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Model predictive control of 2DOF helicopter

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ABSTRACT: This paper deals with model predictive control (MPC) approach for a 2 DOF (degree of freedom) helicopter. The main objective is to stabilize beam of the 2DOF helicopter with respect to pitch and yaw angles. Development of controller for 2DOF helicopter is challenging because of its coupling effects between two axes and also due to its highly nonlinear characteristics. An accurate model of the system is developed first which has a similar response to that of the real plant. This model is used as a reference to design a non-linear MPC for 2 DOF helicopter. The 2DOF helicopter can also be referred as TRMS (twin rotor MIMO system) because it has two rotor each at its head and tail. Irrespective of variations in reference signals and speed, the controller has a good response in terms of accuracy.

KEYWORDS: MIMO system, 2 DOF, Non-linear MPC, TRMS.

INTRODUCTION

Helicopter has several non-linarites and open loop unstable dynamics as well as significant cross-coupling between their control channels, which makes the control of such multi-input multi-output (MIMO) system a challenging task. Conventional approaches to helicopter flight control involves linearization of these non-linear dynamics about a set of pre-selected equilibrium conditions. Based on obtained linear model, classical single-input single-output (SISO) techniques with a PID controller are widely used [1-2]. Of course, this approach will require multi-loop controllers, which make their design inflexible and difficult to tune.

In the proposed work, the above mentioned difficulty is overcome by developing a of a model predictive control (MPC) approach for an experimental aerodynamic test rig-a twin rotor multi-input-multi-output system (TRMS). The control objective is to make the beam of the TRMS move quickly and accurately to the desired positions, i.e., the pitch and the yaw angles [1-3]. Development of controller for this type of system is challenging due to the coupling effects between two axes and also due to its highly nonlinear characteristics. An accurate dynamic model of the system is developed first in order to have a very similar response to that of the real plant. This model is then used as a test-bed to design a non-linear MPC addressing the 2-DOF of the TRMS [3-4].

As helicopter does not require any runway to take-off, they has a significant role in war, transport and so on. Hence a control design is required to use helicopter effectively. In recent years the computers based optimization plays a major role in control system. MPC, as one such optimization technique is used to control the 2DOF helicopter model. MPC are used because the traditional controller like PID LQR have some limitations.

TWIN ROTOR MIMO SYSTEM

The assembly of helicopter model is shown in Fig.1 [5]. It consists of two motor at front and tail end, which is connected by a beam pivoted on its base. The motor present in the head or front position is responsible for up and down movement whereas the motor present in the tail is responsible for clockwise or anticlockwise rotation of 2DOF helicopter. The power is supplied to motor through slip ring. A high resolution encoder is used for both pitch and yaw measurement [6-8].

There are four state variables that can be are measured. They are the vertical angle, horizontal angle, vertical angular velocity and horizontal angular velocity. The vertical and horizontal angles are physically measured using incremental encoders placed at the pivot. Angular velocities of the beam are software reconstructed by differentiating and filtering measured position angles of the beam [9].

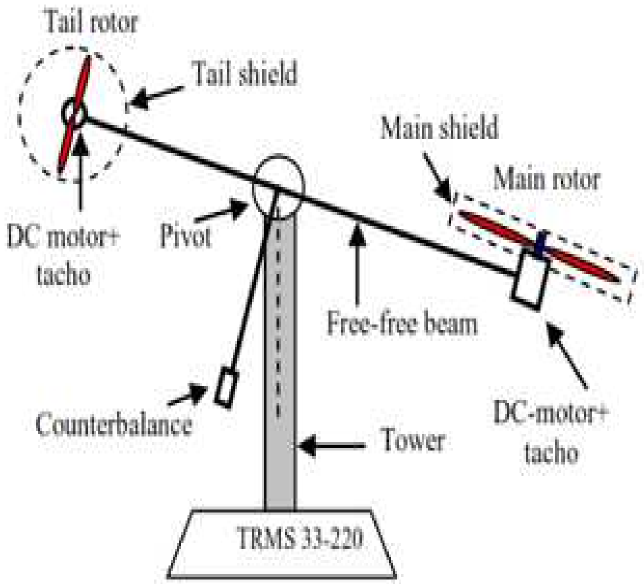

Fig. 1. The twin rotor MIMO system

TABLE 1. Main Differences between Helicopter and TRMS

	TRMS	Helicopter
Location of pivot point	Midway between the two rotor	The main rotor head
Lift generation or vertical control	Speed of control of main rotor	Collective pitch control*
Yaw control	Tail rotor speed	Pitch angle of the tail rotor blades
Cyclical control	No	Yes (directional control)

