International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 20 No. 1 Jan. 2016, pp. 25-37

http://www.ijisr.issr-journals.org/

An accurate method to study the Rayleigh-Bénard problem in a rotating layer saturated by a Newtonian nanofluid

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ABSTRACT: The aim of this paper, is to use a more realistic model which incorporates the effects of Brownian motion and the thermophoresis of nanoparticles for studying the effect of some control parameters on the onset of convective instability in a rotating medium filled of a Newtonian nanofluid layer and heated from below, this layer is assumed to have a low concentration of nanoparticles. The linear study which was achieved in this investigation shows that the thermal stability of Newtonian nanofluids depends of the buoyancy forces, the Coriolis forces generated by the rotation of the system, the Brownian motion, the thermophoresis of nanoparticles and other thermo-physical properties of nanoparticles. The studied problem will be solved analytically by converting our boundary value problem to an initial value problem, after this step we will approach numerically the searched solutions by polynomials of high degree to obtain a fifth order accurate solution.

KEYWORDS: Linear stability, Newtonian nanofluid, Rotating layer, Brownian motion, Thermophoresis, Power series.

1 INTRODUCTION

The nanofluid is considered as a homogeneous fluid containing colloidal suspensions of nano-sized particles named nanoparticles in the base fluid (water, ethylene glycol, oil). The nanoparticles used in nanofluids are generally prepared of metals, oxides, carbides, or carbon nanotubes. The purpose of using nanofluids is to obtain a higher value of heat transfer coefficient compared with that of the base fluid , this remarkable properties make them potentially useful in many practical applications , for example in modern science and engineering including rotating machineries like nuclear reactors, petroleum industry, biochemical and geophysical problems.

In the recent years, the problem of natural convection in a confined medium filled of a Newtonian nanofluid layer has been studied in different situations by several authors [1-7]. When the volumetric fraction of nanoparticles is constant at the horizontal walls limiting the layer, they found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on the relative distribution of nanoparticles between the top and bottom walls.

Today, the problem of natural convection for the nanofluids is studied by some authors [9-14] using a new type of boundary conditions for the nanoparticles which combines the contribution of the Brownian motion and the thermophoresis of nanoparticles instead to impose a nanoparticle volume fraction at the boundaries of the layer. The new model of boundary conditions assumes that the nanoparticle flux must be zero on the impermeable boundaries. D.A. Nield and A.V. Kuznetsov [9] are considered as the first ones who were used this type of boundary conditions for the nanoparticles. Until now, the precedent boundary conditions are used to study only the problem of natural convection in a porous (Darcy or Darcy-Brinkman model) or non-porous medium saturated by a nanofluid using the Galerkin weighted residuals method based only on some test functions.

Our work consists of studying the Rayleigh-Bénard problem in a rotating medium filled of a Newtonian nanofluid layer in the free-free, rigid-free and rigid-rigid cases where the nanoparticle flux is assumed to be zero on the boundaries, our problem will be solved with a more accurate numerical method based on analytic approximations (power series method).

In this investigation we assume that the effect of the rotation in the momentum equation is restricted to the Coriolis force and also the centrifugal acceleration is negligible compared to the buoyancy force.

The used method gives results with an absolute error of the order of 10^{-6} to the critical values characterizing the onset of the convection. To show the accuracy of our method in this study, we will check some results treated by Chandrasekhar [8] concerning the study of the convective instability of the regular fluids in a rotating medium.

2 MATHEMATICAL FORMULATION

We consider an infinite horizontal layer of an incompressible Newtonian nanofluid characterized by a low concentration of nanoparticles, heated uniformly from below and confined between two identical horizontal surfaces where the temperature is constant and the nanoparticle flux is zero on the boundaries, this layer will be subjected to a uniform rotation characterized by an angular velocity $\vec{\Omega} = \Omega\vec{e}_z$ and also acted upon by the gravity force $\vec{g} = -g\vec{e}_z$ (Fig 1).The thermo-physical properties of nanofluid (viscosity, thermal conductivity, specific heat) are assumed constant in the vicinity of the temperature of the cold wall T_c except for the density variation in the momentum equation which is based on the Boussinesq approximations .The asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks).

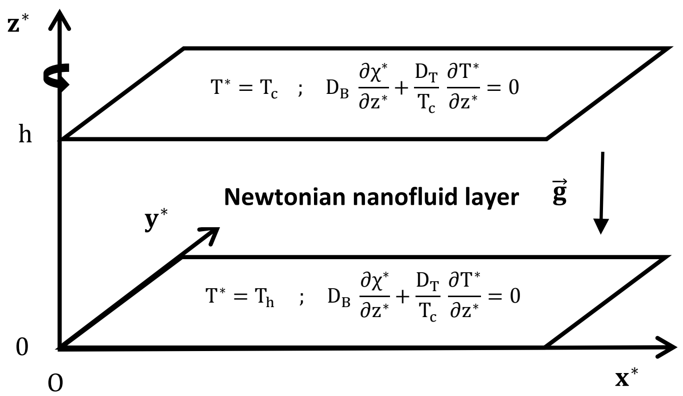

Fig. 1. Physical configuration

Within the framework of the assumptions which were made by Buongiorno [1] and Tzou [2, 3] in their publications for the Newtonian nanofluids, we can write the basic equations of conservation which govern our problem in dimensional form as follows:

$$\vec{\nabla}^*.\vec{V}^* = 0 \tag{1}$$

$$\rho_0\left[\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\vec{V}^*\right] = -\vec{\nabla}^*P^* - 2\rho_0\vec{\Omega}\times\vec{V}^* + \{\rho_0[1-\beta(T^*-T_c)](1-\chi^*) + \rho_p\chi^*\}\vec{g} + \eta\vec{\nabla}^{*2}\vec{V}^* \tag{2}$$

$$(\rho c)\left[\frac{\partial T^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)T^*\right] = \kappa\vec{\nabla}^{*2}T^* + (\rho c)_p\left[D_B\vec{\nabla}^*\chi^*.\vec{\nabla}^*T^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^*T^*.\vec{\nabla}^*T^*\right] \tag{3}$$

$$\frac{\partial \chi^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\chi^* = D_B\vec{\nabla}^{*2}\chi^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^{*2}T^* \tag{4}$$

Where $\vec{\nabla}^*$ is the vector differential operator.

