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SOLVING MODELS OF ELECTRICAL NETWORKS BY AN IMPROVED HOMOTOPY PERTURBATION METHOD

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ABSTRACT: In this paper, an efficient numerical algorithm to find exact solutions for the system of linear equations based on homotopy perturbation method (HPM) is presented. A reliable modification is proposed, and the modified method is employed to solve the system of linear equations generated from some models of electrical networks; the results are compared with those obtained by the conventional methods of solving the system of linear equations. Two examples are given to illustrate the ability and reliability of the improved homotopy perturbation method. The results reveal that the improved method is very simple and effective.

KEYWORDS: Electrical Networks, Homotopy Perturbation Method.

1 INTRODUCTION

Homotopy perturbation method was first established in 1999 by J. Huan He, a Chinese researcher and a Mathematician. It was later improved and developed by him in 2000 and 2001. Many asymptotic techniques including homotopy perturbation method by He (1999, 2000, and 2003), energy method by He (2002), D'Acunto (2006) called this method as He's variational method, modifications of Lindstedt-Poincare method by HE (2000), bookkeeping parameter method, parameterized perturbation method by He (1999). Cai et al (2006) called this method as He's perturbation method, iteration perturbation method by He (2001), and Exp-function method by He (2006) were suggested by Prof. J Huan He during 1999-2008. For a relatively comprehensive survey on the method and its applications, the reader is referred to Prof. He's review article in 2006.

The HPM, proposed first by He, was further developed and improved by scientists and engineers in 2007 by M Gorji, D.D. Ganji and S Soleimani in their New Application of He's homotopy perturbation method. The method, which is a coupling of the traditional perturbation method and homotopy in topology, deforms continuously to significant advantage in that it provides an analytical approximate solution to a wide range of nonlinear problems in applied sciences. The HPM is applied to nonlinear oscillators by He (2004), to bifurcation of nonlinear problems by He (2005), to the system of linear equations by Keramati (2007), to nonlinear partial differential equations of fractional order by Wang (2006), to boundary value problems by He (2006), to solve integral equations by Biazar (2007) and functional integral equations by Abbasbandy (2007), to solve the system of integral equations by Yusufoglu (2007) and to other fields.

Here we present an improvement of the HPM, as developed by Yusufoglu Elcin in 2009, to find exact solutions for the system of linear equations. An accelerating vector based on the HPM is introduced which leads to a rapid convergence and gives exact solutions. Two examples are given to illustrate the efficiency of the improved method.

2 HOMOTOPY PERTURBATION METHOD

Consider the system of linear equations

$$\boldsymbol{Ax} = \boldsymbol{b}, \tag{1}$$

Where

$$\boldsymbol{A} = [a_{ij}], \quad \boldsymbol{x} = [x_j], \quad \boldsymbol{b} = [b_i], \qquad i = 1,2,\dots,n, j = 1,2,\dots,n.$$

To explain the HPM, we reconstitute (1) as

$$L(\boldsymbol{u}) = \boldsymbol{Au} - \boldsymbol{b} = \boldsymbol{0}, \tag{2}$$

With solution $\boldsymbol{u} = \boldsymbol{x}$, and we define the homotopy $H(\boldsymbol{u}, p)$ by

$$H(\boldsymbol{u}, 0) = F(\boldsymbol{u}), \qquad H(\boldsymbol{u}, 1) = L(\boldsymbol{u}), \tag{3}$$

Where $F(\boldsymbol{u})$ is a functional operator with solution, say, $\boldsymbol{u_0}$, which can be obtained easily. We may choose a convex homotopy

$$H(\boldsymbol{u}, p) = (1 - p)F(\boldsymbol{u}) + pL(\boldsymbol{u}) = \boldsymbol{0}, \tag{4}$$

And continuously trace an implicitly defined curve from a starting point $H(\boldsymbol{u_0}, 0)$ to a solution $H(\boldsymbol{x}, 1)$. The embedding parameter p monotonically increases from zero to one as the trivial problem $F(\boldsymbol{u}) = \boldsymbol{0}$ is continuously deformed to the original problem $L(\boldsymbol{u}) = \boldsymbol{0}$. The embedding parameter $p \in [0,1]$ can be considered as an expanding parameter

$$\boldsymbol{u} = \boldsymbol{u_0} + p\boldsymbol{u_1} + p^2\boldsymbol{u_2} + \cdots \tag{5}$$

