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Trig-Disc: An innovative device for determination of values of trigonometric functions with single measurement

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ABSTRACT: Angles and lengths of a triangle engaged the mathematicians from historical time. The branch of mathematics that resulted from these studies is now known as Trigonometry. The discovery that lengths of a right angled triangle and the angles between them have a definite relationship led to the invention of trigonometric functions. The determination of values of various trigonometric functions like sine, cosine, tangent, cotangent, cosecant and secant for various angles in four quarters of a circle was a challenging task. Various methods and tables were generated for determining these values. Efforts were made to invent various devices for these measurements. There exist various U.S. patents [1-7] based on devices developed to visualize, teach and calculate values of various trigonometric functions. In the present paper, we are reporting a very accurate, economic, simple, and portable device designed for measuring the values of Trigonometric functions. The patent application number for the reported device is 1068/DEL/2015. The device consists of (i) a unit radius circular disc (with 360^{o} angle scale indication on its circumference of 1^{o} accuracy, X- and Y-linear scales markings with accuracy of 0.01 unit and marking of four quadrants of the circle), (ii) a corresponding linear scale with positive marking, (iii) a corresponding linear scale with negative marking and (iv) a blank transparent strip. The device can directly measure all the six trigonometric identities (sine, cosine, tangent, cotangent, cosecant and secant) for any value of angle up to the accuracy of 0.01 units just by measurement of only one parameter on one of the provided scale by appropriately placing it on Trig Disc along with blank linear strip in respective quadrants. Single measurement on a linear scale and no division or calculations of values is main feature of the device.

KEYWORDS: Trigonometry, trigonometric functions, trigonometric function values, Trigonometric devices, trigonometric identities

1 INTRODUCTION

Angles and lengths of a triangle engaged the mathematicians from historical time. The branch of mathematics that resulted from these studies is now known as Trigonometry. The discovery that lengths of a right angled triangle and the angles between them have a definite relationship led to the invention of trigonometric functions. The concept of unit circle (a circle of 1 unit radius) is commonly used to determine values of all trigonometric functions. A unit circle is shown below in Figure 1:

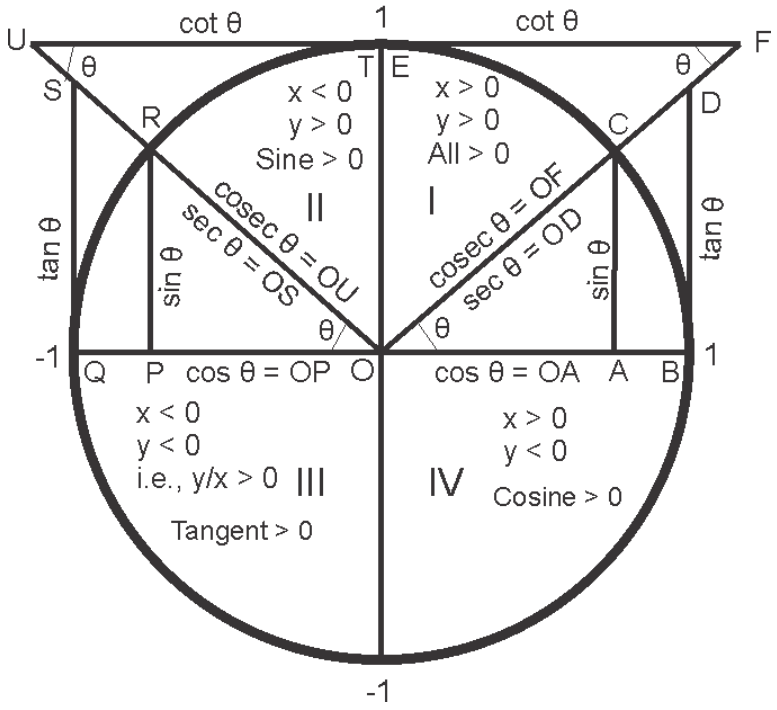

Figure 1: A Unit Circle concept to express all trigonometric functions.

Here, the trigonometric functions sine and cosine are defined in terms of the coordinates of points lying on the unit circle $x^2 + y^2=1$.