If we consider the following dimensionless variables:

$$(x^*;y^*;z^*) = h(x;y;z) \quad ; \quad t^* = \frac{h^2}{\alpha}t \quad ; \quad \vec{V}^* = \frac{\alpha}{h}\vec{V} \quad ; \quad P^* = \frac{\eta\alpha}{h^2}P \quad ; \quad T^* - T_c = (T_h - T_c)T \quad ; \quad \chi^* - \chi_0^* = \chi_0^*\chi$$

Then, we can get from the equations (1)-(4) the following adimensional forms:

$$\vec{\nabla}.\vec{V} = 0 \tag{5}$$

$$P_r^{-1}\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -\vec{\nabla}(P + R_M z) + \sqrt{T_A}(v\vec{e}_x - u\vec{e}_y) + \vec{\nabla}^2\vec{V} + [(1-\chi_0^*)R_aT - R_N\chi - \chi_0^*R_aT\chi]\vec{e}_z \tag{6}$$

$$\frac{\partial T}{\partial t} + (\vec{V}.\vec{\nabla})T = \vec{\nabla}^2 T + N_B L_e^{-1} \vec{\nabla}\chi.\vec{\nabla}T + N_A N_B L_e^{-1} \vec{\nabla}T.\vec{\nabla}T \tag{7}$$

$$\frac{\partial \chi}{\partial t} + (\vec{V}.\vec{\nabla})\chi = L_e^{-1}\vec{\nabla}^2\chi + N_A L_e^{-1}\vec{\nabla}^2 T \tag{8}$$

Such that:

$$P_r = \frac{\eta}{\rho_0\alpha} \quad ; \quad L_e = \frac{\alpha}{D_B} \quad ; \quad N_B = \frac{(\rho c)_p}{(\rho c)}\chi_0^* \quad ; \quad T_A = \left(\frac{2\rho_0\Omega h^2}{\eta}\right)^2 \quad ; \quad \alpha = \frac{\kappa}{(\rho c)} \quad ; \quad R_a = \frac{\rho_0 g\beta h^3(T_h - T_c)}{\eta\alpha}$$

$$R_M = \frac{[\rho_0(1-\chi_0^*) + \rho_p\chi_0^*]gh^3}{\eta\alpha} \quad ; \quad R_N = \frac{(\rho_p - \rho_0)\chi_0^* gh^3}{\eta\alpha} \quad ; \quad N_A = \frac{D_T}{D_B T_c}\left(\frac{T_h - T_c}{\chi_0^*}\right)$$

Where χ_0^* is the reference value for nanoparticle volume fraction.

2.1 Basic Solution

The basic solution of our problem is a quiescent thermal equilibrium state, it's assumed to be independent of time where the equilibrium variables are varying in the z-direction, therefore:

$$\vec{V}_b = \vec{0} \tag{9}$$

$$T_b = 1 \quad ; \quad \frac{d\chi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \quad \text{at} \quad z = 0 \tag{10}$$

$$T_b = 0 \quad ; \quad \frac{d\chi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \quad \text{at} \quad z = 1 \tag{11}$$

If we introduce the precedent results into equations (6)-(8), we obtain:

$$\vec{\nabla}(P_b + R_M z) = [(1-\chi_0^*)R_a T - R_N\chi - \chi_0^* R_a T\chi]\vec{e}_z \tag{12}$$

$$\frac{d^2T_b}{dz^2} + N_B L_e^{-1}\left(\frac{d\chi_b}{dz}\frac{dT_b}{dz}\right) + N_A N_B L_e^{-1}\left(\frac{dT_b}{dz}\right)^2 = 0 \tag{13}$$

$$\frac{d^2\chi_b}{dz^2} + N_A\frac{d^2T_b}{dz^2} = 0 \tag{14}$$

After using the boundary conditions (10) and (11), we can integrate the equation (14) between 0 and z for obtaining:

$$\chi_b = N_A(1 - T_b) + \chi_0 \tag{15}$$

Where χ_0 is the relative nanoparticle volume fraction at $z = 0$, such that:

$$\chi_0 = \frac{\chi_b^*(0) - \chi_0^*}{\chi_0^*}$$

If we take into account the expression (15), we can get after simplification of the equation (13):

$$\frac{d^2T_b}{dz^2} = 0 \tag{16}$$

Finally, we obtain after an integrating of the equation (16) between 0 and z :

$$T_b = 1 - z \tag{17}$$

$$\chi_b = N_A z + \chi_0 \tag{18}$$

2.2 Stability Analysis

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows:

$$T = T_b + T' \quad ; \quad \vec{V} = \vec{V}_b + \vec{V}' \quad ; \quad P = P_b + P' \quad ; \quad \chi = \chi_b + \chi' \tag{19}$$

