right)^{2}+8 b_{2}{ }^{2} b_{3}\left(R_{s}-b_{1}\right)\right) V_{t h}} \\
I_{o d}=\frac{-6 e^{-\frac{b_{0}}{V_{t h}}\left(8 b_{2}{ }^{4}-3 b_{3}{ }^{2}\left(b_{1}-R_{s}\right)^{2}+4 b_{2} b_{4}\left(b_{1}-R_{s}\right)^{2}+8 b_{2}{ }^{2} b_{3}\left(R_{s}-b_{1}\right)\right) V_{t h}^{4}}}{\left(b_{1}-R_{s}\right)^{3}\left(b_{2}\left(b_{1}-R_{s}\right)^{4}+6\left(b_{1}-R_{s}\right)^{2}\left(2 b_{2}{ }^{2}+b_{3}\left(R_{s}-b_{1}\right)\right) V_{t h}+12\left(5 b_{2}^{3}+b_{4}\left(b_{1}-R_{s}\right)^{2}+5 b_{2} b_{3}\left(R_{s}-b_{1}\right)\right) V_{t h}^{2}\right)}  \tag{24}\\
I_{o r}=-e^{\frac{b_{0}}{V_{t h}}} I_{o d}(\eta-1)+\frac{\left(b_{1}-R_{s}\right)\left(b_{0}-I_{o d} R_{s h}\right)+\eta\left(R_{s}+R_{s h}-b_{1}\right) V_{t h}}{\left(b_{1}-R_{s}\right) R_{s h}} \tag{25}
\end{gather*}
$$

The last parameter $R_{s}$ is a root of a polynomial of degree 6 .

### 5.2 Method validation

To validate our method, several tests were performed on a PV cell "H750". A database of I-V measurements were taken with an automated electronic device designed and manufactured by our research team [14].

Tests are made on a basis consists of 64 points measured, ranging from ( $10^{-5} \mathrm{~A} ; 19.03 \mathrm{~V}$ ) to ( $2.8 \mathrm{~A} ; 0.05 \mathrm{~V}$ ). We will, first, find the polynomial coefficients, find the value of the series resistance, and then inject them into the previous expressions. The values obtained are compared with the statistical indicators following: The standard deviation (SD), the normalized root of the mean square error (NRMSE) and the "t-statistics", defined by following relations [15]:

$$
\begin{gather*}
\mathrm{SD}=\sqrt{\frac{1}{\mathrm{~N}-1} \sum_{\mathrm{j}=1}^{\mathrm{N}}\left(\mathrm{~V}_{\mathrm{j}}-\hat{\mathrm{V}}_{\mathrm{j}}\right)^{2}}  \tag{19}\\
\mathrm{NRMSE}=\frac{\left(\frac{1}{\mathrm{~N}} \sum_{\mathrm{j}=1}^{\mathrm{N}}\left(\mathrm{~V}_{\mathrm{j}}-\hat{V}_{\mathrm{j}}\right)^{2}\right)^{1 / 2}}{\frac{1}{\mathrm{~N}} \sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{~V}_{\mathrm{j}}}  \tag{20}\\
\mathrm{NMBE}=\frac{\sum_{\mathrm{j}=1}^{\mathrm{N}}\left(\mathrm{~V}_{\mathrm{j}}-\mathrm{V}_{\mathrm{j}}\right)}{\sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{~V}_{\mathrm{j}}} \tag{12}
\end{gather*}
$$

$$
\begin{equation*}
t=\sqrt{\frac{(N-1) M B E^{2}}{\left(R M S E^{2}-M B E^{2}\right)}} \tag{13}
\end{equation*}
$$

Where MBE and RMSE defined by:

$$
\begin{align*}
\text { RMSE } & =\left(\frac{1}{\mathrm{~N}} \sum_{\mathrm{j}=1}^{\mathrm{N}}\left(\hat{\mathrm{~V}}_{\mathrm{j}}-\mathrm{V}_{\mathrm{j}}\right)\right)^{1 / 2}  \tag{14}\\
\text { MBE } & =\frac{1}{\mathrm{~N}} \sum_{\mathrm{j}=1}^{\mathrm{N}}\left(\hat{\mathrm{~V}}_{\mathrm{j}}-\mathrm{V}_{\mathrm{j}}\right) \tag{15}
\end{align*}
$$

With $\wedge_{\mathrm{V}}^{\mathrm{j}}, \quad \mathrm{V}_{\mathrm{j}}$ are respectively the estimated and measured voltage value at the iteration $j$, and N is the number of measures.

The values of the polynomial coefficients found are presented in table 1, depending of degree of polynomial and corresponding statistical errors. Much polynomial degree's taken; the statistical indicators provide information on short-term performance of the fit, and regarding the surplus or under-estimation of fit. A higher value of NRMSE means bad performance of the polynomial model with the coefficients found. A positive value of NMBE implies an overestimate, while negative values indicate an underestimation.

