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Performance of Multiuser MIMO-OFDM downlink system with ZF-BF and MMSE-BF linear precoding

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ABSTRACT: The forthcoming wireless communication networks, commonly referred to as fourth generation (4G) systems, are expected to support extremely high data rates as close as possible to the theoretical channel capacity while satisfying quality of service (QoS) constraints. The development of these systems must take into account the problem of limited radio resources and the harshness of wireless channel conditions. Two emerging technologies that are potential candidates for 4G wireless networks are multiuser multiple-input multiple-output (MU-MIMO) wireless systems and orthogonal frequency division multiplexing (OFDM). The MU-MIMO technique allows the spatial multiplexing gain at the base station to be obtained without the need for multiple antenna terminals, thereby allowing multiple users to receive data over the downlink simultaneously. The use of OFDM provides protection against intersymbol interference (ISI) and allows high data rates to be achieved. Linear precoding schemes for MU-MIMO wireless systems, e.g., zero forcing beamforming (ZF-BF) and minimum mean squared error beamforming (MMSE-BF), have been widely concerned for their high performance in single-carrier MU-MIMO networks where a base station attempts to communicate simultaneously with multiple users. In this paper, we evaluate and extend the ZF-BF and MMSE-BF schemes from single-carrier MU-MIMO to multicarrier MU-MIMO architecture based on OFDM, i.e., MU-MIMO-OFDM system, assuming the availability of channel state information (CSI) at the transmitter. Numerical results demonstrate that both introduced linear precoding strategies provide a higher sum-rate capacity improvement compared to a conventional MU-MIMO-OFDM system where the users are served on a time division multiple access (TDMA) basis.

KEYWORDS: Multiuser MIMO, OFDM, zero forcing, minimum mean squared error, beamforming, precoding.

1 INTRODUCTION

The main challenge of the forthcoming wireless communication systems, commonly referred to as fourth generation (4G) systems, is to satisfy the increasing demand for high data rates while satisfying quality of service (QoS) constraints. In particular, the use of multiple-input multiple-output (MIMO) wireless systems can improve the capacity by a factor dependent on the minimum number of transmit and receive antennas, if perfect channel state information (CSI) is available at the base station [1]. On other hand, orthogonal frequency division multiplexing (OFDM) is a popular method for high data rate wireless transmission. It is an effective technique to mitigate the effects of intersymbol interference (ISI) in frequency-selective channels by converting a frequency-selective channel into a parallel collection of frequency flat subchannels [2]. Combining MIMO antenna configurations with OFDM results in a powerful architecture, MIMO-OFDM, that is able to exploit spatial as well as frequency diversity and allow high data rates with large degrees of freedom available in the wireless environment [3].

Recently, multiuser MIMO (MU-MIMO) has been considered a key technology for system capacity improvement in modern wireless networks without the need for multiple antennas and expensive signal processing at user equipments. In contrast to single-user MIMO (SU-MIMO), where a base station can only communicates with a single user, MU-MIMO allows

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multiple mobile stations to be served simultaneously by means of space division multiple access (SDMA) [4]. Information theory reveals that if CSI is fully known at the transmitter, dirty paper coding (DPC) is the optimal transmit strategy for the MU-MIMO broadcast channel from a system capacity point of view [5]. However, deploying this technique in practice is hard to implement because of the high computational complexity it requires, especially when the number of users is large. Therefore, suboptimal linear precoding strategies such as zeroforcing beamforming (ZF-BF) [6], [7] and minimum mean square error beamforming (MMSE-BF) [8] have been investigated to provide the capacity gain promised by DPC while removing the multiuser interference among the simultaneously transmitted users.

In this paper, ZF-BF and MMSE-BF linear precoding schemes are evaluated and extended, in light of the available CSI at the transmitter, from single-carrier to multicarrier MU-MIMO systems where the users transmit strictly using OFDM (i.e., frequency is not used for multiple access). Simulations results have show that compared to a conventional MU-MMO-OFDM based time division multiple access (TDMA) strategy, where the BS transmits to the best user at each time slot, considerable sum-rate capacity improvement can be achieved by both proposed linear precoding techniques. The remainder of this paper is organized as follows. Section 2 presents the MU-MIMO-OFDM system model. Section 3 presents the proposed precoding schemes. Numerical results are shown in Section 4. Finally, Section 5 summarizes the main outcomes of the paper.