In the framework of the Oberbeck-Boussinesq approximations, we can neglect the terms which are coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (6), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (19) into equations (5)-(8) the following linearized equations:

$$\vec{\nabla}.\vec{V'} = 0 \tag{20}$$

$$P_r^{-1}\frac{\partial \vec{V}'}{\partial t} = -\vec{\nabla}P' + \sqrt{T_A}\left(v'\vec{e}_x - u'\vec{e}_y\right) + (R_a T' - R_N \chi')\vec{e}_z + \vec{\nabla}^2\vec{V}' \tag{21}$$

$$\frac{\partial T'}{\partial t} - w' = \vec{\nabla}^2 T' - N_A N_B L_e^{-1}\frac{\partial T'}{\partial z} - N_B L_e^{-1}\frac{\partial \chi'}{\partial z} \tag{22}$$

$$\frac{\partial \chi'}{\partial t} + N_A w' = N_A L_e^{-1}\vec{\nabla}^2 T' + L_e^{-1}\vec{\nabla}^2\chi' \tag{23}$$

After application of the curl operator twice to the equation (21) and using the equation (20), we obtain the following equations:

$${P_r}^{-1}\frac{\partial F'}{\partial t} = \vec{\nabla}^2 F' + \sqrt{T_A}\frac{\partial w'}{\partial z} \tag{24}$$

$${P_r}^{-1}\frac{\partial}{\partial t}\vec{\nabla}^2 w' = \vec{\nabla}^4 w' + R_a\vec{\nabla}_2^2 T' - R_N\vec{\nabla}_2^2\chi' - \sqrt{T_A}\,\frac{\partial F'}{\partial z} \tag{25}$$

Where:

$$\vec{\nabla}_2^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right) \;\; ; \;\; F' = \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)$$

Analyzing the disturbances into normal modes, we can simplify the equations (22) - (25) by assuming that the perturbation quantities are of the form:

$$(w', T', \chi', F') = \left(\mathcal{w}(z), \mathcal{T}(z), \mathcal{X}(z), \mathcal{F}(z)\right)\exp\left[i\left(k_x x + k_y y\right) + \sigma t\right] \tag{26}$$

After introducing the expressions (26) into equations (22) - (25), we obtain:

$${P_r}^{-1}\sigma\mathcal{F} = (D^2 - k^2)\mathcal{F} + \sqrt{T_A}D\mathcal{w} \tag{27}$$

$${P_r}^{-1}\sigma(D^2 - k^2)\mathcal{w} = (D^2 - k^2)^2\mathcal{w} - k^2 R_a\mathcal{T} + k^2 R_N\mathcal{X} - \sqrt{T_A}\,D\mathcal{F} \tag{28}$$

$$\sigma\mathcal{T} - \mathcal{w} = (D^2 - k^2)\mathcal{T} - N_A N_B L_e^{-1}D\mathcal{T} - N_B L_e^{-1}D\mathcal{X} \tag{29}$$

$$\sigma\mathcal{X} + N_A\mathcal{w} = N_A L_e^{-1}(D^2 - k^2)\mathcal{T} + L_e^{-1}(D^2 - k^2)\mathcal{X} \tag{30}$$

Where:

$$k = \sqrt{k_x^2 + k_y^2} \quad ; \quad D = d/dz$$

The equations (27) - (30) will be solved subject to the following boundary conditions:

- For the rigid-rigid case;

$$\mathcal{w} = D\mathcal{w} = \mathcal{T} = D(\mathcal{X} + N_A\mathcal{T}) = \mathcal{F} = 0 \qquad \text{at} \quad z = 0\,;1 \tag{31}$$

- For the free-free case;

$$\mathcal{w} = D^2\mathcal{w} = \mathcal{T} = D(\mathcal{X} + N_A\mathcal{T}) = D\mathcal{F} = 0 \qquad \text{at} \quad z = 0\,;1 \tag{32}$$

- For the rigid-free case;

$$\mathcal{w} = D\mathcal{w} = \mathcal{T} = D(\mathcal{X} + N_A\mathcal{T}) = \mathcal{F} = 0 \qquad \text{at} \quad z = 0 \tag{33}$$

$$\mathcal{w} = D^2\mathcal{w} = \mathcal{T} = D(\mathcal{X} + N_A\mathcal{T}) = D\mathcal{F} = 0 \qquad \text{at} \quad z = 1 \tag{34}$$

2.3 METHOD OF SOLUTION

In this study we assume that the principle of exchange of stability is valid, as we are interested in a stationary stability study $(\sigma = 0)$, then the equations (27)-(30) become:

$$\sqrt{T_A} D\mathcal{W} + (D^2 - k^2)\mathcal{F} = 0 \tag{35}$$

$$(D^2 - k^2)^2 \mathcal{W} - k^2 R_a \mathcal{T} + k^2 R_N \mathcal{X} - \sqrt{T_A}\, D\mathcal{F} = 0 \tag{36}$$

$$\mathcal{W} + (D^2 - k^2)\mathcal{T} - N_A N_B L_e^{-1} D\mathcal{T} - N_B L_e^{-1} D\mathcal{X} = 0 \tag{37}$$

$$N_A \mathcal{W} - N_A L_e^{-1}(D^2 - k^2)\mathcal{T} - L_e^{-1}(D^2 - k^2)\mathcal{X} = 0 \tag{38}$$

We can solve the equations (35)-(38) which are subjected to the conditions (31)-(34), by making a suitable change of variables that makes the number of variables equal to the number of boundary conditions to obtain a set of ten first order ordinary differential equations which we can write it in the following form:

$$\frac{d}{dz} u_i(z) = a_{ij} u_j(z)\ ;\ \ 1 \le i, j \le 10 \tag{39}$$

With:

$$a_{ij} = a_{ij}(k, R_a, T_A, N_B, L_e, R_N, N_A)$$

The solution of the system (39) in matrix notation can be written as follows:

$$U = BC \tag{40}$$

Where:

$B = \left(\left(b_{ij}(z)\right)_{\substack{1\le i\le 10\\ 1\le j\le 10}}\right)$: is a square matrix of order 10×10.

