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Inventory Model for Dynamic Demand and Product Cost with Limited Storage Space Using L.P.P/I.P.P. Technique

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ABSTRACT: In global market, where demand as well as the cost fluctuates rapidly, whereas the inventory holding capacity and capital to be invested doesn't change at the same pace; long term planning seems reasonable to avoid any loss either due to overstocking the inventory or due to stock-outs. This paper presents proposes a deterministic mathematical model to effectively control the inventory holdings. Proposed model considers the periodic fluctuations in demand and cost that affect the cost and quantity of inventory holding under the constraint that available storage space is limited. It minimizes the total cost of inventory in a given period of time and without any shortages.

KEYWORDS: EOQ model, Inventory Control, Shortages, Stock holding, Carrying Cost.

1 INTRODUCTION

Inventory is the stock to be held by an organization (may be a retailer/manufacturer/charity organization etc.) to meet the future demand. As any denial in fulfilling the demand is not a healthy sign for the growth of business outlet, because each denial results in loss of goodwill and a potential consumer. To meet all the demands that may occur at any period of time a very careful and effective strategy should be adopted so that while maintaining and managing the items in store. This strategic management of the inventory is called inventory control.

Many businesses have limited resource. In such a case if a large part of the capital is tied up in wrong kind of inventory then it may be catastrophic for the organization. Also inventory may be old, worn out, shop worn, obsolete, or of the wrong sizes or colours, or there may be an imbalance among different product lines that reduces the customer's appeal of the total operation. Therefore it is essential to ensure that the organization has the right amount of goods on hand to avoid stock-outs, to prevent shrinkage (spoilage/theft), and to provide proper accounting.

In today's open market the demand and/or cost does not remain constant but keeps changing with time. It actually varies when the selling price of any item is changed [1]. So the problem arises that how much should be kept in stock prior to the arrival of next demand?

Also, if there are some items are left from the previous period then how to decide for the next level of inventory. So it is necessary to pre-decide the inventory level and manage it effectively at any instant of time. Since if supplier/ manufacturer doesn't check the inventory level on time than they may be at loss either due to over-stocking it or by underestimating it.

2 INVENTORY MODEL FOR DYNAMIC DEMAND AND PRODUCT COST WITH LIMITED STORAGE SPACE USING L.P.P/I.P.P. TECHNIQUE [SINGLE ITEM]

In the basic Barabas EOQ Model [2], [3] demand of the item as well as the cost of product is assumed to be constant. But in today's open market, the cost and demand is not static rather fluctuate rapidly. It actually varies when the selling price of any item is changed [1]. It may also vary if the paying capacity of the consumer changes.

Again in Barabas EOQ Model, the inventory left (if any) during the previous period has not been accounted for, as it has been assumed that next order is placed when inventory level reaches zero with zero lead time. Thinking of it that cost doesn't change and inventory is replenished as soon as it is ordered, than why does a firm need extra expenses on holding the inventory? It would be better that organization orders for the item, only when a demand arises and thus saving the capital, which can be used for other productive work. Practically it is not possible. In fact, the Cost and demand both changes rapidly due to various reasons. Also, there may be inventory left from the previous period which should be utilized in the next time period. Organization just cannot throw the leftover inventory except for the case where it is expired or spoiled for one or the other reasons.

One more point of concern is the storage capacity. Holding an inventory needs space and acquiring more space is not that easy due to various constraints. Definitely, any organization cannot have unlimited storage space. Thus, one has to keep this constraint in mind, while developing a model. Thus, a long term inventory plan may be more beneficial for any organization. Therefore, it is more desirable for a supplier/ manufacturer to have prior knowledge of the state of inventory holdings, so that he can manage his inventory level effectively, at any instant of time. Since if supplier/ manufacturer doesn't check the inventory level on time than they may be at loss either due to over-stocking it or by running short of it.

Many models have been proposed for time dependent demand rate over the years such as Baker & Urban, (1988)[4] presented the EOQ model in which the demand is a multivariate function of price, time, and level of inventory;

Mandal & Maiti's (1999)[5] model with stock dependent demand. This study proposes a new inventory control model which addresses all the above cited problems and takes into account the fluctuation in demand rate and product cost that affects the inventory and still minimizes the total cost of inventory in a given period of time.

The model that has been formulated in this study considers all the above factors and is based on the following assumptions

- Multiple items are being stocked.
- Demand is deterministic and changes with time.
- Cost of a product also fluctuates.
- One period is divided into 'n' time zones. 'n' refers to the number of times demand or price or both changes during the period under study.
- Storage capacity is limited.
- Shortages are not allowed.
- There may be stock holdings from the previous time zone and it is carried to the next and utilized.