- Cosine of the angle θ is defined to be the horizontal coordinate x.
- Sine of the angle θ is defined to be the vertical coordinate y.
- The values of all six trigonometric functions can be represented as the lengths of various sides of triangles associated with the unit circle definition of cosine and sine.
- From Layout (in I Quadrant), see that OB, OE and OC all have length 1.
- ∠ COA = ∠ DOB = ∠ OFE = θ, and ∠ OAC = ∠ OBD = ∠ FEO (all three are right angles).
- By definition the point C has coordinates (cos θ, sin θ). This means cos θ = OA and sin θ = AC.
- Triangles OAC and OBD are similar. (since, ∠COA = ∠DOB = θ, and ∠OAC = ∠OBD). Hence, the ratios of corresponding sides are therefore equal.
- AC/OA = BD/OB, that is, sin θ/cos θ = BD/1. This means that tan θ = BD.
- Also, OC/OA = OD/OB, that is, 1/cos θ = OD/1. This means that sec θ = OD.
- Triangles OBD and FEO are similar. Therefore OB/BD = EF/OE, that is, 1/tanθ = EF/1. This means that cot θ = EF.
- Finally, using the similar triangles OAC and FEO, we get OC/AC = OF/OE, that is, 1/sin θ = OF/1, and so this means that cosec θ = OF.
- Hence, we get
  - Cos θ = OA Sin θ = AC Tan θ = BD
  - Sec θ = OD Cot θ = EF Cosec θ = OF

2 EXISTING DEVICES FOR MEASURING VALUES OF TRIGONOMETRIC FUNCTIONS

Since values of trigonometric functions are used in wide variety of applications in navigation, engineering, and physics, it is necessary to have some simple devices for determining these values. For use on demand basis, trigonometric tables were developed long ago in seventeenth century (such as Bernegger tables) to have easy access to the values for trigonometric functions. In modern days, calculators and computers are easiest ways to know these values. However, this has not hindered people to innovate and design tools for calculating values for trigonometric functions. To meet this requirement various visual teaching aids for measuring trigonometry identities have come into being.

U.S. patent no. 4435162, for example, discloses a trigometric visualizer which comprises of a pair of circular discs which are connected at their centers for their relative coaxial rotation on one another. The upper disc comprises of a larger circle

while lower disc has a circle, which is one half of that of upper circle. It uses two circles and measures two functions i.e. sine and cosine only as compared to the present invention which uses only one unit circle to measure all six identities [1].

U.S. patent no. 4655714, discloses a visual aid for teaching trigonometric functions, which comprises of a device wherein two planar membranes are pivotally collected. The relative rotational movement of said two members aids the measurement of angles. One said member features rectilinear scales for secants and cosecants, which are mutually perpendicular, calibrated and collinear with the pivot point. While in present invention all the six identities are measured using single unit circle [2].

U.S. patent no. 20140215840, discloses a device which has a tangent calculator that enables the user to read positive and negative tangent values in the configuration described. A sine/cosine calculator has enabled the user to do positive and negative sine and cosine readings, as well as per unit ratios of triangle sides. An angle tracking device tracks angles and can do tangent readings. It is mentioned in the patent that since the tangent of 90^{o} equals to plus infinity, and the tangent of -90^{o} equals to minus infinity, the Tangent Calculator was restricted to readings between −1.7 and +1.7, corresponding to angles between -60^{o} and $+60^{o}$ However, in comparison, using the present invention values of tangent up to 75^{o} (in first quadrant) can easily be measured and for values for angles more than 75^{o}, combination of more than one linear scale can be used [3].

U.S. patent 3556397, discloses a trigonometric calculator with a scale member having circular degree scale and a cursor member mounted to pivot on it at the center of the degree scale. Coordinate axes at the center of the scale lengthen one unit in each positive and negative direction and are calibrated from zero to one. The sine and cosine functions may be directly read from the coordinate axes for any angle indicated by the cursor member on the degree scale. In comparison the present invention provides a simpler method of directly measuring the values of all the six trigonometric functions (sin, cos, tan, cot, sec, cosec) based on measurement of only one length on the provided scale along with blank linear strip. [4]

There are many other devices developed for determining trigonometric functions, which were patented [5-7].