$U = \left(\left(u_i(z)\right)_{1\le i\le 10}\right)^T$: is the unknown vector column of our problem.

$C = \left(\left(c_j\right)_{1\le j\le 10}\right)^T$: is a constant vector column.

If we assume that the matrix B is written in the following form:

$$B = \left(\left(u_i^j(z)\right)_{\substack{1\le i\le 10\\ 1\le j\le 10}}\right) \tag{41}$$

Therefore, the use of five boundary conditions at $z = 0$, allows us to write each variable $u_i(z)$ as a linear combination for five functions $u_i^j(z)$, such that:

$$b_{ij}(0) = u_i^j(0) = \delta_{ij} \tag{42}$$

Where δ_{ij} is the Kronecker delta symbol.

After introducing the new expressions of the variables $u_i(z)$ in the system (39), we will obtain the following equations:

$$\frac{d}{dz} u_i^j(z) = a_{il} u_l^j(z)\ ;\ \ 1 \le i, l, j \le 10 \tag{43}$$

For each value of j, we must solve a set of ten first order ordinary differential equations which are subjected to the initial conditions (42), by approaching the variables $u_i^j(z)$ with power series defined in the interval [0,1] and truncated at the order N, such that:

$$u_i^j(z) = \sum_{p=0}^{p=N} d_p^{i,j} z^p \tag{44}$$

A linear combination of the solutions $u_i^j(z)$ satisfying the boundary conditions (31), (32) or (33) and (34) at $z = 1$ leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_a, k, T_A, N_B, L_e, R_N, N_A) = 0 \quad (45)$$

If we give to each control parameter $(T_A, N_B, L_e, R_N, N_A)$ its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number R_a which corresponds to a fixed wave number k and verifies the dispersion relation (45). After that, we will find a set of points (k, R_a) which help us to plot our curve and find the critical value (k_c, R_{ac}) which characterizes the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

2.4 Validation of the Method

The main aim in this investigation is to study the influence of a uniform rotation on the convective instability of the Newtonian nanofluids in a confined medium filled of a Newtonian nanofluid layer for different cases of boundary conditions: free-free, rigid-free and rigid-rigid cases. Our study shows that the thermal stability of Newtonian nanofluids depends on five parameters : T_A, N_B, L_e, R_N and N_A .

The truncation order N which corresponds to the convergence of our method is determined, when the five digits after the comma of the critical thermal Rayleigh number R_{ac} for a Newtonian nanofluid ($N_B = 0.01, L_e = 100, R_N = 1, N_A = 0.1$) remain unchanged (Tables 1 and 3).

To validate our method, we compared our results with those obtained by Chandrasekhar [8] concerning the Rayleigh-Bénard problem in a rotating medium filled of a regular fluid layer (Tables 4 and 5). To make this careful comparison, we must take into consideration the restrictions $L_e^{-1} = R_N = N_A = N_B = 0$ in the governing equations of our problem.

Table 1. The stationary instability threshold of the Newtonian nanofluid for different values of the Taylor number in the free-free case.

N	$T_A = 100$		$T_A = 400$		$T_A = 1000$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
15	2.57044	805.42899	3.14490	1162.22820	3.70512	1661.28391
16	2.57044	805.42870	3.14491	1162.22717	3.70515	1661.28256
17	2.57044	805.42938	3.14491	1162.22781	3.70515	1661.28128
18	2.57044	805.42939	3.14491	1162.22799	3.70515	1661.28295
19	2.57044	805.42936	3.14491	1162.22788	3.70515	1661.28217
20	2.57044	805.42936	3.14491	1162.22791	3.70515	1661.28242
21	2.57044	805.42936	3.14491	1162.22791	3.70515	1661.28236
22	2.57044	805.42936	3.14491	1162.22791	3.70515	1661.28237
23	2.57044	805.42936	3.14491	1162.22791	3.70515	1661.28237
24	2.57044	805.42936	3.14491	1162.22791	3.70515	1661.28237
25	2.57044	805.42936	3.14491	1162.22791	3.70515	1661.28237
Exact value	**2.57044**	**805.42936**	**3.14491**	**1162.22791**	**3.70515**	**1661.28237**

Table 2. The stationary instability threshold of the Newtonian nanofluid for different values of the Taylor number in the rigid-free case

N	$T_A = 100$		$T_A = 400$		$T_A = 1000$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
21	2.84015	1188.87687	3.19644	1454.44537	3.62143	1866.88793
22	2.84015	1188.87684	3.19644	1454.44470	3.62144	1866.87576
23	2.84015	1188.87685	3.19644	1454.44490	3.62143	1866.87944
24	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87862
25	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87873
26	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87874
27	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87873
28	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87873
29	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87873
30	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87873
31	2.84015	1188.87685	3.19644	1454.44486	3.62144	1866.87873
Exact value	**2.84015**	**1188.87685**	**3.19644**	**1454.44486**	**3.62144**	**1866.87873**

Table 3. The stationary instability threshold of the Newtonian nanofluid for different values of the Taylor number in the rigid-rigid case