When $p \to 1$, Eq. (4) corresponds to Eq. (2) and Eq. (5) becomes the approximate solution of Eq. (2), i.e.,

$$\boldsymbol{x} = \lim_{p\to 1}(\boldsymbol{u_0} + p\boldsymbol{u_1} + p^2\boldsymbol{u_2} + \cdots) = \sum_{k=0}^{\infty} \boldsymbol{u_k}. \tag{6}$$

Taking $F(\boldsymbol{u}) = \boldsymbol{u} - \boldsymbol{w_0}$, we substitute (5) into (4) and equate the terms with identical powers of p, obtaining:

$$p^0: \quad \boldsymbol{u_0} - \boldsymbol{w_0} = \boldsymbol{0}, \quad \boldsymbol{u_0} = \boldsymbol{w_0}, \tag{7}$$

$$p^1: \ (\boldsymbol{A} - \boldsymbol{I})\boldsymbol{u_0} + \boldsymbol{u_1} - \boldsymbol{w_0} - \boldsymbol{b} = \boldsymbol{0}. \ \ \boldsymbol{u_1} = \boldsymbol{b} - (\boldsymbol{A} - \boldsymbol{I})\boldsymbol{u_0} + \boldsymbol{w_0}, \tag{8}$$

$$p^2: \ (\boldsymbol{A} - \boldsymbol{I})\boldsymbol{u_1} + \boldsymbol{u_2} = \boldsymbol{0}. \ \ \boldsymbol{u_2} = -(\boldsymbol{A} - \boldsymbol{I})\boldsymbol{u_1}, \tag{9}$$

And in general

$$\boldsymbol{u}_{n+1} = -(\boldsymbol{A} - \boldsymbol{I})\boldsymbol{u_n}, \quad n = 1,2,\dots, \tag{10}$$

If we take $\boldsymbol{u_0} = \boldsymbol{w_0} = \boldsymbol{0}$, then we have

$$\boldsymbol{u_1} = \boldsymbol{b} \tag{11}$$

$$\boldsymbol{u_2} = -(\boldsymbol{A} - \boldsymbol{I})\boldsymbol{u_1} = -(\boldsymbol{A} - \boldsymbol{I})\boldsymbol{b}, \tag{12}$$

$$\boldsymbol{u_3} = (\boldsymbol{A} - \boldsymbol{I})^2\boldsymbol{b}, \tag{13}$$

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$$\boldsymbol{u}_{n+1} = (-1)^n(\boldsymbol{A} - \boldsymbol{I})^n\boldsymbol{b}, \tag{14}$$

Hence, the solution can be of the form

$$\boldsymbol{u} = [\boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{I}) + (\boldsymbol{A} - \boldsymbol{I})^2 - \cdots]\boldsymbol{b}, \tag{15}$$

The convergence of the series (15) has been proved by Keramati. In practice, all terms of series (6) cannot be determined and so we use an approximation of the solution by the truncated series:

$$\boldsymbol{u_m} = \sum_{k=0}^{m-1} \boldsymbol{u_k} \tag{16}$$

3 AN IMPROVEMENT TO HPM

In this section we propose a scheme to accelerate the rate of HPM applied to system of linear equations. We define new homotopy $H(\boldsymbol{u}, p, \alpha)$ by

$$H(\boldsymbol{u}, o, \alpha) = F(\boldsymbol{u}), \qquad H(\boldsymbol{u}, 1, \alpha) = L(\boldsymbol{u}), \tag{17}$$

and typically, a convex homotopy as follows

$$H(\boldsymbol{u}, p, \alpha) = (1-p)F(\boldsymbol{u}) + pL(\boldsymbol{u}) + p(1-p)\alpha = \boldsymbol{0} \tag{18}$$

Where $\alpha = [\alpha_i]^t$ is called the accelerating vector, and for $\alpha = \boldsymbol{0}$ we define $H(\boldsymbol{u}, p, \boldsymbol{0}) = H(\boldsymbol{u}, p)$, which is the standard HPM.