** At constant rotor speed, the pitch angles of all the blades of the main rotor are*

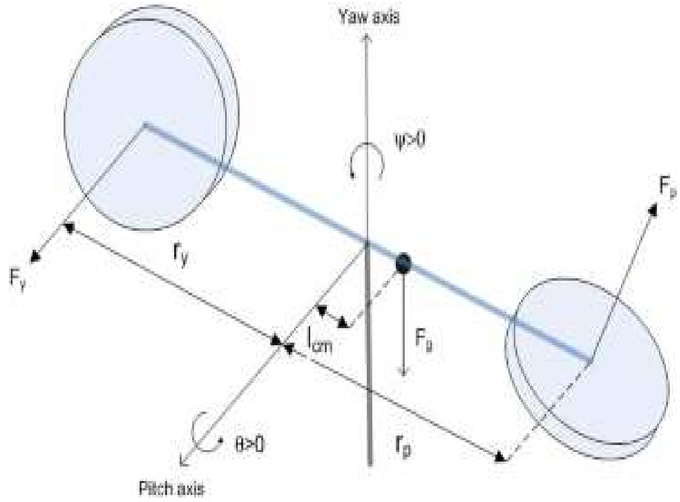

Fig. 2. Dynamics of 2 DOF Helicopter

The Euler Lagrange method is used to derive the nonlinear equations describing the motions of the helicopter. The potential energy due to gravity is

$$V = m_{heli} g l_{cm} \sin\theta \quad (1)$$

The total kinetic energy is

$$T = T_{r,p} + T_{r,y} + T_t \quad (2)$$

From the Eq. (2) is the sum of the rotational kinetic energies acting from the pitch,$T_{r,p}$, and from the yaw ,$T_{r,y}$, along with the translational kinetic energy generated by the center of mass, T_t.

The potential and kinetic energy expressed here are used to derive the equations of motions. Nonlinear equation of motion for the 2 DOF Helicopter, the Euler-Lagrange equations are

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial q_1} - \frac{\partial L}{\partial q_1} = Q_1$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial q_1} - \frac{\partial L}{\partial q_2} = Q_2 \quad (3)$$

Where L is the Lagrange variable and is the difference between the kinetic and potential energy of the system,

L= T−V.

The generalized coordinates are

$$q = [\ q1\ q2\ q3\ q4\ ]^T$$
$$= \left[\ \theta\ \varphi\ \ \dot{\theta}\ \ \dot{\varphi}\ \right]^T \quad (4)$$

And the generalized forces are

$$Q_1 = \tau_P\left(V_{m,p}, V_{m,y}\right) - B_p\dot{\theta}$$

$$Q_1 = \tau_y\left(V_{m,p}, V_{m,y}\right) - B_y\dot{\varphi} \quad (5)$$

Equation (5) includes the viscous rotary friction acting about the pitch and yaw axes, B_p and B_y . The torques applied at the pitch and yaw axes are coupled. The torque applied at the pitch and yaw axis from the motors is

$$\tau_P\left(V_{m,p}, V_{m,y}\right) = K_{pp}V_{m,p} + K_{py}V_{m,y}$$

$$\tau_y\left(V_{m,p}, V_{m,y}\right) = K_{yp}V_{m,p} + K_{yy}V_{m,y} \quad (6)$$

Where $V_{m,p}$ is the input pitch motor voltage and $V_{m,y}$ is the input yaw motor voltage. The torques acting on the pitch and yaw axes are coupled. The torque constants used in the equation (6) are

$$K_{pp} = K_{f,p} r_p$$

$$K_{yy} = K_{f,y} r_y$$

$$K_{py} = \frac{K_{t,y}}{R_{m,y}}$$

$$K_{yp} = \frac{K_{t,p}}{R_{m,p}} \quad (7)$$

Where $K_{f,p}$ and $K_{f,y}$ are the thrust force constants of the pitch and yaw motor/propeller actuators found experimentally, $K_{t,p}$ and $K_{t,y}$ are the current torque constants of the pitch and yaw motors, and $R_{m,p}$, $R_{m,y}$ are the electrical resistances of the pitch and yaw motors.