N	$T_A = 100$		$T_A = 400$		$T_A = 1000$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
22	3.15254	1740.78625	3.27530	1880.49260	3.48211	2136.47610
23	3.15249	1740.79954	3.27486	1880.57516	3.47817	2136.92631
24	3.15251	1740.79543	3.27499	1880.55699	3.47900	2136.91107
25	3.15251	1740.79644	3.27497	1880.55985	3.47891	2136.88477
26	3.15251	1740.79624	3.27497	1880.55968	3.47889	2136.89773
27	3.15251	1740.79627	3.27497	1880.55960	3.47891	2136.89377
28	3.15251	1740.79627	3.27497	1880.55964	3.47890	2136.89466
29	3.15251	1740.79627	3.27497	1880.55963	3.47890	2136.89452
30	3.15251	1740.79627	3.27497	1880.55963	3.47890	2136.89453
31	3.15251	1740.79627	3.27497	1880.55963	3.47890	2136.89453
32	3.15251	1740.79627	3.27497	1880.55963	3.47890	2136.89453
Exact value	**3.15251**	**1740.79627**	**3.27497**	**1880.55963**	**3.47890**	**2136.89453**

Table 4. The comparison of critical values of Rayleigh number and the corresponding wave number with Chandrasekhar [8] for different values of the Taylor number in the free-free and rigid-rigid cases

T_A	Chandrasekhar free-free case		Present study free-free case			Chandrasekhar rigid-rigid case		Present study rigid-rigid case		
	k_c	R_{ac}	k_c	R_{ac}	N	k_c	R_{ac}	k_c	R_{ac}	N
0	2.2214	657.511	2.22144	657.51138	17	3.117	1707.762	3.11632	1707.76177	28
10	2.270	677.1	2.27012	677.07685	21	3.10	1713	3.12087	1712.67407	28
100	2.594	826.3	2.59354	826.28956	19	3.15	1756.6	3.16081	1756.34730	27
500	3.278	1275	2.27756	1274.56710	19	3.30	1940.5	3.31925	1940.19924	29
1000	3.710	1676	3.71043	1676.11802	19	3.50	2151.7	3.48471	2151.34119	30

Table 5. The comparison of critical values of Rayleigh number and the corresponding wave number with Chandrasekhar [8] for different values of the Taylor number in the rigid-free case

T_A	Chandrasekhar rigid-free case		Present study rigid-free case		
	k_c	R_{ac}	k_c	R_{ac}	N
0	2.682	1100.65	2.68232	1100.64960	22
6.25	2.68	1108.5	2.69450	1107.72808	22
31.25	2.70	1136.5	2.74103	1135.40837	23
62.50	2.79	1169.5	2.79489	1168.72026	23
187.5	2.975	1291.7	2.97549	1290.80447	23

According to the above results, we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. Briefly, the convergence of the results depends greatly on the truncation order N of the power series and also of the Taylor number T_A . Finally, to ensure the accuracy of our obtained critical values for the studied nanofluids, we will take as truncation order:

- N = 22 : for the free-free case .
- N = 27 : for the rigid-free case .
- N = 30 : for the rigid-rigid case.

3 RESULTS AND DISCUSSION

To study the effect of a parameter $(T_A, N_B, L_e, R_N, N_A)$ on the onset of the convective instability in a rotating medium filled of a Newtonian nanofluid layer, we must plot in Figs 2-5 the variation of the critical thermal Rayleigh number R_{ac} as a function of the Taylor number T_A for different values of this parameter and compare the obtained results with those of the regular fluids which are characterized by:

$$L_e^{-1} = R_N = N_A = N_B = 0$$

Generally the variation in the critical thermal Rayleigh number R_{ac} with the Taylor number T_A is an increasing function whatever the value taken for the parameters N_B , L_e , R_N and N_A , so the presence of the Coriolis forces allows us to reduce the effect of the buoyancy forces , hence the Taylor number T_A has a stabilizing effect.The precedent figures confirm that the presence of the friction on the horizontal walls is a factor producing the thermal stability of the system, such that :

$$R_{ac}^{rr} > R_{ac}^{rf} > R_{ac}^{ff}$$

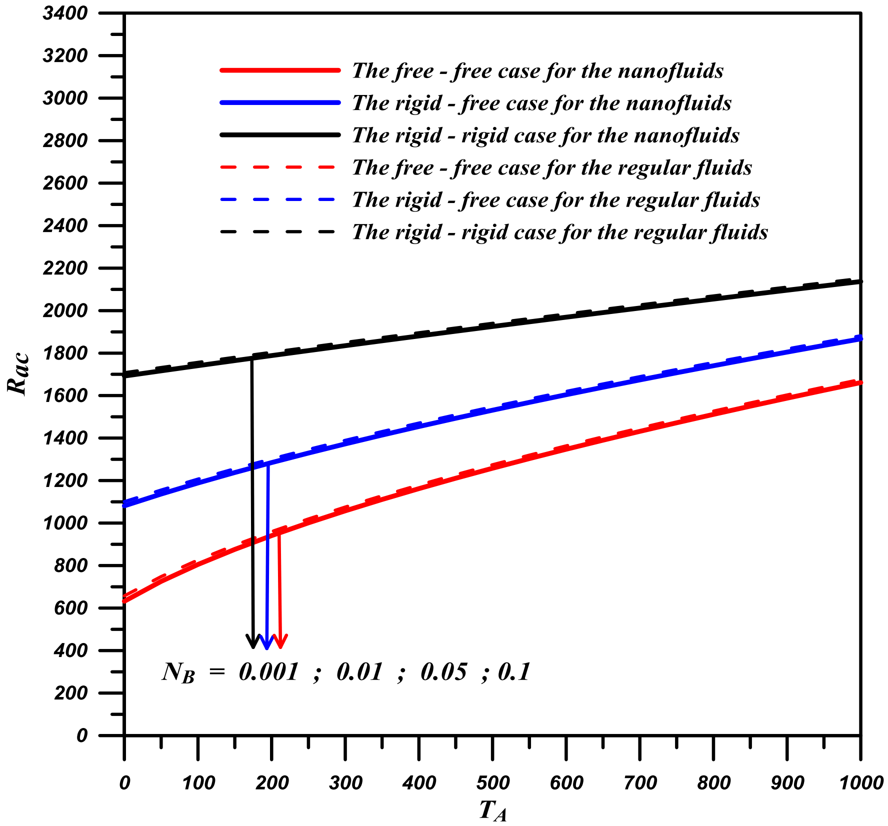

Fig. 2. Plot of R_{ac} as a function of T_A for different values of N_B in the case where L_e=100 , R_N=1 and N_A=0.1