3 PRESENT INVENTION

3.1 TRIG DISC: A DEVICE FOR DETERMINATION OF VALUES OF TRIGONOMETRIC FUNCTIONS WITH SINGLE MEASUREMENT

In most of the above-mentioned devices [1-7], there are methods, which are slightly complicated or need some calculations for determining the values of trigonometric functions. Also, the compilation of all the values of trigonometric angles from 0^{o} to 360^{o} forms a very bulky booklet to carry and to use. Hence it is felt that there is a need for an invention, which obviates the drawbacks mentioned above. This motivated us to do the reported work.

3.2 DEVICE DESIGNING

Based on the concept of unit circle we have designed a small handy innovative plastic device consisting of a plastic unit radius circular disc, (with 360^{o} angle scale indication on its circumference of 1^{o} accuracy, X- and Y- linear scales markings with accuracy of 0.01 unit, marking of four quadrants of the circle and markings of Sin θ and Cos θ), a linear scales of positive markings with accuracy of 0.01 unit, a linear scale of negative marking with accuracy of 0.01 unit and a blank linear transparent strip without any markings. The device is shown in the Figure 2.

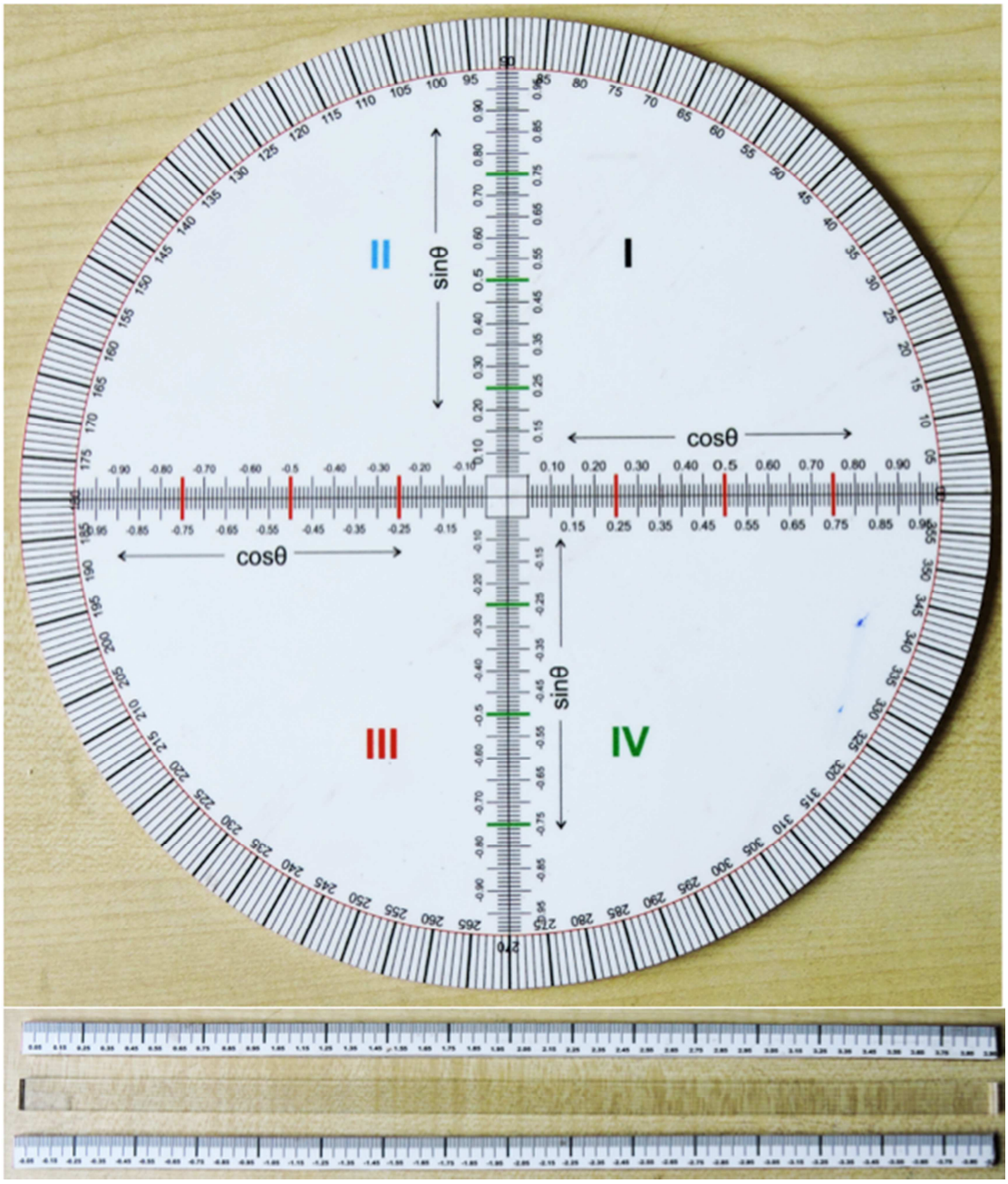

Figure 2: Trig Disc consisting of (i) a plastic unit radius circular disc, (ii) a linear scales with positive markings (iii) a linear scale with negative markings and (iv) a transparent strip.