NOTATIONAL REMARK

Boldface letters denote matrix-vector quantities while non-bold letters are used for scalars. The operation $(.)^T$ and $(.)^H$ represent the transpose and the Hermitian transpose of a matrix, respectively. E(.) denotes the expectation operator, *Tr*(.) is the trace and C is the set of complex numbers. $\mathbf{I}$ is the identity matrix and $\|a\|$ denotes the Euclidean norm of a vector ***a***.

2 SYSTEM MODEL

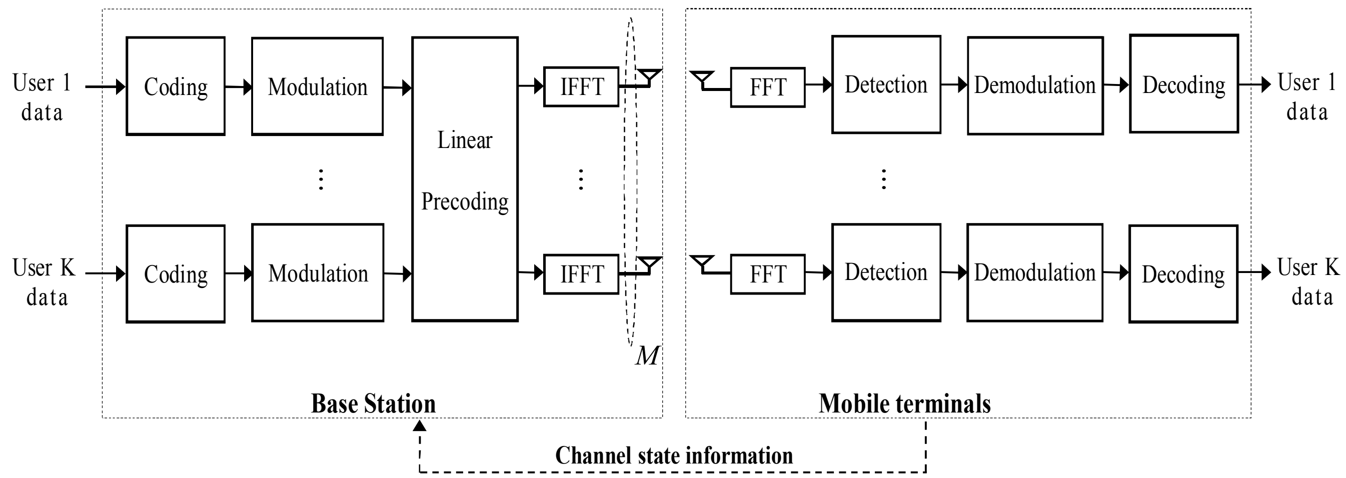

Fig. 1. Block diagram of MU-MIMO-OFDM downlink system with N_T transmit antennas and K single antenna mobile users

We consider the downlink of a single-cell MU-MIMO-OFDM system in which a single base station (BS) equipped with M transmit antennas communicates with $K \le M$ mobile users, each equipped with a single receive antenna. The system operates in a total bandwidth W that is exploited by means of N_c OFDM subcarriers. The BS broadcasts to all K users simultaneously over all OFDM subcarriers. The system block diagram is depicted in Fig. 1. Let $s_{k,n}$ be the data symbol of user k over the nth subcarrier. The $M\times 1$ overall data vector of transmitted symbols from the BS antennas on subcarrier n for all K users is

$$\mathbf{x}_n = \left[s_{1,n} \; s_{2,n} \cdots s_{K,n} \right]^T . \tag{1}$$

Assuming perfect frequency synchronization between the transmitter and receiver and cyclic prefix duration exceeding the channel delay spread, the received signal, $y_{k,n}$, for user k on subcarrier n for an arbitrary OFDM symbol is given by