By substituting Eq. (5) in Eq. (18) and equating the terms with identical powers of p, we obtain

$$p^0: \ \boldsymbol{u_0} - \boldsymbol{w_0} = \boldsymbol{0}, \quad \boldsymbol{u_0} = \boldsymbol{w_0}, \tag{19}$$

$$p^1: \ (\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_0} + \boldsymbol{u_1} - \boldsymbol{w_0} - \boldsymbol{b} + \boldsymbol{\alpha} = \boldsymbol{0}, \quad \boldsymbol{u_1} = \boldsymbol{b} - (\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_0} + \boldsymbol{w_0} - \boldsymbol{\alpha}, \tag{20}$$

$$p^2: \ (\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_1} + \boldsymbol{u_2} - \boldsymbol{\alpha} = \boldsymbol{0}, \quad \boldsymbol{u_2} = -(\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_1} + \boldsymbol{\alpha} \tag{21}$$

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$$p^3: \ (\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_2} + \boldsymbol{u_3} = \boldsymbol{0}, \quad \boldsymbol{u_3} = -(\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_2}, \tag{22}$$

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$$p^{n+1}: \ (\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_n} + \boldsymbol{u_{n+1}} = \boldsymbol{0}, \quad \boldsymbol{u_{n+1}} = -(\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_n}, \tag{23}$$

If we take $\boldsymbol{u_0} = \boldsymbol{w_0} = \boldsymbol{0}$, then we have

$$\boldsymbol{u_1} = \boldsymbol{b} - \boldsymbol{\alpha}, \tag{24}$$

$$\boldsymbol{u_2} = -(\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u_1} + \boldsymbol{\alpha} = -(\boldsymbol{A}-\boldsymbol{I})(\boldsymbol{b}-\boldsymbol{\alpha}) + \boldsymbol{\alpha} \tag{25}$$

$$\boldsymbol{u}_3 = -(\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u}_2, \tag{26}$$

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$$\boldsymbol{u}_{n+1} = -(\boldsymbol{A}-\boldsymbol{I})\boldsymbol{u}_n, \tag{27}$$

We try to find the parameters α as such that

$$\boldsymbol{u}_2 = \boldsymbol{0}, \tag{28}$$

Hence from (25) we should have

$$-(\boldsymbol{A}-\boldsymbol{I})(\boldsymbol{b}-\alpha) + \alpha = 0, \tag{29}$$

Or

$$\boldsymbol{A\alpha} = (\boldsymbol{A}-\boldsymbol{I})\boldsymbol{b}, \tag{30}$$

From Eq. (30) we conclude that

$$\boldsymbol{\alpha} = (\boldsymbol{I} - \boldsymbol{A}^{-1})\boldsymbol{b}, \tag{31}$$

Thus from (27), we have $\boldsymbol{u_3} = \boldsymbol{u_4} = \cdots = \boldsymbol{0}$, and the exact solution will be obtained as

$$\boldsymbol{u} = \boldsymbol{u_0} + \boldsymbol{u_1} = \boldsymbol{0} + \boldsymbol{u_1} = \boldsymbol{u_1}, \tag{32}$$

4 DERIVATION FROM THE CIRCUIT

In figs 1 and 2, the unknown currents are i_1, i_2 and i_3 in the electrical networks. To obtain it, we label the current as shown, choosing directions arbitrarily. If a current will come out negative, this simply means that the current flows against the direction of our arrow. The current entering each battery will be the same as the current leaving it. The equations for the currents result from Kirchoff's laws:

Kirchoff's Current Law (KCL): At any point of a circuit, the sum of the in-flowing currents equals the sum of the out-flowing currents.

Kirchoff's Voltage Law (KVL): In any closed loop, the sum of all voltage drops equals the impressed electromotive force.

Example 4.1

Consider the derived system of equations (See Appendix 1)

$$\begin{aligned} 5i_1 - i_2 + 2i_3 &= 3 \\ -i_1 + 4i_2 + i_3 &= 8 \\ -2i_1 + i_2 + 3i_3 &= 2 \end{aligned} \tag{33}$$

The true solution by other conventional methods is $(0.81,\ 2.07,\ 0.51)^t$

By improved homotopy,

We solve the system $\boldsymbol{Au} = \boldsymbol{b},$ where

$$\boldsymbol{A} = \begin{bmatrix} 5 & -1 & 2 \\ -1 & 4 & 1 \\ -2 & 1 & 3 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

Now by (30), i.e. $\boldsymbol{A\alpha} = (\boldsymbol{A} - \boldsymbol{I})\boldsymbol{b},$ we have

$$\begin{bmatrix} 5 & -1 & 2 \\ -1 & 4 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 2 \\ -1 & 3 & 1 \\ -2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

or

$$\begin{bmatrix} 5 & -1 & 2 \\ -1 & 4 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 23 \\ 6 \end{bmatrix}$$

Hence by (31) i.e. $\boldsymbol{\alpha} = (\boldsymbol{I} - \boldsymbol{A}^{-1})\boldsymbol{b},$

The solution of this system gives as $\alpha = (2.19, 5.93, 1.49)^t$. Thus the solution of Eq (33) becomes $\boldsymbol{u} = \boldsymbol{u_1} = \boldsymbol{b} - \boldsymbol{\alpha} = (3, 8, 2)^t - (2.19, 5.93, 1.49)^t = (0.81, 2.07, 0.51)^t$, which is the exact solution of Example 1.