Thus the main torque generated by the pitch motor on the pitch axis is $\tau_{pp} = K_{pp} V_{m,p}$and, similarly, the main torque acting on the yaw axis is $\tau_{yy} = K_{yy} V_{m,y}$. The torque generated by the yaw motor that acts on the pitch axis is $\tau_{py} = K_{py} V_{m,y}$. Likewise, the pitch motor generates a rotary force about the yaw axis $\tau_{yp} = K_{yp} V_{m,p}$.

Evaluating the Euler–Lagrange expressions in equation (3) using the coordinates defined in (4) and the forces in (5) results in the nonlinear equation of motion.

$$\left(J_{eq,p} + \ m_{heli} l_{cm}^2\right)\ddot{\theta} = K_{pp} V_{m,p} + K_{py} V_{m,y} - m_{heli} g l_{cm} \cos\theta - B_p \dot{\theta} - \ m_{heli} l_{cm}^2 \sin\dot{\theta} \cos\theta \dot{\varphi}^2$$

$$\left(J_{eq,p} + m_{heli} l_{cm}^2 \cos\theta^2\right)\ddot{\varphi} = K_{yy} V_{m,y} + K_{yp} V_{m,p} - B_y \dot{\varphi} + 2 m_{heli} l_{cm}^2 \sin\theta \cos\theta \dot{\varphi} \dot{\theta} \quad (8)$$

The equivalent moment of inertia about the center of mass in equation (8) equals

$$J_{eq,p} = J_{m,p} + J_{body,p} + J_p + J_y$$

$$J_{eq,y} = J_{m,y} + J_{body,y} + J_p + J_y + J_{shaft} \quad (9)$$

Where $J_{m,p}$ and $J_{m,y}$ are the moment of inertias of the motor rotor given in the specifications and

$$J_{body,p} = \frac{m_{body,p} L_{body}^2}{12}$$

$$J_{body,y} = \frac{m_{body,y} L_{body}^2}{12}$$

$$J_{shaft} = \frac{m_{shaft} L_{shaft}^2}{3}$$

$$J_p = \left(m_{m,p} + m_{shield}\right) r_p^2$$

$$J_y = \left(m_{m,y} + m_{shield}\right) r_y^2 \quad (10)$$

The equations of motion can be packaged in the matrix form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \quad (11)$$

With the inertia, damping, gravitational, and applied torque matrices

$$D(q) = \begin{bmatrix} J_{eq,p} + m_{heli} l_{cm}^2 & 0 \\ 0 & J_{eq,y} + m_{heli} l_{cm}^2 \sin\theta \cos\theta^2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} B_p & m_{heli} l_{cm}^2 \sin\theta \cos\theta \dot{\varphi} \\ -2 m_{heli} l_{cm}^2 \sin\theta \cos\theta \dot{\varphi} & B_y \end{bmatrix}$$

$$g(q) = \begin{bmatrix} m_{heli} g l_{cm} \cos\theta \\ 0 \end{bmatrix}$$

$$\tau = \begin{bmatrix} K_{pp} V_{m,p} + K_{py} V_{m,y} \\ K_{yp} V_{m,p} + K_{yy} V_{m,y} \end{bmatrix}$$

NON-LINEAR MODEL PREDICTIVE CONTROL

The non-linear model of the 2DOF helicopter model is obtained from Euler-Lagrange method. The input variables are the voltages of main rotor and tail rotor and the output variables are the pitch angle and yaw angle. The plant has two channels and there is an interaction between these channels. In order to reveal in full the plant behavior the system should be considered as multivariable. The obtained non-liner equation form Eq. 8 are

$$\dot{\theta} = 2.361\theta V_{m,p} + 0.0787\theta V_{m,y} - 29.2985 sin\theta - 9.26\theta - 0.555\dot{\varphi}^2 cos2\theta$$

$$\ddot{\theta} = 2.361 V_{m,p} + 0.0787 V_{m,y} - 29.2985 cos\theta - 9.26\dot{\theta} - 0.555\dot{\varphi}^2 sin\theta cos\theta$$