$$y_{k,n} = \mathbf{h}_{k,n}\mathbf{x}_n + \eta_{k,n}, \tag{2}$$

where $\mathbf{h}_{k,n} \in \mathbb{C}^{1xM}$ represents the channel gain response corresponding to user k over the subcarrier n and $\eta_{k,n}$ is a zero-mean additive white Gaussian noise (AWGN) sample with variance σ_η^2. The base station (BS) has full and instantaneous knowledge of the channel state information (CSI) of all K users. The transmitter is subject to an average power constraint P_T, i.e., $Tr(\mathrm{E}(\mathbf{x}_n\mathbf{x}_n^H)) \le P_T$, which implies that the total transmit power is not dependent on the number of transmit antennas.

The transmitter multiplies the data symbol, $s_{k,n}$, for each user k on each subcarrier n by a precoding vector $\mathbf{w}_{k,n} \in \mathbb{C}^{Mx1}$ so that the transmitted signal on each subcarrier n is a linear function hat can be written as

$$\mathbf{x}_n = \sum_{k=1}^{K} \sqrt{P_{k,n}}\mathbf{w}_{k,n}s_{k,n}, \tag{3}$$

where $P_{k,n}$ denotes the power allocated to the k th user on the n th subcarrier satisfying,

$$\sum_{n=1}^{N_c}\sum_{k=1}^{K} \left\|\mathbf{w}_{k,n}\right\|^2 P_{k,n} = P_T, \tag{4}$$

and thus, the resulting received signal for user k on subcarrier n may be rewritten as

$$y_{k,n} = \sum_{j=1}^{K}\mathbf{h}_{k,n}\mathbf{w}_{j,n}s_{j,n} + \eta_{j,n} \tag{5}$$

$$= \sqrt{P_{k,n}}\mathbf{h}_{k,n}\mathbf{w}_{k,n}s_{k,n} + \sum_{j=1,\, j\neq k}^{K}\sqrt{P_{j,n}}\mathbf{h}_{k,n}\mathbf{w}_{j,n}s_{j,n} + \eta_{k,n}, \tag{6}$$

where the second-term in (6) corresponds to the multi-user interference that represents the major impairment in this scenario. The challenge now is how to perform the precoding operation in order to eliminate all multiuser interference.

3 PRECODING TECHNIQUES FOR MU-MIMO-OFDM SYSTEMS

In MU-MIMO-OFDM downlink system, a BS communicates simultaneously with multiple receivers using the SDMA technique. To achieve this goal precoding strategies should be designed, in light of the available CSI, in order to increase system capacity and/or reduce the complexity of the receiver. In this section we present and evaluate linear precoding schemes using either ZF-BF and MMSE-BF. For comparison we also present a conventional MU-MIMO-OFDM based TDMA system where the base station transmits only to best user at a given time slot.

3.1 ZERO-FORCING BEAMFORMING

Let us define the $K \times M$ channel gain matrix and $M \times K$ precoding matrix, on subcarrier n, for all K users, respectively, as

$$\mathbf{H}_n = \left[\mathbf{h}_{1,n}^T\ \mathbf{h}_{2,n}^T \cdots \mathbf{h}_{K,n}^T\right]^T \tag{7}$$

$$\mathbf{W}_n = \left[\mathbf{w}_{1,n}\ \mathbf{w}_{2,n} \cdots \mathbf{w}_{K,n}\right]. \tag{8}$$

In Zero-Forcing Beamforming (ZF-BF) the precoder is designed to achieve the zero interference condition between the users,

$$\text{i.e.,} \begin{cases} \mathbf{h}_{k,n}\mathbf{w}_{j,n} = 0, \text{ for } j \neq k \\ \mathbf{h}_{k,n}\mathbf{w}_{j,n} = 1, \text{ for } j = k \end{cases}. \quad (9)$$

