Insight into Fractal Image Compression: A Multifacet View

Mekala Ramesh¹, V. Palanisamy², B. Gopinath³, and Thangavel Rukmangadhan

¹Computer Science and Engineering, Info Institute of Engineering, Coimbatore, TamilNadu, India

²Info Institute of Engineering, Coimbatore, TamilNadu, India

³Electronics and Communication Engineering, Info Institute of Engineering, Coimbatore, TamilNadu, India

⁴Department of Electronics, Sri RamaKrishna Mission Vidhyalaya College of Arts and Science, Coimbatore, TamilNadu, India

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ABSTRACT: This paper presents a survey on practical as well as theoretical advancements in fractal image compression (FIC) practice. It is a lossy compression scheme comprises Affine Contractive Transforms, Iterated Functions represented as fractals. It is with a high compression ratio, but entails more time for encoding. Although many schemes are published to speed up encoding, they do not easily satisfy the encoding time or the reconstructed image quality requirements. FIC method incorporating optimization techniques results in better performance of the practical applications.

Keywords: FIC, CMT, IFS, image partitioning, Domain Pool Selection.

1 INTRODUCTION

Basically images are classified as natural images (from natural sources) and artificial images (created artificially). In current scenario, it is required to process images from natural sources more than that of artificial one since it occupies more space comparatively. Thus, there is a necessity for a better Image Processing Technique (IPT) to be identified to optimize the performance. The IPT encompasses formation and enhancement of image. Enhancement is categorized into visualization, analysis and management. Image management plays a vital role with inclusion of Compression, Archiving, Retrieval and Communication, in which the compression is much more important. Thus, the Image Compression Technique (ICT) is concentrated by considering the size of compressed file versus quality of decompressed image. ICT is classified into lossless compression (Run Length Encoding (RLE), Huffman Coding (HC), Lempel-Ziv-Welch (LZW)) and lossy compression (Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT), Fractal Image Compression (FIC)).FIC scheme is chosen because of its self- similarity, resolution independence and fractal interpolation features. To optimize the performance of FIC each of its features has to be studied from different point of view.

2 FRACTAL IMAGE COMPRESSION

M. F. Barnsley [1][2] proposed the FIC by representing image as a collection of Affine Transformations, Iterated Functions and derived the Contractive Mapping Transform (CMT) applied to IFS's called the Collage Theorem. Y. Fisher [7] employed the Partitioned or Local Iterated Function System (PIFS/LIFS) to describe images utilizing the property of self-similarity. M. F. Barnsley [3] converted PIFS/LIFS fractal image compression to Recurrent Iterated Function System (RIFS). Fractal geometry, [6] denotes partitioned image as a collection of shrunken copies, a structure equivalent to RIFS. First practical FIC approach implemented by A. E. Jacquin [4][5] based on the Baseline Fractal Image Compression (BFIC) principle, partitions an image into Range Blocks (uniform size, non-overlapping) and the Domain Block (optional size, overlapping) and suggested the square shape of blocks by insisting the domain size as twice that of the range size.

2.1 THE COLLAGE THEOREM

Fractal coding represents a signal X as a contractive transform T, chosen such that the fixed point X_T of T is close to X and X_T may be recovered from T by the iterative process.

Ever since the distortion $||e_T||$ (where $e_T = x - x_T$) introduced by the fractal approximation can usually not be directly optimized for these reasons, the standard approach is to optimize T to minimize the collage error $||e_T||$ (where $e_c = x - Tx$),

which is usually computationally tractable. The collage theorem guarantees the $\|e_{\tau}\|$ made small by finding T such that $\|e_{c}\|$ is sufficiently small. The most common form of the collage theorem is $\|e_{\tau}\| \leq (1-\alpha)^{-1} \|e_{c}\|$, where T is a contractive transform with Lipschitz factor α (i.e. $\|Tx - Ty\| \leq \alpha \|x - y\|$). In image coding terms this implies that a transform 'T', for which the fixed

point X_T is close to an original image x, may be found by designing the transform 'T' such that the collage- Tx is close to x, achieved by minimizing the collage error for each range block individually.