Hence $i_1 = 0.81;\ i_2 = 2.07;\ i_3 = 0.51$

Example 4.2

Consider the derived system of equations below (see Appendix 2)

$$\begin{aligned} i_1 + i_2 + 3i_3 &= -4 \\ -2i_1 + 6i_2 + i_3 &= 33 \\ 4i_1 - 2i_2 + i_3 &= 3 \end{aligned} \tag{34}$$

The true solution by other conventional methods is $(6.83,\ 8.87, -6.57)^t$

By improved homotopy,

We solve the system $\boldsymbol{Au} = \boldsymbol{b},$ where

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 6 & 1 \\ 4 & -2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -4 \\ 33 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

Now by (30), i.e. $\boldsymbol{A\alpha} = (\boldsymbol{A} - \boldsymbol{I})\boldsymbol{b}$, we have

$$\begin{bmatrix} 1 & 1 & 3 \\ -2 & 6 & 1 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 \\ -2 & 5 & 1 \\ 4 & -2 & 0 \end{bmatrix} \begin{bmatrix} -4 \\ 33 \\ 3 \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & 1 & 3 \\ -2 & 6 & 1 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 42 \\ 176 \\ -82 \end{bmatrix}$$

Hence by (31), i.e. $\boldsymbol{\alpha} = (\boldsymbol{I} - \boldsymbol{A^{-1}})\boldsymbol{b}$,

The solution of the system gives as $\alpha = (-4, 33, 3)^t$. Thus the solution of Eq (34) becomes, $\boldsymbol{u} = \boldsymbol{u_1} = \boldsymbol{b} - \boldsymbol{\alpha} = (-4,\ 33,\ 3)^t - (-10.83,\ 24.13,\ 9.57)^t = (6.83,\ 8.87,\ -6.57)^t$, which is the exact solution of Example 2

Hence, $i_1 = 6.83;\ i_2 = 8.87;\ i_3 = -6.57$

5 Conclusion

Here a modification to the HPM was first proposed in which accelerating parameters were introduced to solve the system of linear equations. a new homotopy therefore was constructed named the accelerating vector. This accelerating vector of course leads to a fast convergent rate, since only one iteration leads to exact solution. The examples analyzed illustrate the ability and reliability of the improved method presented in this paper and reveal that the improvement of HPM is a simple and very effective tool for calculating the exact solutions.