The ZF-BF precoding matrix for each subcarrier n is given by the pseudo-inverse of the channel gain matrix $\mathbf{H}_n$ [7], that is,

$$\mathbf{W}_n = \mathbf{H}_n^H \left(\mathbf{H}_n \mathbf{H}_n^H\right)^{-1}, \quad (10)$$

where the precoding vector $\mathbf{w}_{k,n}$ for each user k on subcarrier n is obtained by normalizing the k th column of $\mathbf{W}_n$. The achievable sum-rate capacity using ZF-BF over all subcarriers is expressed as,

$$R_{\text{ZF-BF}} = \frac{1}{N_c} \sum_{n=1}^{N_c} \sum_{k=1}^{K} \log_2 \left(1 + \frac{P_{k,n}}{\sigma_\eta^2} \gamma_{k,n} \right), \quad (11)$$

where $P_{k,n}$ denotes the power allocated to user k on subcarrier n. The optimal power allocation that achieves the maximum sum-rate capacity is given by the waterfilling algorithm [9].

3.2 Minimum Mean Squared Error Beamforming

The ZF-BF precoder completely eliminates multi-user interference at the expense of noise enhancement and thus the system can be treated as a group of parallel SU-MIMO communications at each subcarrier n. However, if some of the channels are in bad condition the system needs large power to compensate the bad channel condition. The minimum mean-square-error beamforming (MMSE-BF) precoder can reach a good tradeoff between noise and interference and is suitable to be used to overcome this problem. In presence of CSI at the transmitter, the MMSE-ZF precoder at each subcarrier n is given as follows:

$$\mathbf{W}_n = \mathbf{H}_n^H \left(\mathbf{H}_n \mathbf{H}_n^H + \beta \mathbf{I}\right)^{-1}, \quad (12)$$

where β is a regularization factor commonly chosen as $\beta = M\sigma_\eta^2/P_T$ motivated by the results in [10] showing that, for single carrier systems, the performance of MMSE-BF is certainly significantly better at low-medium SNR and converges to that of ZF-BF at high SNR. However, MMSE-BF does not provide parallel channels and thus power allocation techniques cannot be performed in a straightforward manner.

The achievable sum-rate capacity using MMSE-BF over all subcarriers with equal power allocation is given by

$$R_{\text{MMSE-BF}} = \frac{1}{N_c} \sum_{n=1}^{N_c} \sum_{k=1}^{K} \log_2 \left(1 + \text{SINR}_{k,n}\right), \quad (13)$$

where $\text{SINR}_{k,n}$ represents the signal to interference plus noise ratio of user k on subcarrier n, and can be expressed as

$$\text{SINR}_{k,n} = \frac{\left|\mathbf{h}_{k,n}\mathbf{w}_{k,n}\right|^2}{\sum_{j=1, j\neq k}^{K} \left|\mathbf{h}_{k,n}\mathbf{w}_{j,n}\right|^2 + K\sigma_\eta^2/P_T}. \quad (14)$$

3.3 Time Division Multiple Access

In a conventional MU-MIMO-OFDM network where user multiplexing takes place using TDMA techniques, the base station selects the best user at a time who will be allocated all the spectrum and power resources [11]. In this scenario it is easy to show that, once a user has been selected, the precoding operation on each subcarrier n is simply implemented by means of maximum ratio transmission (MRT) [12], that is,

$$\mathbf{x}_n = \sqrt{P_{k,n}}\mathbf{w}_{k,n}s_{k,n}, \quad (15)$$

where $\mathbf{w}_{k,n} = \mathbf{h}_{k,n}^H$. The maximum sum-rate capacity achieved by sending to the user with the largest channel norm is

$$R_{\text{TDMA}} = \max_{k\in\{1,\cdots,K\}} \frac{1}{N_c}\sum_{n=1}^{N_c}\log_2\left(1+\frac{P_T}{\sigma_\eta^2}\left\|\mathbf{h}_{k,n}\right\|^2\right). \quad (16)$$

4 NUMERICAL RESULTS

In this section, we evaluate the performance of ZF-BF and MMSE-BF linear precoding schemes in MU-MIMO-OFDM scenarios, assuming the availability of CSI at transmitter, and compare them to conventional MU-MMO-OFDM based TDMA system. The simulations consider the use of parameters currently found in the latest WLAN standard IEEE 802.11n. The system has been configured to operate at 5.25GHz carrier frequency on a bandwidth of W=20MHz with N_c=64 OFDM subcarriers, where the subchannel gains are independent and identically distributed for each user. The channel profile used to generate the frequency-selective channel responses correspond to profiles B (residential) from channel models developed within the IEEE 802.11n standard [13]. The base station is assumed to communicate with a total of $K = M$ mobile users, each equipped with a single receive antenna.