Now consider the following variables:

n: Number of time zones in a given time period (may be a financial year).
t: Time period which takes the values 1,2,3……n in one financial year.
I: Maximum number of units that can be stocked at a time.
I_0: Initial stock holding.
I_t: Number of units in stock at the end of the period t. (t = 0, 1, 2…..n)
h: Inventory carrying cost.
d_t: Demand of the product in the period t: (t = 1, 2…..n)
C_t: Cost of unit item in the period t: (t = 1, 2…..n)
X_t: Number of units to be ordered in a time period t. (t = 0, 1, 2…..n)

Since inventory is being carried from one period to the next, it is necessary to define the stock.

$$Closing\ stock\ of\ period\ t = opening\ stock\ of\ period\ t\ +\ Quantity\ ordered\ in\ period\ t - demand$$

Symbolically, $I_t = I_{t-1} + X_t - d_t$

Obviously, $Opening\ stock\ in\ any\ period = Closing\ stock\ of\ previous\ period.$

Also $I_t \geq 0$ 2.1

Thus we get the constraint $I_{t-1} + X_t \geq d_t$ 2.2

Since storage space is limited therefore $I_t \leq I$ 2.3

Since number of units cannot be negative, therefore, $X_t \geq 0$ 2.4

The inventory problem is to minimize the cost of inventory and obtain the optimal order quantity. Thus it can be formulated as:

$$Min\, T_c = h\sum_{t=0}^{n} I_t + \sum_{t=1}^{n} C_t X_t$$

This is the linear objective function. Combining it with constraints equations 2.1, 2.2, 2.3 and 2.4; inventory problem can be modeled as follows

$$Min\ T_c = h\sum_{t=0}^{n} I_t + \sum_{t=1}^{n} C_t X_t$$

Subject to the constraints:

$$I_t \leq I$$

$$I_{t-1} + X_t \geq d_t$$

$$I_t \geq 0,\ X_t \geq 0$$

This is a linear programming problem and can be solved by Simplex method proposed by George Dantzig[6] in 1947. Numerical examples presented in this study are solved with the help of “lp_solve 5.5.2.0”[7] (http://lpsolve.sourceforge.net/5.5/). lp_solve is a software to solve optimization problem using simplex method. It was originally developed by Michel Berkelaar at Eindhoven University of Technology. Henri Gourvest, developed easy to use graphical interface called LPSolve IDE for this program. This is very simple and user friendly.

NUMERICAL EXAMPLE [2.1]: If 15 units of a beverage are already in store and it is possible to hold up to 90 units of a beverage in store from one period to the next at a carrying cost of Rs. 2 per unit. Demand and cost for four consecutive periods of the beverage is estimated as below:

Table 1. Table of Demand and Cost of beverage

Period	Demand (unit)	Cost per unit
1	100	Rs. 15
2	80	Rs 14
3	120	Rs 17
4	110	Rs 18

It is required to determine the storage schedule which will meet the stated demands over the four time periods at minimum cost.

SOLUTION: This problem can be modeled as

$$Min\ T_c = 2\left(I_0 + I_1 + I_2 + I_3 + I_4\right) + 15X_1 + 14X_2 + 17X_3 + 18X_4$$

Subject to the constraints:

$$I_0 = 20\ , I_1 \leq 90\ , I_2 \leq 90, I_3 \leq 90, I_4 \leq 90$$

$$I_0 + X_1 \geq 100\ , I_1 + X_2 \geq 80\ \ I_2 + X_3 \geq 120, I_3 + X_4 \geq 100$$

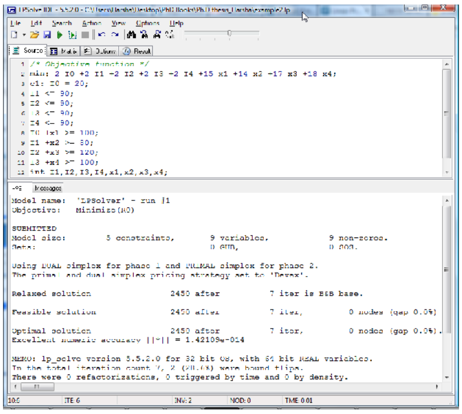

Fig 2.1: Window image showing objective function

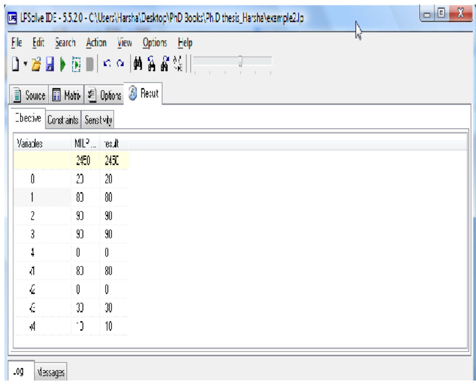

Fig 2.2: Window image showing result

$I_t \geq 0$, $X_t \geq 0$

This is solved using lp_solve 5.5.2.0 [7] [Refer Window images 2.1 and 2.2 for solution]