$$\dot{\varphi} = \frac{1}{0.0432 \ + 0.04733\, cos^2\theta}[0.072\varphi V_{m,y} + 0.0219\varphi V_{m,p} - 0.318\varphi^2 + 0.0958 sin\theta cos\theta \varphi\dot{\theta}]$$

$$\ddot{\varphi} = \frac{1}{0.0432 \ + 0.04733\, cos^2\theta}[0.072 V_{m,y} + 0.0219 V_{m,p} - 0.318\varphi + 0.0958 sin\theta cos\theta \dot{\varphi}\dot{\theta}$$

TABLE 2. Parameters used in non-linear equation

Symbol	***Description***
$\boldsymbol{\theta}, \boldsymbol{\varphi}$	*Pitch and Yaw angle*
$\dot{\boldsymbol{\theta}}, \dot{\boldsymbol{\varphi}}$	*Vertical and horizontal angular velocity*
$\boldsymbol{J_{eq,p}}$,$\boldsymbol{J_{eq,y}}$	*Equivalent moment of inertia of pitch and yaw*
$\boldsymbol{B_{eq,p}}$, $\boldsymbol{B_{eq,y}}$	*Equivalent viscous damping of pitch and yaw*
$\boldsymbol{K_{pp}}$, $\boldsymbol{K_{py}}$	*Thrust torque constant acting on pitch/yaw axis from yaw/pitch*
$\boldsymbol{V_{m,p}}$,$\boldsymbol{V_{m,y}}$	*Control voltages of pitch and yaw motor*
$\boldsymbol{m_{heli}}$	*Total moving mass of helicopter*
$\boldsymbol{l_{cm}}$	*Length oh helicopter*

MODEL PREDICTIVE CONTROL

MPC is an optimal control strategy based on numerical optimization. Further control inputs and further plant response are predicted using a system model and optimized at regular intervals with respect to a performance index. Despite being very simple to design and implement, MPC algorithms can control large scale systems with many control variables and most importantly MPC provides a systematic method of dealing with constraints on inputs and states. In MPC these constraints are accounted for explicitly by solving a constrained optimization problem in real time to determine the optimal predicted inputs. Nonlinear plant dynamics cab be similarly incorporated in the prediction model [4].

The MPC approach can be explained in the following steps:

Step 1: The system output is predicted over a prediction horizon N_p

Step 2: A set of future control signals, $\bar{u}$ are choosen, over a control horizonN_c, which minimizes the future errors between the predicted output and the desired output.

Step 3: Use the first element of $\bar{u}$ as current input, repeat all the steps at the next time-sample

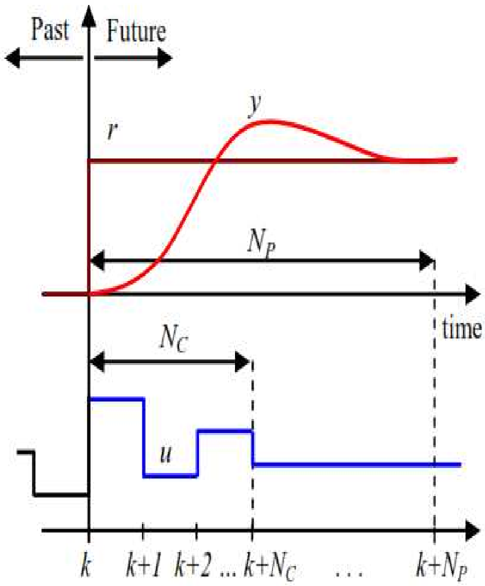

Fig. 3. The model predictive approach

The MPC calculation is based on current measurenemts and predictions of the future values of the outputs. The objective of the MPC control calculations is to determine a sequence of control moves(that is, manipulated input changes) so that the predicted response move to the set point in an optimal manner. The general block diagram of MPC is shown in the Fig. 4.