The device can be used to easily find (directly measure) the values of all the six trigonometric functions (sine, cosine, tangent, cot, secant, cosec) just by measurement of only one length on one of the provided scales (positive or negative) by appropriately placing it on Trig Disc along with blank linear transparent strip in respective quadrants (first, second, third and fourth quadrants) for any angle. The device and scales are designed in Corel draw. Using this instrument, the value of trigonometric functions can be determined up to an accuracy of 0.01 units.

3.3 MEASUREMENTS OF TRIGONOMETRIC VALUES USING TRIG DISC

(i) Sine: To find the value of sine of an angle in first quadrant place blank linear strip parallel to x-axis at the angle on the circumference of Trig Disc whose value need to be measured. Look at the intersection of blank linear strip on y-axis. This gives the value of Sine of the angle as shown in Figure 3. For example, Sin 30^{o} = 0.5. The value of all sine angles from 0 to 90^{o} in first quadrant thus can be measured.

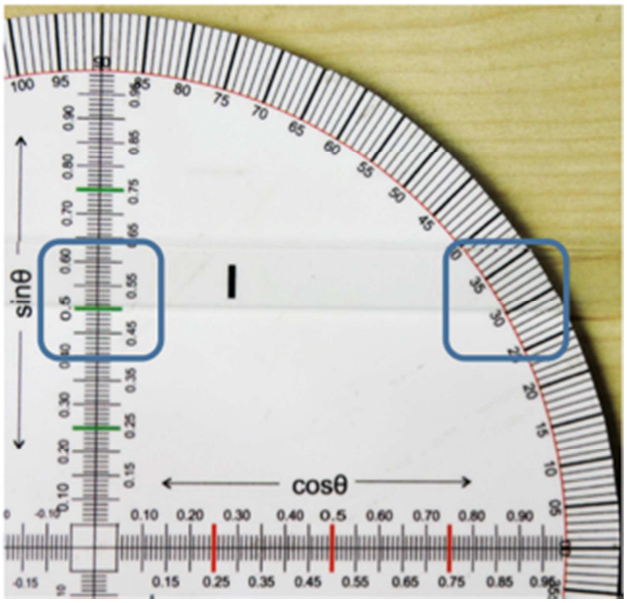

Figure 3: Measurement of Sine values

(ii) Cosine: To find the value of cosine of an angle in first quadrant place blank linear strip parallel to y-axis at the angle on the circumference of Trig Disc whose value need to be measured. Look at the intersection of blank linear strip on x-axis. This gives the value of Cosine of the angle as shown in Figure 4. e.g. Cos 75^{o} = 0.26. The value of all cosine angles from 0 to 90^{o} in first quadrant can thus be measured.