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APPENDIX 1

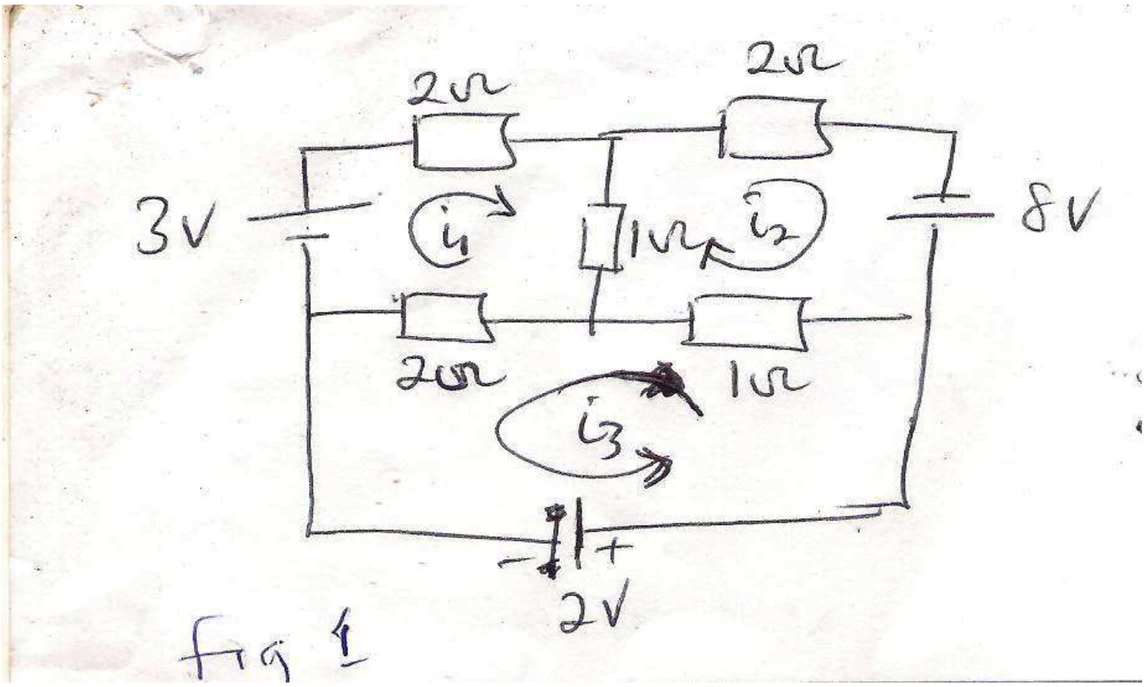

Fig 1: Electrical network model

The derived equation from fig 1:

$$5i_1 - i_2 + 2i_3 = 3$$
$$-i_1 + 4i_2 + i_3 = 8$$
$$-2i_1 + i_2 + 3i_3 = 2$$

APPENDIX 2

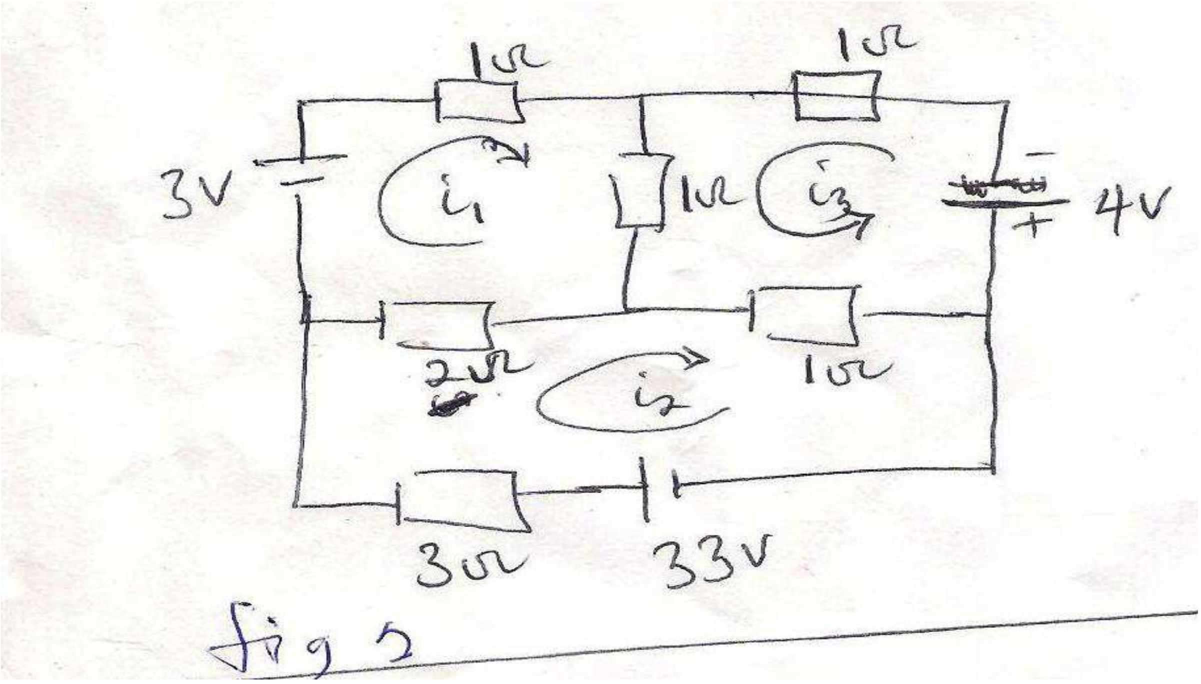

Fig 2: Electrical network model

The derived equation from fig 2:

$$i_1 + i_2 + 3i_3 = -4$$
$$-2i_1 + 6i_2 + i_3 = 33$$
$$4i_1 - 2i_2 + i_3 = 3$$