A similar bound is possible for eventual contractivity while a tighter collage bound is possible by imposing certain restrictions, consisting primarily of requiring DC subtraction in the block transform and setting the domain increment to be equal to the range block size. In spite of the considerable improvement over the usual collage theorem bound, this bound is still rather loose. The majority of existing fractal coding schemes restricts T to be an affine transform Tx = Ax + b, where A is a linear transform (encapsulating the combined effects of the spatial contractions, isometry operations, and scalings of the individual domain to range mappings) and b is an offset vector (comprises of the offsets in each of the individual domain to range mappings). In this case $e_c = (I - A)e_T$, and bounds $(1 + ||A||)^{-1}||e_c|| \le ||e_T|| \le (1 - ||A||)^{-1}||e_c||$ may be derived, in terms of an operator norm ||A|| consistent with the vector norm, by noting that that $|||u|| - ||v||| \le ||u - v|| \le ||u|| + ||v||$ for arbitrary vectors u and v.

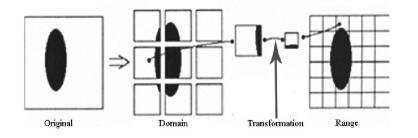


Fig.1 Fractal Image Compression Process

2.2 PERSPECTIVE OF FRACTAL IMAGE COMPRESSION

To explore FIC [23] four perspectives are identified.

- Iterated Function Systems (IFS): the metric spaces operators have fractal subsets as attractors of IFSs. Each block is approximated by the sum of DC components and a scaled copy of an image block taken from the VQ codebook.
- Self Vector Quantization: fractal encoding is same as mean-removed shape-gain vector quantization (MRSG-VQ) except the explicit code book availability in which an image.

- Self-Quantized Wavelet Subtrees (SQWS): wavelet transform coding organizes the Haar wavelet coefficients in a tree structure as well as subtrees are obtained by scaled copies approximation of other subtrees closer to the root of the wavelet tree.
- Convolution Transform Coding (CTC): operations carried out in searching, a matching image region are equivalent to a convolution operation. Only one of the convolution coefficients is selected for the fractal code. This leads to hybrid codes which hold the strongest prospects for the best rate-distortion techniques.

3 FIC APPROACH

The fundamental principle behind FIC is Formation of range blocks by image partition techniques, Selection of domain pool by improvements, Class of block transforms applied on domain pools, Searching the suitable domain pool for formation of particular range block and Optimizing the search strategies

3.1 IMAGE PARTITIONING TECHNIQUES

Partitioning of an image is employed with either Right-Angled approach or Triangular/Polygonal approach.

1) Right-Angled Partition Approach

- The fixed size square blocks [8] provide simplest possible range partition.
- Adaptive square blocks [9] introduced adaptivity in partitioning images with large blocks in low detail regions and small blocks with significant detail regions.
- Quad tree partition [10] employs a recursive splitting of image quadrants represented by a tree structure with each non-terminal node has four descendents which can be extended to variants.
- Irregular block partition [11] tiled an image by right-angled irregular shapes employing various merging strategies on a fixed square block / quadtree, either by top-down or by bottom-up approach.
- The Horizontal-Vertical (HV) partition [12][13] splits the image block is by horizontal or vertical line boundary approximation of prominent edges with adaptive size.

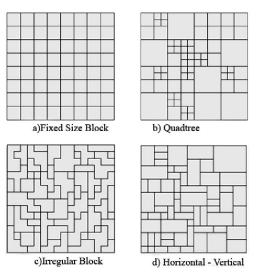


Fig2. Right-Angled Partition approach

2) Triangular and Polygonal Partition approach

- Polygonal block partition [14] [15] investigated splitting the image into two main triangles by the insertion of suitable diagonal, progressively smaller triangles where ever necessary by a 1-side and 3-side split from a vertex of the triangle to a point on the opposite side.
- Delaunay triangulation partition [16][17] initiated from set of seed points, and extended to additional seed points in regions of high image variance by the insertion of line segments at various angles employing recursive splitting and finally merging triangles to form quadrilaterals.