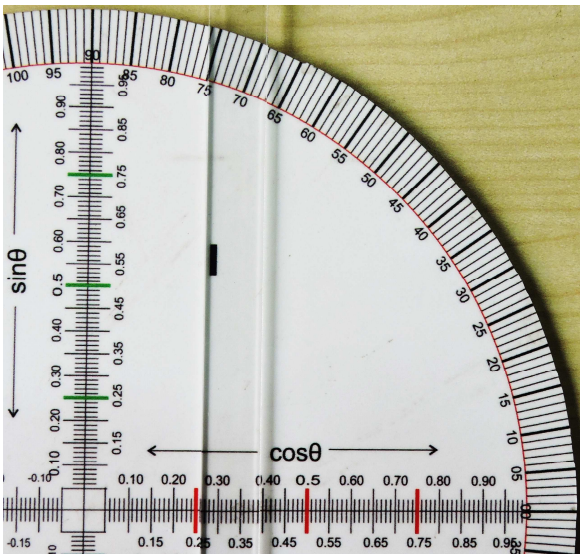

Figure 4: Measurement of Cosine values

(iii) Tangent: To find the value of tangent of an angle in first quadrant place positive marking linear scale perpendicular to x-axis at its unit value and blank linear strip along the line from center and the angle on the circumference of Trig Disc whose value need to be measured. Look at the intersection of positive marking linear scale and blank linear strip. This gives the value of tangent of the angle as shown in Figure 5. e.g. Tan 30^{o} = 0.58 It shall be noted that for measuring values of tangent of angles more than 75^{o}, larger scales shall be used.

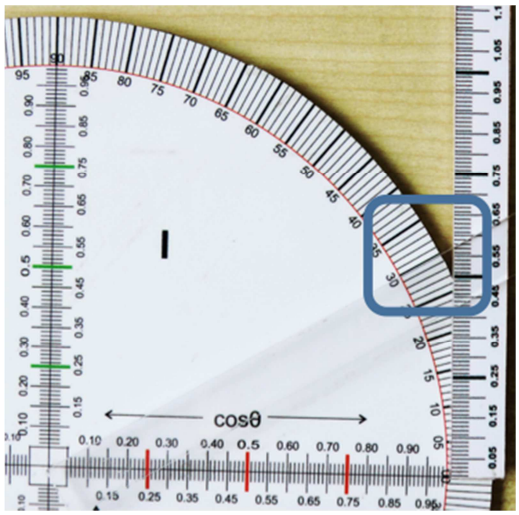

Figure 5: Measurement of Tangent values

(iv) Secant: To find the value of secant of an angle in first quadrant place blank linear strip perpendicular to x-axis at its unit value and positive marking linear scale along the line from center and the angle on the circumference of Trig Disc whose value need to be measured. Look at the intersection of positive marking linear scale and blank linear strip. This gives the value of secant of the angle as shown in Figure 6. e.g. $Sec30^{o}$=1.15. It shall be noted that for measuring values of secant of angles more than 75^{o}, larger scales shall be used.

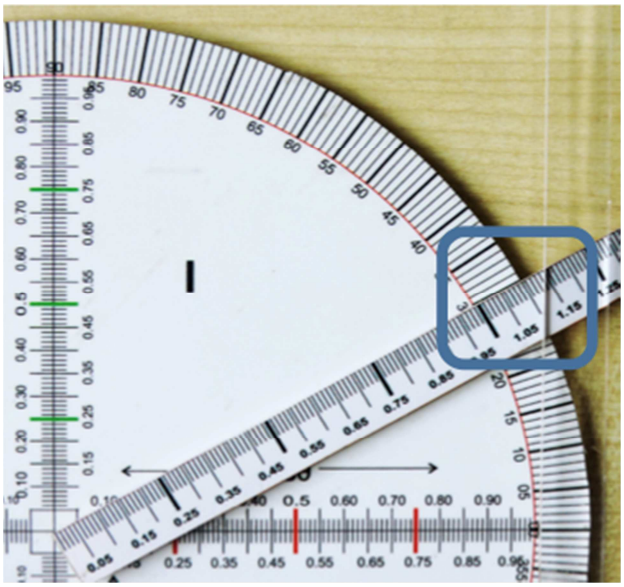

Figure 6: Measurement of Secant values

(v) Cotangent: To find the value of cot of an angle in first quadrant place blank linear strip along the line from center and the angle on the circumference of Trig Disc whose value need to be measured and positive marking linear scale perpendicular to y-axis at its unit value. Look at the intersection of positive marking linear scale and blank linear strip. This gives the value of cotangent of the angle as shown in Figure 7. e.g. $Cot30^{o}$=1.73. It shall be noted that for measuring values of cotangent of angles less than 15^{o}, larger scales shall be used.