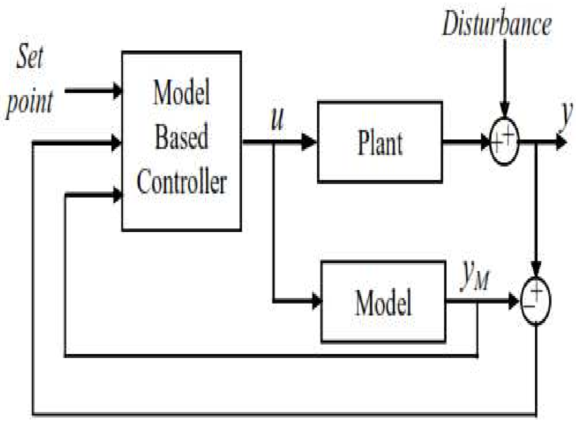

Fig. 4. Basic block diagram of MPC

SIMULATION RESULTS

An accurate mathematical model of the TRMS has been formulated. Based on this mathematical model a reference model of MPC is developed. MPC is used to achieve robust performance and stability in the presence of bounded modelling errors. Even though the uncertainties, disturbance and measured noises exist, the MPC does the job of i) To maintain system stability ii) To reduce the effect of disturbance and noise in the real control system. The control scheme of 2-DOF helicopter model is implemented using MPC controller is shown in the Fig. 5. The plant model is imported to the MPC controller in MPC tool box. The control interval of MPC is 0.1. The prediction and control horizon of MPC is respectively 500 and 50. Input and output constraint of the helicopter model is also updated to MPC controller.

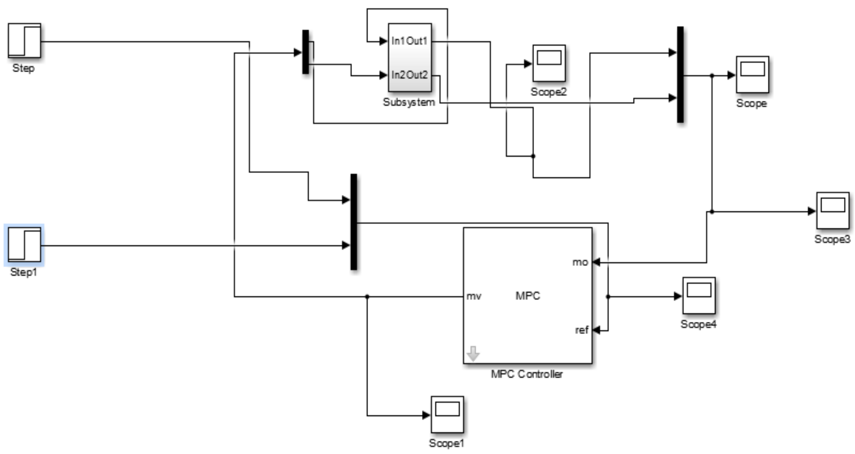

Fig. 5. Simulation model

From the non-linear equation of 2DOF helicopter, the non-linear model is implemented in MATLAB simulation model. The non-linear plant model is shown in the Fig. 6 MPC tool box is shown in the Fig.7. AT first the plant model is loaded in the MPC toolbox and a suitable controller is designed by varying the weight of the MPC Controller till the desired output is obtained.