• Overlapped block partition [18][19] extended a quadtree fixed block size range partition employing multiple domain transforms by block overlapping with extended wavelet domain fractal coding. These methods require interpolation during the block transformation.

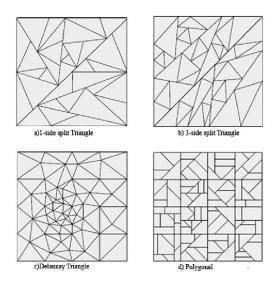


Fig.3. Triangular and Polygonal Partition approach

4 DOMAIN POOL SELECTION TECHNIQUES

The selection of domain pool depends on the type of partition technique adopted. A domain block undergoes a symmetry operation before mapping it onto a range block. This increases the size of the domain pool.

- Global Code Book [13] established by fixed square block/quadtree partition for all range blocks or for a particular class of range blocks in the image.
- Local Code Book [20] incurred by masking of range block at centre, for each range block is being restricted to a spiral search path followed outwards from the range block position.
- Synthetic code book [21][22] *extracted independently from low resolution image approximation without any iteration.*
- Hybrid code book [23] represented by combining mappings from domain blocks with fixed Vector Quantization codebook.

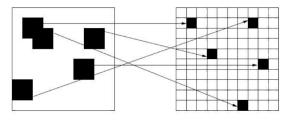


Fig.4. Mapping from Domain to Range Block

5 BLOCK TRANSFORM TECHNIQUES

The convergence properties on decoding are determined by the type of block transform selected in Fractal encoding scheme. Quantized parameters of encoding include the most of the information in the compressed representation.

5.1 REPRESENTATION OF TRANSFORM

The range partition is represented by Quadtree or Horizontal-Vertical adaptive partitions. Domain partition is represented by Discrete Values and spiral search based on the corresponding Vector Quantization technique [20].

The class of block transform is chosen depending upon the partition approach used and the type of domain pool selected.

- Affine Transformation [16] [24] applied on nonrectangular partitions by matching the vertices of transformed domain blocks with the vertices of the range blocks, compiled of skewing, stretching, rotation, scaling, translations, dilations and shears.
- Wavelet Based Fractal Transform (WBFT) [1] realized by connecting IFS and the theory of Multi Resolution Analysis (MRA).
- Second order Transform [7] extended affine transformation including multiple fixed blocks, quadratic blocks and also cubic blocks.
- Discrete Cosine Transform (DCT) [25] [26] formed basis vector for image blocks through mutual orthogonality.

6 SEARCHING TECHNIQUES

Searching is a method match up each range block to a domain block with the minimal difference under an affine transformation.

- Boss, fisher and jacob [27] proposed subdivision of a square domain/range block into four quadrant canonical ordering and a range block is compared only with the domain blocks belongs to the same category.
- D.Saupe [28] adopted a spatial constraint on the domain pool for each range, performed contractive mapping between domain block with a higher variance than the range block called domain pool reduction.
- B. Hurtgen [29] adopted four quadrants technique of Fisher, and calculated average intensities of quadrants of any block and compared with the average intensity of overall block to classify them into 360 classes.
- Nearest neighbour search [30] utilised a preprocessing stage representing hyperplane induced partition to arrange the blocks in tree structure.
- Invariant representation [31] of image block is equivalent to the multidimensional nearest neighbour search method based on feature extraction.
- Efficient distance computation [32] constructed an invariant representation from Hadamard transform coefficients in zig-zag scan order and computated the inner products between domain and range blocks.
- Classifications [21] rely on features of least approximate invariant representation.
- Distance bounds [34] excluded impossible matches calculating distance inequalities within image blocks.
- Clusters [35] for small range blocks are generated for classes of Fisher scheme using KD-tree and nearest neighbour search method.
- Adaptive approximation [36] converted the domain and range matching to nearest neighbour search, approximated it by orthogonal projections.
- Deferring Range/Domain Comparison (DRDC) [37] created domain codebook by comparing with preset block, then classified the blocks according to an approximation error and represented using KD-tree as a search key.
- Pyramid algorithm [38] applied a multiresolution tree search at each level of resolution domains progressing as pyramid.