Table 1. Polynomial coefficients of polynomial model

|  | $\mathbf{n}=\mathbf{4}$ | $\mathbf{n}=\mathbf{5}$ | $\mathbf{n}=\mathbf{6}$ | $\mathbf{n}=\mathbf{7}$ | $\mathbf{n}=\mathbf{8}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{b}_{\mathbf{0}}$ | -0.0377 | -0.1679 | -0.1501 | -0.14906 | -0.15014 |
| $\mathbf{b}_{\mathbf{1}}$ | -0.3228 | 0.1993 | -0.05146 | -0.11632 | 0.17493 |
| $\mathbf{b}_{\mathbf{2}}$ | 0.10426 | -0.1003 | 0.03899 | 0.08579 | -0.16862 |
| $\mathbf{b}_{\mathbf{3}}$ | -0.01058 | 0.0173 | -0.01069 | -0.02339 | 0.06344 |
| $\mathbf{b}_{\mathbf{4}}$ | $3.36 e-4$ | $3.147 \mathrm{e}-5$ | 0.00136 | 0.00305 | -0.01218 |

Table 2. Statistical indicators of polynomial model

|  | $\mathbf{n}=\mathbf{4}$ | $\mathbf{n}=\mathbf{5}$ | $\mathbf{n}=\mathbf{6}$ | $\mathbf{n}=\mathbf{7}$ | $\mathbf{n}=\mathbf{8}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{S D}(\mathbf{1 0} \mathbf{- 9})$ | 0.25 | 0.84 | 3.6 | 5.86 | 3.21 |
| $\mathbf{t}\left(\mathbf{1 0}^{-\mathbf{1}}\right)$ | 1.26 | 4.52 | 6.53 | 3.65 | 2.43 |
| NMRSE | 1.02 | 0.81 | 0.27 | 0.74 | 0.58 |
| NMBE | 0.39 | 0.14 | 0.36 | 0.95 | 0.74 |

After finding the coefficients of the polynomial, the determination of intrinsic electrical parameters is the next step. A polynomial of degree six has six roots, and numerical resolution was made with the predefined functions of same mathematical software. The values found of electrical parameters and those statistical indicators are grouped in table 3. The obtained values of electrical parameters depends on the degree of the polynomial, the statistical indicators shows that the results obtained by this new technique are relatively acceptable.

Table 3. Electrical parameters and statistical indicators of electrical model

|  | $\mathrm{n}=4$ | $\mathrm{n}=5$ | $\mathrm{n}=6$ | n=7 | $\mathrm{n}=8$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\eta$ | 1.22 | 1.42 | 1.33 | 1.41 | 1.39 |
| $\mathrm{I}_{0 \mathrm{r}} 10^{-6}(\mathrm{~A})$ | 2.06 | 1.95 | 1.85 | 1.82 | 1.87 |
| $\mathrm{I}_{0 \mathrm{~d}} 10^{-12}(\mathrm{~A})$ | 1.13 | 0.73 | 1.29 | 2.15 | 2.43 |
| $\mathrm{R}_{\mathrm{s}}(\mathbf{\Omega})$ | 1.92 | 1.05 | 0.26 | 0.45 | 0.29 |
| $\mathrm{R}_{\text {sh }}(\Omega)$ | 216.25 | 165.62 | 114.25 | 124.58 | 110.29 |
| SD ( $10^{-9}$ ) | 10.59 | 9.93 | 9.44 | 12.84 | 9.68 |
| t (10 ${ }^{-1}$ ) | 9.83 | 8.97 | 8.23 | 9.16 | 8.74 |
| NMRSE | 1.08 | 0.52 | 0.46 | 0.95 | 0.73 |
| NMBE | 0.85 | -0.74 | 0.82 | 1.004 | 0.86 |

Indeed, the found values for the statistical indicators with ( $n=6$ ) were the lowest. So, the corresponding electrical parameters are considered afterwards. Regarding convergence, it converges always because the polynomial model used can fit any other model, changing his degree as showed in table 2 . This technique is very fast, and it can be used at real time, to extract models parameters. Dependency between the degree of the polynomial and values of the parameters found, is not a problem, it is rather an advantage of the method, because this dependency ensures convergence and improved accuracy. These results were confirmed by characteristics $(\mathrm{I}=\mathrm{f}(\mathrm{V}), \mathrm{P}=\mathrm{g}(\mathrm{V})$ ) of the PV cell :



Fig. 4. (I-V) and (P-V) characteristics of electrical model (equations 1, 3) and measures
The determination of PVC electrical parameters allows deducing the behavior of these cells. Among the elements of this behavior are dynamic resistance ( $\mathrm{R}_{\mathrm{d}}=\mathrm{V} / \mathrm{I}$ ) and internal resistance ( $\left.\mathrm{R}_{\mathrm{in}}=\left(\mathrm{V}_{\mathrm{co}}-\mathrm{V}\right) / \mathrm{I}\right)$ of the PV cell (Figure 5).