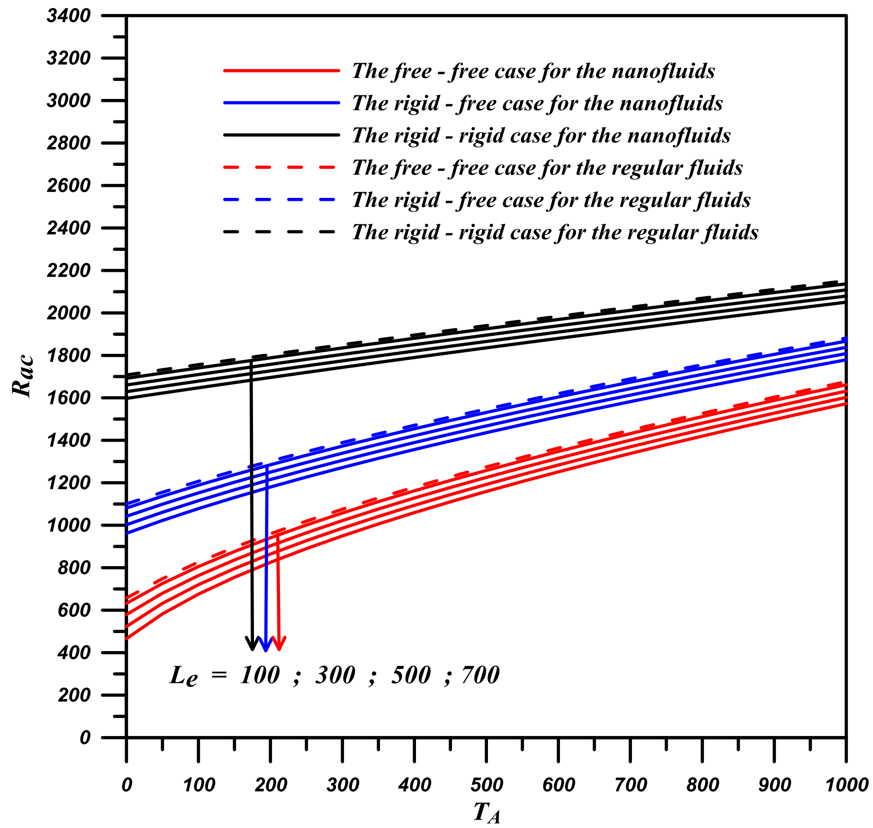

Fig. 3. Plot of R_{ac} as a function of T_A for different values of L_e in the case where N_B=0.01 , R_N=1 and N_A=0.1

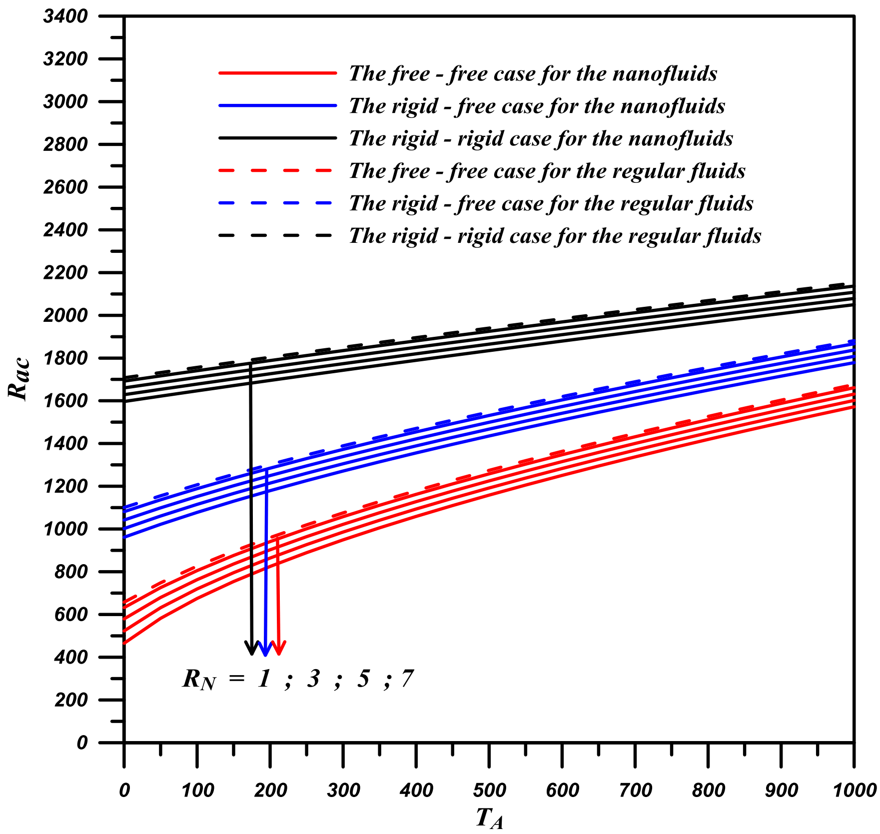

Fig. 4. *Plot of R_{ac} as a function of T_A for different values of R_N in the case where N_B=0.01 , L_e=100 and N_A=0.1*