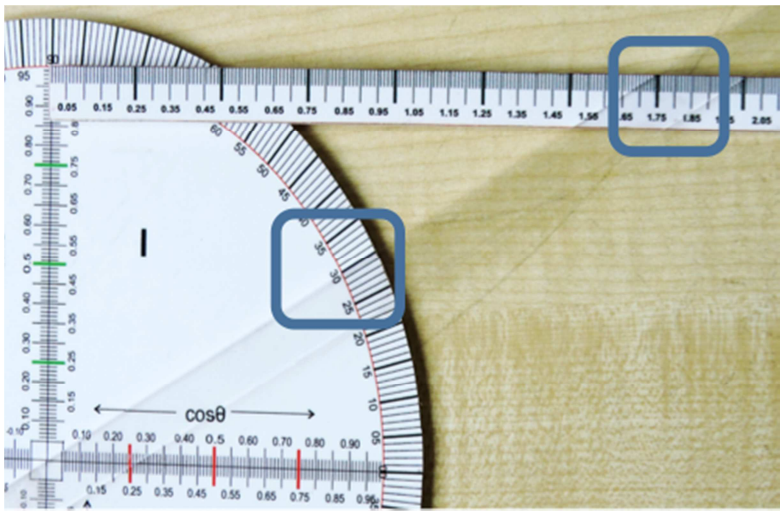

Figure 7: Measurement of Cotangent values

(vi) Cosec: To find the value of cosec of an angle in first quadrant place positive marking linear scale along the line from center and the angle on the circumference of Trig Disc whose value need to be measured and blank linear strip perpendicular to y-axis at its unit value. Look at the intersection of positive marking linear scale and blank linear strip. This gives the value of cotangent of the angle as shown in Figure 8. e.g. Cosec30^{o}=2.00. It shall be noted that for measuring values of cotangent of angles less than 15^{o}, larger scales shall be used.

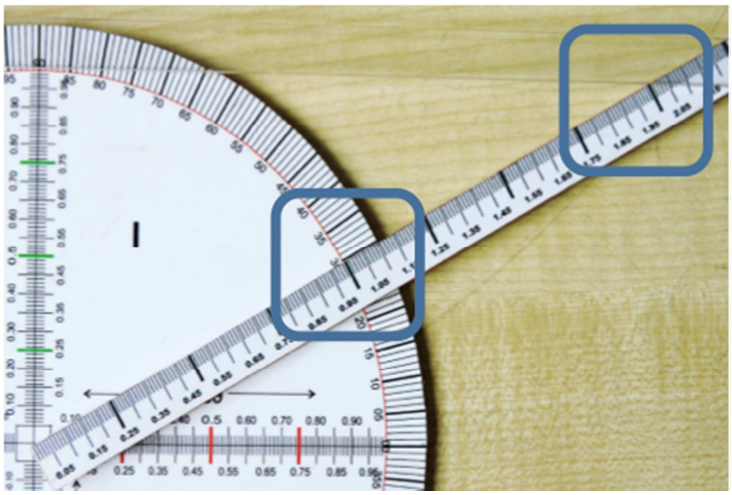

Figure 8: Measurement of Cosecant values

Similarly, by appropriately placing one of the scale and blank strip the values of all six trigonometric functions, namely, sine, cosine, tangent, cotangent, cosecant and secant can be determined in other quadrants of the circle. The appropriate method of placing the correct scale and blank strip in any of the four quarter is described in the Figure 9.