Fig. 5. Dynamic and internal resistances of PVC

## 6 Conclusion

The electric model used in this study for photovoltaic cells, is the double exponential model. These intrinsic electrical parameters are determined with polynomial model coefficients. This extraction is done by calling a polynomial model, to compare with the theoretical model. The equality of these two models allows setting the parameters sought. The results obtained by this technique show that it is effective for the parameters extraction of theoretical models describing the behavior of photovoltaic cells. The proposed method is always accurate and converges independently of the initial values, and it is very fast to use in real time applications.

## References

[1] Sze, Physics of Semiconductor Devices, Wiley, New York, (1969).
[2] A. Ortiz-Conde, F.J. G. Sánchez, J. Muci, "New method to extract the model parameters of solar cells from the explicit analytic solutions of their illuminated I-V characteristics", Solar Energy Materials and Solar Cells 90, pp. 352-361, 2006.
[3] V. Mikhelashvili, G. Eisenstein, V. Garber, S. Fainleib, G. Bahir, D. Ritter, M. Orenstein and A. Peer, "On the extraction of linear and nonlinear physical parameters in nonideal diodes", J. Appl. Phys., vol. 85, pp. 6873-6883, 1999.
[4] S. Yadir, S. Assal, M. khaidar, M. Sidki, M. Benhmida, A. Malaoui. "Extraction of Solar Cell Physical Parameters Model with Double Exponential from Illuminated I-V Experimental Curve". Global Journal of Physical Chemistry, pp 236-240, 2011.
[5] A. Jain and A. Kapoor, "Exact analytical solutions of the parameters of real solar cells using Lambert Wfunction", Solar Energy Materials and Solar Cells, vol. 81, pp. 269-277, 2004.
[6] T. Easwarakhanthan, J. Bottin, I. Bouhouch and C. Boutrit, 'Nonlinear Minimization Algorithm for Determining the Solar Cell Parameter with Microcomputers', International Journal of Sustainable Energy, Vol. 4, No1, pp. 1-12, 1986.
[7] A. Ortiz-Conde, F. J. García Sánchez and M. Guzmán, "Exact analytical solution of channel surface potential as an explicit function of gate voltage in undoped-body MOSFETs using the Lambert W function and a threshold voltage definition therefrom", Solid-State Electronics, vol. 47, pp. 2067-2074, 2003.
[8] Y. Nesterov, 'Introductory Lectures on Convex Optimization: A Basic Course', Applied Optimization, Kluwer Academy Publishers, Boston, 2004.
[9] T.C. Banwell, "Bipolar Transistor Circuit Analysis Using the Lambert W-Function", IEEE Trans. Circuits and Systems I, vol. 47, pp. 1621-1633, 2000.
[10] A. Caralli, A. Ortiz-Conde and F.J. García Sánchez, "Percentage Area Difference (PAD) as a measure of distortion and its use in Maximum Enclosed Area (MEA), a new ECG signal compression algorithm", Int. Caracas Conf. on Cir. Dev. and Sys., (Aruba), pp. 1035-1- 1035-5, 2002.
[11] Abdessamad Malaoui, Abdelmajid Elmansouri. « Deux nouvelles méthodes complémentaires pour l'extraction optimale des paramètres électriques des jonctions ». Revue des Energies Renouvelables CDER, Vol. 13, N², 2010.
[12] B.T. Polyak, 'Introduction to Optimization on Multistep Gradient Methods', 1987.
[13] O. Aomari, A. Malaoui, et al. "Implementation of a new analytical technique to determine the electrical parameters of junction models". Global Journal of Physical Chemistry, pp 68-72, 2011.
[14] Abdessamad Malaoui, "Implementation and tests of an automatic system to improve electrical energy in photovoltaic installations," International Journal of Innovation and Applied Studies, vol. 8, no. 1, pp. 328-340, 2014.
[15] A. Malaoui, "Automatisation en température par microcontrôleur d'un banc de mesure ultrasonore: Applications au contrôle qualité en agroalimentaire.", thèse de doctorat de l'Université de Provence, 2005.
[16] Oueriagli, A., Kassi, H., Holchandani, S. \& Leblanc, R.M. Analysis of dark current-voltage characteristics of AI/chlorophyll a/Ag sandwich cells., J. Appl. Phys., 71, 5523-553, 1992.