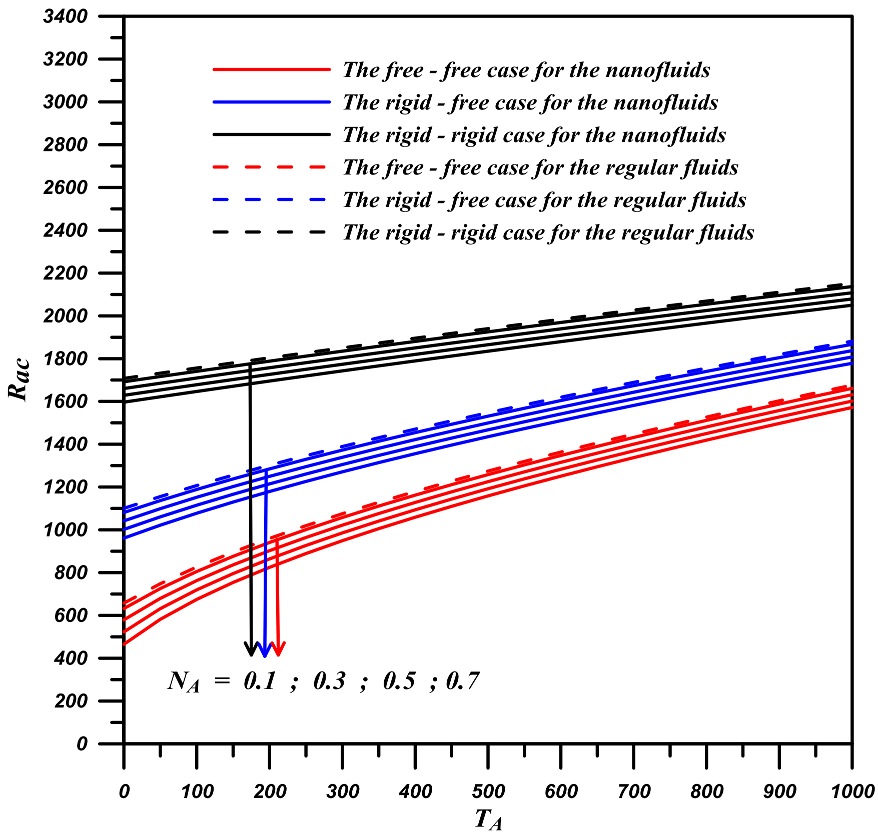

Fig. 5. *Plot of R_{ac} as a function of T_A for different values of N_A in the case where N_B=0.01 , L_e=100 and R_N=1*

The Fig 2 shows that the modified particle-density increment N_B has almost no effect on the convective instability of the nanofluids, this result may be explained by its low value $(N_B \sim 10^{-3} - 10^{-1})$ which appears only in the perturbed energy equation (22) as a product with the inverse of the Lewis number $(L_e \sim 10^2 - 10^3)$ near the temperature gradient and the volume fraction gradient of nanoparticles, so the effect of this parameter on the onset of convection in nanofluids will be very small which we can neglect it .

From the Figs 3 and 4 we conclude that an increase either in the Lewis number L_e or in the concentration Rayleigh number R_N allows us to accelerate the onset of the convection, hence they have a destabilizing effect .Therefore, to ensure the stability of the system, we can use the nanofluids which are having a less thermal diffusivity or containing less dense nanoparticles.

In this investigation, we find also that an increase in the volume fraction of nanoparticles destabilizes the nanofluids, because an increase in this parameter, increases also the Brownian motion and the thermophoresis of nanoparticles, which cause the destabilizing effect, this result confirm that the regular fluids are more stable than the nanofluids.

When the modified diffusivity ratio N_A increases, the temperature difference between the horizontal plates also increases. The Fig 5 shows that an increase in the modified diffusivity ratio N_A allows us to decrease the critical thermal Rayleigh number R_{ac} , this result can be explained by the increase in the buoyancy forces which destabilizes the system.

4 CONCLUSIONS

In this paper, we have examined the effect of a uniform rotation on the onset of convection in a confined medium filled of a Newtonian nanofluid layer, heated uniformly from below and cooled from above for free-free, rigid-rigid and rigid -free boundaries in the case where the nanoparticle flux is zero on the boundaries. The contribution of the Brownian motion and the thermophoresis of nanoparticles in the equation expressing the buoyancy effect coupled with the conservation of nanoparticles have a major effect on the onset of convection compared with their contributions in the thermal energy equation .

The resulting eigenvalue problem is solved analytically and numerically using the power series method. The behavior of various parameters like the Taylor number T_A , the modified particle-density increment N_B, the Lewis number L_e , the concentration Rayleigh number R_N and the modified diffusivity ratio N_A on the onset of convection has been analysed. The results can be summarized as follows:

I. The presence of the Coriolis forces allows us to stabilize the Newtonian nanofluids, such that an increase in the Taylor number T_A induces also an increase in the critical thermal Rayleigh number R_{ac}.

II. The presence of the friction on the horizontal walls is a factor producing the thermal stability of the system, where the rigid-rigid case is the more stable case compared with the rigid-free and free-free cases , such that:

$$R_{ac}^{rr} > R_{ac}^{rf} > R_{ac}^{ff}$$

III. To ensure the stability of the system, we can use the nanofluids which are having a less thermal diffusivity, a low concentration of nanoparticles or consisting of less dense nanoparticles.

IV. An increase either in the volume fraction of nanoparticles, in the buoyancy forces, in the Brownian motion or in the thermophoresis of nanoparticles allows us to destabilize the nanofluids.