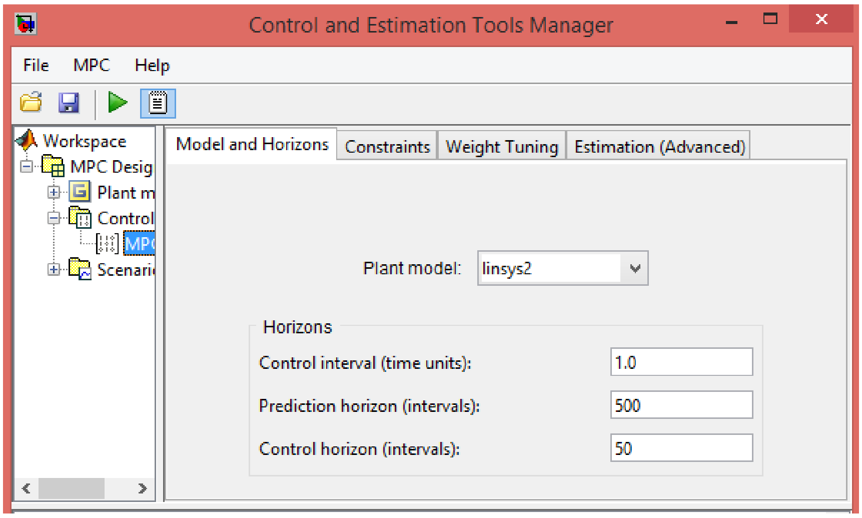

Fig. 7. MPC toolbox

The performance of PID and MPC controller is compared in the presence of disturbance, Fig. 8 represent the waveform of pitch angle with step and sine as disturbance. A step disturbance along with sine disturbance is introduced into the plant model at 40Sec. and 70 Sec. The response of PID has initial peak overshoot, whereas there is no such peak overshoot in the response of MPC but both the controller have same settling time of 20 Sec. At the time of step disturbance, there is high oscillation in the response of PID compared to the response of MPC controller. Both the response settle within 10 Sec. after the introduction of step disturbance. At 70 Sec. a sine disturbance is introduced into the plant. Due to this disturbance there is a peak overshoot in the response of PID controller but MPC has a less overshoot compared to PID. The settling time is almost same for both the response after the introduction of sine disturbance

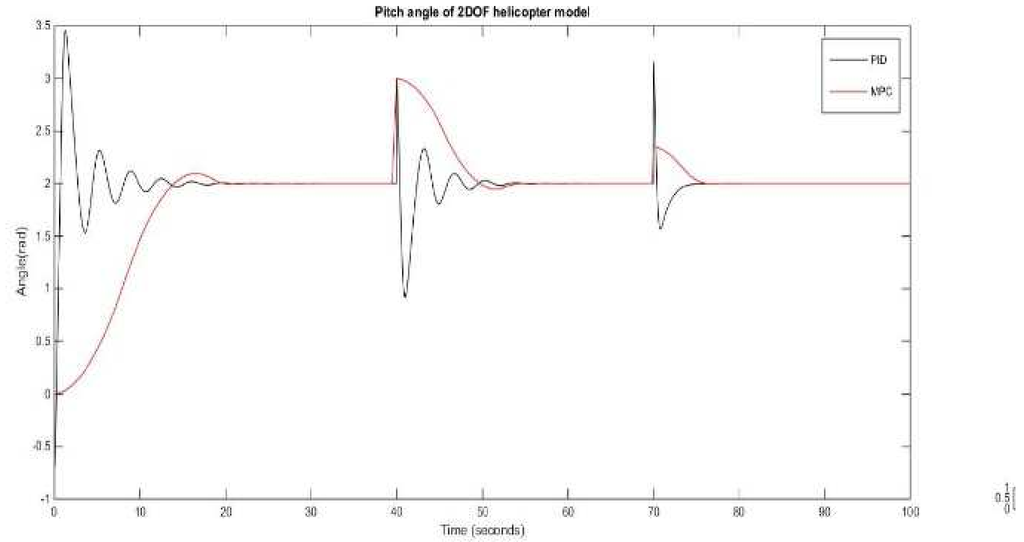

Fig. 8. Output response of pitch anglewith step, sine and Sine as disturbance

The output waveform of yaw angle with step and sine as disturbance is shown in the Fig. 9. There are high initial oscillation in the response of PID controller compared to the response of MPC. The settling time of the response of MPC is 20 Sec. whereas the response of PID take 18 Sec. to settle. A step disturbance in applied to the plant at 40 Sec. due to this disturbance there is oscillation in the response of both PID and MPC controller. After 10 Sec. of introduction of the disturbance the response of MPC and PID settles to the desired angle. At the instant of sine disturbance there is a high peak overshoot in the response of PID controller compared to the response of MPC. The response of MPC settles within 8 Sec. after the introduction of disturbance, whereas the response of PID takes almost 12 Sec. to settle to its desired angle. The experiment setup is shown in the Fig. 10. .