Fig. 2 shows a performance comparison in terms of sum-rate capacity as a function of the average SNR for MMSE-BF, ZF-BF and the conventional TDMA network in a system with $M=4$ transmit antennas and $K = M$ mobile users. As expected, it can be seen that the performance of MMSE-BF is certainly significantly better at low-medium SNR regime and converges to that of ZF-BF at high SNR regime. Moreover, the gain in sum-rate capacity gap between both linear precoding schemes and the conventional TDMA system exhibits a linear increase with SNR values reaching more that 10 bits/s/Hz for an SNR=30 dB.

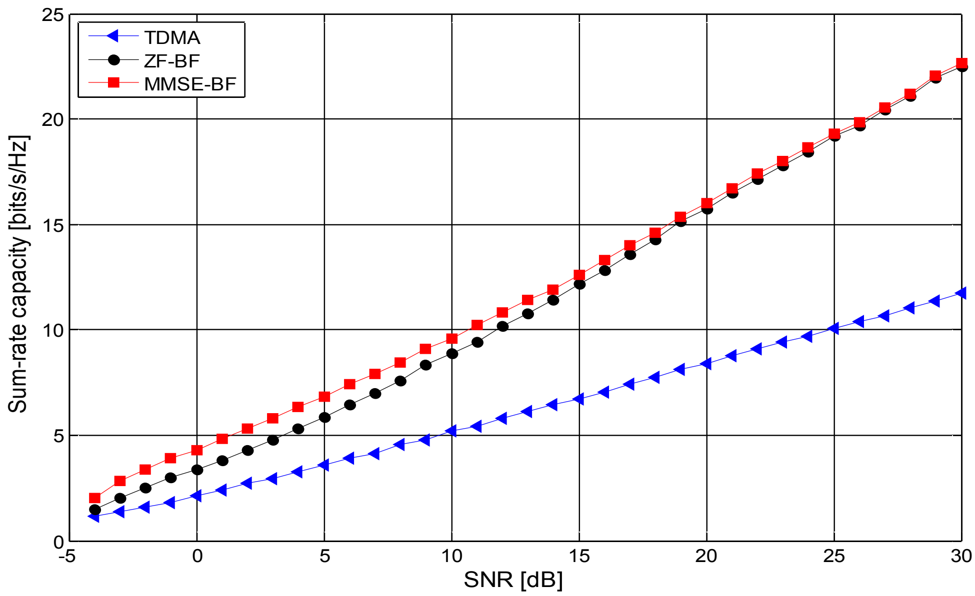

Fig. 2. Sum-rate capacity versus the average SNR. K = M = 4

In Fig. 3 we compare the sum-rate capacity versus the number of transmit antennas M for $K=M$ mobile terminals and an average SNR=10 dB. It can be observed that, increasing the number of transmit antennas M, i.e., increasing the number of simultaneously transmitting users K, has the detrimental effect of providing a linear sum-rate capacity growth for both linear precoding schemes, where the sum-rate capacity of MMSE-BF outperforms that of the ZF-BF. However, for the conventional case, where the best user is scheduled, the sum-rate capacity saturates at 7.7 bits/s/Hz. This can be explained by the fact that adding more transmit antennas does not improve the performance of the TDMA technique.