$I_1 = 80$, $I_2 = 90$, $I_3 = 90$, $I_4 = 0$, $X_1 = 80$, $X_2 = 80$, $X_3 = 30$, $X_4 = 10$

$Min\ T_c =$ Rs. 2450

NUMERICAL EXAMPLE [2.2]: Demand and product cost for an item for 2 periods are given below:

Table 2. Table of Demand and Cost for an item

Period	Demand (unit)	Cost per unit
1	75	Rs 6
2	124	Rs 8

Carrying cost is Rs 2.5 per period and initially there are 25 items in the store. If a store can hold a maximum of 100 units, from one period to another, than find the inventory that minimizes the total cost for the given time.

SOLUTION**:** The problem takes the following l.p.p. form.

$$Min\ T_c = \ 2.2\left(I_0 + I_1 + I_2\right) + 6X_1 + 8X_2$$

Subject to the constraints:

$$I_0 = 25,,\ I_1 \leq 100,\ I_2 \leq 100,$$

$$I_0 + X_1 \geq 75,\ I_1 + X_2 \geq 124,\ I_t \geq 0,\ X_t \geq 0$$

Solving by '*lp_solve 5.5.2.0'* [7] We get

$I_1 = 100, I_2 = 0,\ X_1 = 50,\ X_2 = 24$ and. Min T_c = Rs 804.50.

3 INVENTORY MODEL FOR DYNAMIC DEMAND AND PRODUCT COST WITH LIMITED STORAGE SPACE USING L.P.P/I.P.P. TECHNIQUE [MULTIPLE ITEMS]

The model presented in the last section deals with single item inventory holdings. It can be reformulated for the multiple item inventory holding, which will be more applicable as most of the time an organisation deals with many items inventory holding.

Assumptions**:** All the assumptions remain the same as the previous one except for the first one that is now 'multiple items are being stocked.

Now consider the following variables:

n: Number of time zones in a given time period.

t: Time period which takes the values 1,2,3……*n* in one financial year.

p: Product number; which takes the values 1,2,3……*k*, if there are *k* different types of product to be stored.

I_p: Maximum number of t units of the p^{th} product that can be stocked at a time.

I_{0p}: Initial stock holding for p^{th} product.

I_{tp}: Number of units of the p^{th} product in stock at the end of the period *t*. (*t* = 0, 1, 2…..*n*)

h_p: Inventory carrying cost of the p^{th} product.

d_{tp}: Demand of the p^{th} product in the period t: (*t* = 1, 2…..*n*)

C_{tp}: Cost of unit item in the period *t*: (*t* = 1, 2…..*n*)

X_{tp}: Number of units to be ordered in a time period t. (*t* = 0, 1, 2…..*n*)

Since inventory is being carried from one period to the next, it is necessary to define the stock.

$$Closing\ stock\ of\ period\ t = opening\ stock\ of\ period\ t\ +\ Quantity\ ordered\ in\ period\ t - demand$$

Symbolically,

$$I_{tp} = I_{(t-1)p} + X_{tp} - d_{tp}$$

Obviously,

$$Opening\ stock\ in\ any\ period = Closing\ stock\ of\ previous\ period.$$

Also $I_{tp} \geq 0$ 3.1

Thus we get the constraint $I_{(t-1)p} + X_{tp} \geq d_{tp}$ 3.2

Since storage space is limited therefore $I_{tp} \leq I_p$ 3.3

Since number of units cannot be negative, therefore, $X_{tp} \geq 0$ 3.4

The inventory problem is to minimize the cost of inventory and obtain the optimal order quantity. Thus it can be formulated as:

$$Min\, T_c = \sum_{p=1}^{k}\left[h_p \sum_{t=0}^{n} I_{tp}\right] + \sum_{p=1}^{k}\sum_{t=1}^{n} C_{tp}X_{tp}$$

This is the linear objective function. Combining it with constraints equations 3.1, 3.2, 3.3 and 3.4; inventory problem can be modeled as follows

$$Min\, T_c = \sum_{p=1}^{k}\left[h_p \sum_{t=0}^{n} I_{tp}\right] + \sum_{p=1}^{k}\sum_{t=1}^{n} C_{tp}X_{tp}$$

Subject to the constraints:

$I_{tp} \leq I_p$, $I_{(t-1)p} + X_{tp} \geq d_{tp}$, $I_{tp} \geq 0$, $X_{tp} \geq 0$

This is a linear programming problem and can be solved by Simplex method proposed by George Dantzig in 1947.