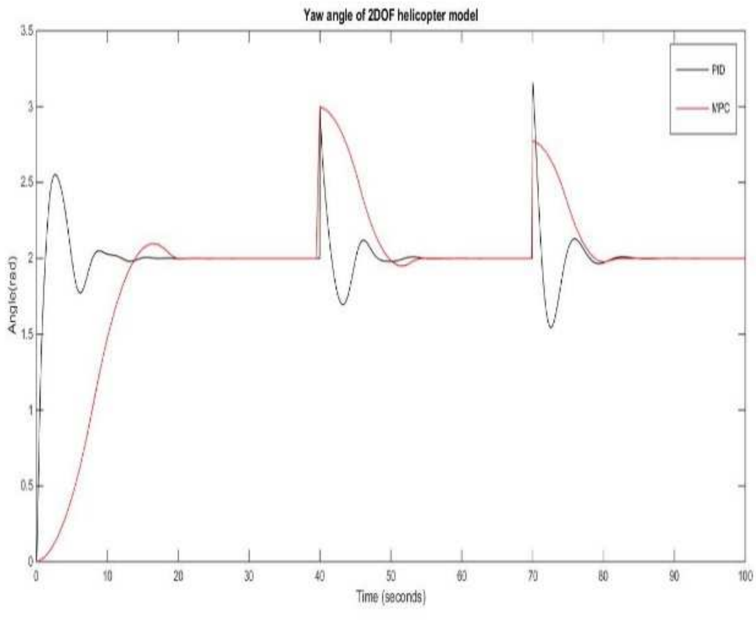

Fig. 9. Output response of yaw angle with step and sine noise as disturbance

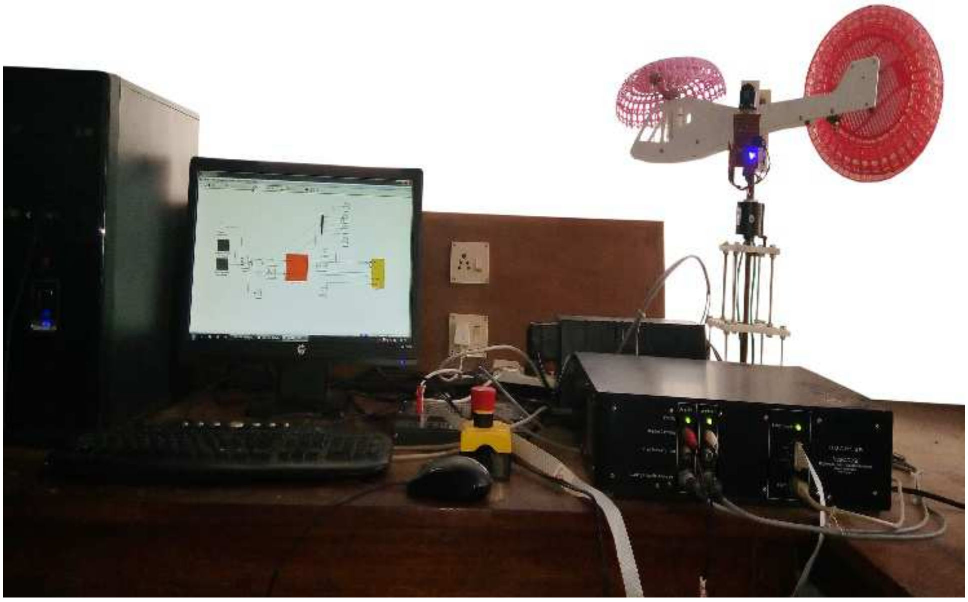

Fig. 10. Experimental setup

TABLE 3. Comparing the Performance of PID and MPC With Respect To Pitch Angle

Disturbance	Controller	Settling Time (Sec)	% overshoot
Step	PID	15	60
	MPC	15	50
Sine	PID	8	65
	MPC	8	20

TABLE 4. Comparing the Performance of PID and MPC With Respect To Yaw Angle

Disturbance	*Controller*	*Settleing Time (Sec)*	*% overshoot*
Step	*PID*	*15*	*50*
	MPC	*15*	*50*
Sine	*PID*	*12*	*60*
	MPC	*12*	*25*

CONCLUSION

As the helicopter have a high non-linearity and strong coupling effect between two axes it is difficult to design a controller. The non-linear equations of 2DOF helicopter is derived using Newton's law of motion. There is a high initial oscillation in the response of PID compared to the response of MPC controller (the percentage peak overshoot in the response of PID is about 160% of its initial value, whereas MPC has a peak overshoot of 105% of its initial value) and there is also a high oscillation in the response of PID compared the response of MPC at the time of disturbance. Moreover MPC provide a smooth control compared to PID controller. Hence MPC controller provide a better control compared to PID controller.

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