7 OPTIMIZING TECHNIQUES

The FIC suffers from the high computation time dedicated on encoding, considering the quality measures and adopting suitable Nature Inspired Algorithms reduces the encoding time thus increases the performance.

7.1 QUALITY MEASURES

A quality measure with good objective calculates the difference between original and distorted image.

Parameters of objective measurement are [39]:

• Compression Ratio (CR) is the ratio between the size of uncompressed image and the size of compressed image.

- Encoding Time (ET) is the time taken to obtain the details of all the domain blocks including all rotations and reflections of each domain block to find a suitable PIFS
- Mean square Error (MSE) or signal to noise ratio (SNR) measures the average squared deviation between original and coded image. The large value of MSE means poor image quality.
- Peak-Signal to Noise-Ratio (PSNR) indicates a smaller difference between the original and reconstructed image. An objective image quality/ distortion measure makes the computation easy. large value of PSNR means good image quality.
- Maximum Difference (MD) the maximum difference between the pixels in original and compressed image among all differences.
- Average Difference (AD) lower difference between the pixels in original and compressed image gives a more noise reduced clean image.
- Mean Absolute Error (MAE) minimum for better image quality.
- Normalized Absolute Error (NAE) measures the deviation of decompressed image from the original image with the value of zero being the perfect fit. Large value of NAE indicates poor image quality.
- Normalized Correlation (NK) assesses the similarity / closeness between two images is quantified in terms of correlation function. Difference measure and correlation measure complement each other. The large value of NK means that image is of good quality

7.2 NATURE INSPIRED ALGORITHMS

Mimicking the nature for optimization technology:

- Genetic Algorithm [40] [41] adopted genetic recombination of chromosomes and survival of the fittest by selection, crossover, mutation using fitness function. GA with a hybrid select mechanism classified the image blocks into three classes, i.e. smooth, vertical/horizontal edge, and diagonal/subdiagonal edge. According to their discrete cosine transformation (DCT) coefficients, population of every generation is separated into two clans: a superior clan and an inferior clan and appropriate parents are selected to reduce the number of MSE computations by preserving the retrieved image quality.
- Particle Swarm Optimization [42] [43] the analogy of swarm of birds mimics the behavior of individuals, each individual makes decision using own experience along with other individuals' experiences. Every range block of an image, an initial swarm of random values of domain blocks and its isometry are generated. Domain block is evaluated to calculate MSE and fittest solution is attained.
- Ant Colony Optimization [44] [45] employs ants self organizing principles with highly coordinated behavior in search of food, and return to residence in minimum possible time with the help of pheromone. The shortest route was found by the Double Bridge experiment.
- Biogeography Based Optimization [46] observed the behavioral nature of biological species with a habitat for immigration, emigration, and mutation. The distribution of plants and animals over time and space is chosen by habitat suitability index (HSI).
- Firefly Algorithm [47] exercises the flashing behavior of firefly. The flashing light produced by bioluminescence process is correlated with the objective function that to be optimized.
- Cuckoo Search [48] based on brood parasitism an aggressive reproduction strategy for generation surviving.
- Bat Algorithm [49] depending upon microbats echolocation extensively as sensor to detect obstacles and prey.
- Bacterial Foraging Optimization [50] aspired the Escherichia coli bacteria's social foraging behavior with a control system to search for food and avoid to enter into noxious substances in a salutatory search motion.

8 CONCLUSION

In this paper, we performed a survey on FIC compression scheme and its principle, describing main ideas behind each category, and comparing their strength and weakness. Most pure fractal based scheme is not viable with the current scenario, the need for improved performance and wide commercial usage, demand newer and better hybrid schemes to be developed by incorporating fractal compression and speed up techniques.

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