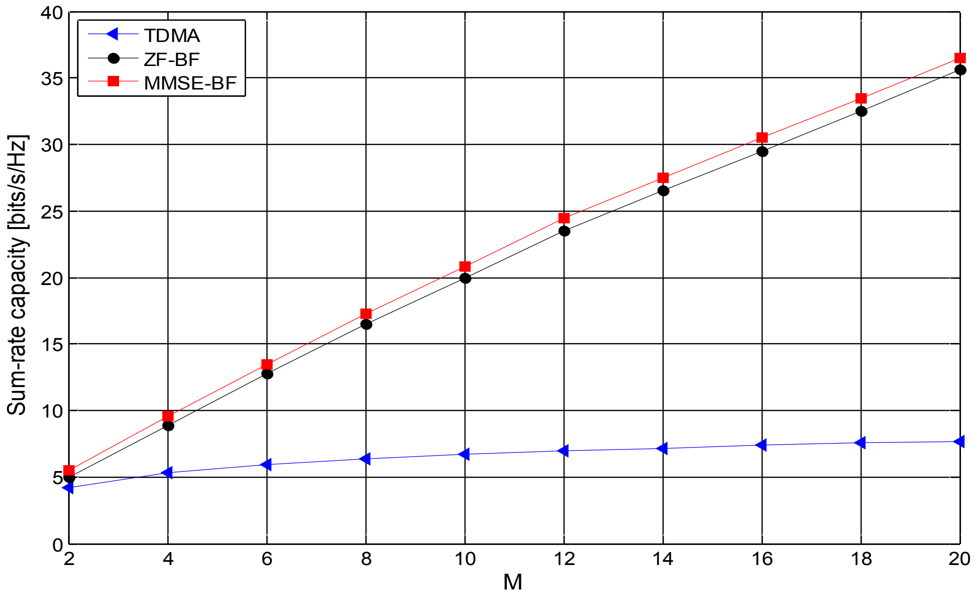

Fig. 3. Sum-rate capacity versus the number of transmit antennas M. K = M users, SNR=10 dB

5 Conclusion

In this paper, zeroforcing beamforming (ZF-BF) and minimum mean square error (MMSE-BF) linear precoding schemes have been analyzed and extended, from single-carrier MU-MIMO to a multicarrier MU-MIMO architecture based on OFDM. Simulation results have shown that, when channel state information (CSI) is available at the transmitter and for practical average SNR values, the performance of both proposed linear precoding techniques is significantly better than the conventional TDMA network and achieve a linear sum-rate growth with the number of transmit antennas. In addition, for a fixed number of transmit antennas at the base station, the performance of MMSE-BF is significantly better at low-medium SNR regime and converges to that of ZF-BF at high SNR regime.

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References

[1] E.Telatar, "Capacity of multi-antenna Gaussian channels*," Eur. Trans.Telecommun*., vol. 10, no. 6, pp. 585–595, Nov. 1999.
[2] R. van Nee and R. Prasad, *OFDM for wireless multimedia communications*. Artech House, 2000.
[3] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications a key to gigabit wireless," *Proceedings of the IEEE,* vol. 92, no. 2, pp. 198–218, 2004.
[4] D. Gesbert, M. Kountouris, R. Heath, C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm," *IEEE Sig. Proces. Mag*., vol. 24, no. 5, Sep. 2007.
[5] M. Costa, "Writing on dirty paper," *IEEE Trans. Info. Theory*, vol. 29,pp. 439–441, May. 1983.
[6] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inform.Theory*, vol. 49, no. 7, pp. 1691–1706, Jul. 2003.
[7] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 528–541, Mar. 2006.
[8] M. Joham, W. Utschick, and J. A. Utschick, "Linear transmit processing in MIMO communications systems," *IEEE Sig. Proces. Mag.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.
[9] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Sum power iterative water-filling for multi-antenna gaussian broadcast channels," *IEEE Trans. on Information Theory*, vol. 51, no. 4, pp. 1570–1580, April 2005.

[10] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.
[11] D. Tse and P. Viswanath, Fundamentals of wireless communications. *Cambridge University Press*, 2005.
[12] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. on Inf. Theory*, vol. 48, no. 6, Jun. 2002.
[13] J. Kermoal, L. Schumacher, K. Pedersen, P. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE JSAC*, vol. 20, no. 6, pp. 1211–1226, Aug 2002.