V. The regular fluids are more stable than the nanofluids.

VI. The used method to solve the convection problem gives more accurate results, because the absolute error of the obtained critical values which characterize the onset of the convection is of the order of 10^{-6} , Hence, we can used our results as a reference to validate other results of the similar problems .

NOMENCLATURE

Symbols :

D_B	Brownian diffusion coefficient (m^2/s)
D_T	Thermophoretic diffusion coefficient (m^2/s)
g	Acceleration due to gravity (m/s^2)
h	Layer depth (m)
κ	Thermal conductivity of Nanofluid $(W/K.m)$
k_x^*	Wave number in x^* direction (m^{-1})
k_y^*	Wave number in y^* direction (m^{-1})
k_c^*	Critical wave number (m^{-1})
L_e	Lewis number
N_A	Modified diffusivity ratio
N_B	Modified particle-density increment
P^*	Pressure (Pa)
P_r	Prandtl number
R_a	Thermal Rayleigh number
R_{ac}	Critical Rayleigh number
R_M	Density Rayleigh number
R_N	Concentration Rayleigh number
$\vec{V}^*$	Velocity vector (m/s)
T_A	Taylor number
T^*	Temperature (K)
t^*	Time (s)
u^*, v^*, w^*	Velocity components (m/s)
x^*, y^*, z^*	Cartesian coordinates (m)

Greek symbols :

α	Thermal diffusivity of nanofluid (m^2/s)
β	Thermal expansion coefficient of base fluid (K^{-1})
Ω	Angular velocity $(rad.s^{-1})$
η	Viscosity of nanofluid $(Pa.s)$
ρ	Nanofluid density (kg/m^3)
ρ_0	Fluid density at reference temperature(kg/m^3)
(ρc)	Heat capacity of nanofluid $(J/m^3.K)$
$(\rho c)_p$	Heat capacity of nanoparticles $(J/m^3.K)$
σ^*	Growth rate of disturbances (s^{-1})
χ^*	Volume fraction of nanoparticles

Superscripts :

$*$	Dimensional variable
$'$	Perturbation variable
ff	Free - Free case
rf	Rigid - Free case
rr	Rigid - Rigid case

Subscripts :

c	Cold
h	Hot
ac	Critical number
b	Basic solution
f	Base fluid
p	Nanoparticle

REFERENCES

[1] J. Buongiorno, "Convective transport in nanofluids," Journal of Heat Transfer, vol.128, pp.240-250, 2006.

[2] D.Y. Tzou, "Thermal instability of nanofluids in natural convection," International Journal of Heat and Mass Transfer, vol.51, pp.2967-2979, 2008.

[3] D.Y. Tzou, "Instability of nanofluids in natural convection", ASME Journal of Heat Transfer, 130, 2008, 372-401.

[4] D.A. Nield and A.V. Kuznetsov, "The onset of convection in a horizontal nanofluid layer of finite depth," European Journal of Mechanics B/Fluids, vol.29, pp.217-233, 2010.

[5] Dhananjay Yadav, R. Bhargava and G.S. Agrawal, "Numerical solution of a thermal instability problem in a rotating nanofluid layer," International Journal of Heat and Mass Transfer, vol.63, pp.313-322, 2013.

[6] Dhananjay Yadav, R. Bhargava and G.S. Agrawal, "Thermal instability in a nanofluid layer with a vertical magnetic field," J .Eng. Math, vol.80, pp.147-164, 2013.

[7] Dhananjay Yadav, R. Bhargava, G. S. Agrawal, Gyeong S. Hwang, Jinho Lee and M. C. Kim, "Magneto-convection in a rotating layer of nanofluid," Asia-Pac. J. Chem. Eng., DOI: 10.1002/apj.1796.

[8] S. Chandrasekhar, Hydrodynamic and hydromagnetic stability, Oxford University Press, Oxford, 1961.

[9] D.A. Nield and A.V. Kuznetsov, "Thermal instability in a porous medium layer saturated by a nanofluid: a revised model," International Journal of Heat and Mass Transfer, vol.68, pp.211-214, 2014.

[10] Shilpi Agarwal, "Natural Convection in a Nanofluid-saturated rotating porous layer: a more realistic approach," Transp. Porous Med., vol.104, pp.581-592, 2014.

[11] I.S. Shivakumara, M. Dhananjaya and Chiu-On Ng, "Thermal convective instability in an Oldroyd-B nanofluid saturated porous layer," International Journal of Heat and Mass Transfer, vol.84, pp.167-177, 2015.

[12] Shilpi Agarwal and Puneet Rana, "Thermal stability analysis of rotating porous layer with thermal non-equilibrium approach utilizing Al_2O_3–EG Oldroyd-B nanofluid," Microfluid Nanofluid, DOI 10.1007/s10404-015-1554-8.

[13] A.Wakif , Z.Boulahia and R.Sehaqui, "A Realistic Approach for Studying the Effect of an Internal Heat Source on the Onset of Convection in a Newtonian Nanofluid Layer: rigid-rigid case," IOSR-Journal of Mathematics,vol.11,issue 5 , Ver. IV, pp.21-30, 2015.
http://iosrjournals.org/iosr-jm/papers/Vol11-issue5/Version-4/D011542130.pdf

[14] A.Wakif and R.Sehaqui, "A Realistic Study of the Rayleigh-Bénard Problem in the Newtonian Nanofluids with a Uniform Heat Source: Free-Free Case," International journal of science and research (IJSR), vol.4, issue 10, p. 611-615, 2015.
http://www.ijsr.net/archive/v4i10/SUB158772.pdf