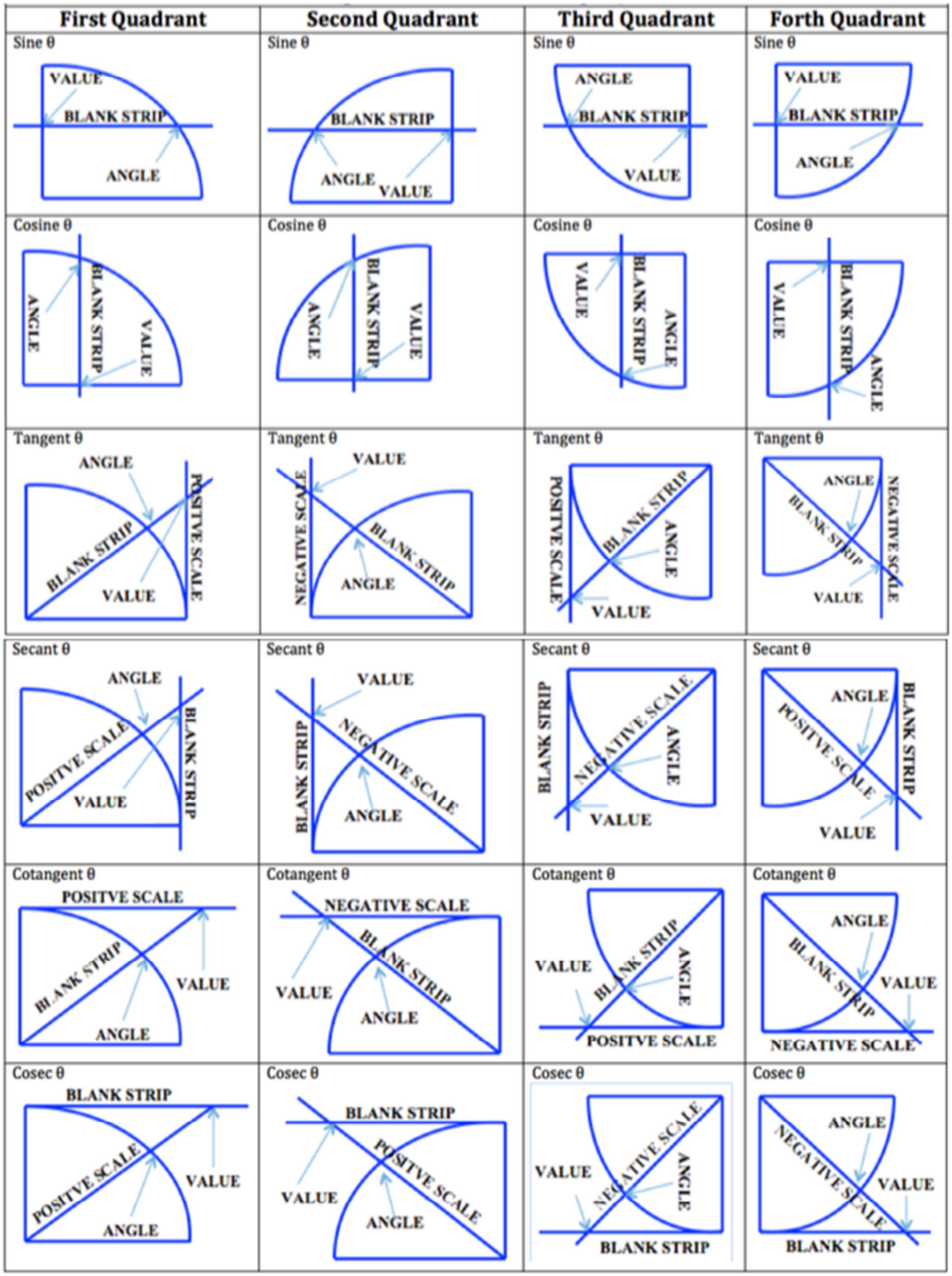

Figure 9: Method of placing type of scale and blank strip on Trig Disc for measuring values of various trigonometric functions in different quadrants of the circle.

3.4 PATENT APPLICATION NUMBER

For the present invention the patent application has been filled in India with application number 1068/DEL/2015 on 27 May 2016.

4 CONCLUSION

The present invention of Trig Disc provides a simple, accurate, easy and economical method of determining values of six trigonometric functions sine, cosine, tangent, cotangent, cosecant and secant in form of a handy portable device. The device consists of (i) a unit radius circular disc (with 360^{o} angle scale indication on its circumference of 1^{o} accuracy, X- and Y-linear scales markings with accuracy of 0.01 unit and marking of four quadrants of the circle), (ii) a corresponding linear scale with positive marking, (iii) a corresponding linear scale with negative marking and (iv) a blank transparent strip. With the device, the values of trigonometric functions can be determined up to an accuracy of 0.01 units with only one direct measurement on one of the provided scale by appropriately placing it on Trig Disc along with blank linear strip in respective quadrants and this too without need of any calculations. The patent application number for the reported device is 1068/DEL/2015.

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