NUMERICAL EXAMPLE [3.1]: A retailer selling grocery item stores five items for future use. Variation in demand and cost of items are given below:

Table 3. Table showing retailer's grocery demand and storage capacity

Product No.	Time period	Demand units	Cost / unit	Initial Inventory I_{0p}	Holding capacity (units)	Carrying cost
1	1	100	Rs 32	30	80	Rs.1
	2	110	Rs 34			
	3	90	Rs 31			
2	1	65	Rs 5	35	95	Rs. 1.5
	2	130	Rs 6			
	3	100	Rs 7			
3	1	68	Rs 15	28	72	Rs. 0.5
	2	83	Rs 13			
	3	70	Rs 14			

Retailer wants to have inventory policy so that he can fulfil the demands without investing much money on it. Plan the inventory policy.

SOLUTION: Following l.p.p. problem will be formulated:

$$Min\ T_c = \left(I_{01}+I_{11}+I_{21}+I_{31}\right)+1.5\left(I_{02}+I_{12}+I_{22}+I_{32}\right)+0.5\left(I_{03}+I_{13}+I_{23}+I_{33}\right)$$

$$+32\mathrm{X}_{11}+34X_{21}+31\ \mathrm{X}_{31}+5\ \mathrm{X}_{12}+6\ \mathrm{X}_{22}+7\ \mathrm{X}_{32}\ +15X_{13}+13X_{23}+14X_{33};$$

Subject to the constraints:

$I_{01}=30$; $I_{02}=35$; $I_{03}=28$; $I_{11}\le 80$; $I_{21}\le 80$; $I_{31}\le 80$; $I_{12}\le 95$; $I_{22}\le 95$; $I_{32}\le 95$;

$I_{13}\le 72$; $I_{23}\le 72$; $I_{33}\le 72$; $I_{01}+X_{11}\ge 100$; $I_{11}+X_{21}\ge 110$; $I_{21}+X_{31}\ge 90$;

$I_{02}+X_{12}\ge 65$; $I_{12}+X_{22}\ge 130$; $I_{22}+X_{32}\ge 90$; $I_{03}+X_{13}\ge 68$;

$I_{13}+X_{23}\ge 83$; $I_{23}+X_{33}\ge 70$;

And $I_{11}\ge 0$, $I_{21}\ge 0$, $I_{31}\ge 0$, $I_{12}\ge 0$, $I_{22}\ge 0$, $I_{32}\ge 0$, $I_{13}\ge 0$, $I_{23}\ge 0$, $I_{33}\ge 0$,

$X_{11}\ge 0$ $X_{21}\ge 0$, $X_{31}\ge 0$, $X_{12}\ge 0$, $X_{22}\ge 0$, $X_{32}\ge 0$, $X_{13}\ge 0$, $X_{23}\ge 0$, $X_{33}\ge 0$;

Solving by '*lp_solve 5.5.2.0*' [7]

$I_{11}=80$, $I_{21}=80$, , $I_{31}=0$, $I_{12}=95$, $I_{22}=90$, $I_{32}=0$, $I_{13}=72$, $I_{23}=70$, $I_{33}=0$,

$X_{11}=70$ $X_{21}=30$, $X_{31}=10$, $X_{12}=30$, $X_{22}=35$, $X_{32}=0$, $X_{13}=40$, $X_{23}=11$, $X_{33}=0$

and Min T_c = Rs 5278

Thus inventory plan for the retailer is as follows:

Table 4. Table showing inventory plan for the retailer

Product No.	Time period	Units to be ordered	Inventory units	Minimum Inventory Cost
1	1	$X_{11} = 70$	$I_{11} = 80$	Min T_c = Rs 5278
	2	$X_{21} = 30$	$I_{21} = 80$	
	3	$X_{31} = 10$	$I_{31} = 0$	
2	1	$X_{12} = 30$	$I_{12} = 95$	
	2	$X_{22} = 35$	$I_{22} = 90$	
	3	$X_{32} = 0$	$I_{32} = 0$	
3	1	$X_{13} = 40$	$I_{13} = 72$	
	2	$X_{23} = 11$	$I_{23} = 70$	
	3	$X_{33} = 0$	$I_{33} = 0$	

4 RESULTS AND DISCUSSION

Inventory planning and control is an essential evil of any organization. In this study it has been thoroughly studied and two models are proposed to help anyone keeping efficient merchandise. Proposed models use simple mathematics that even a person with little subject knowledge can understand easily. The models are deterministic and simplicity of the model is its strength.

The models can predict the economic order quantity for more than one period at a time; therefore, it is fit for long range inventory planning and control.

The models have a limitation that it requires demand to be estimated well before time. But if demand is predicted correctly than model seems to be a good enough to keep inventory control.

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